How does a vernier work?
And what is the difference between a Type A and Type B vernier?

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There are a number of good descriptions of the vernier in books on sextants and scientific instruments, but it can nevertheless be a bit tricky to see how it works. Here I will go into considerable detail, but will try to avoid being too technical or entirely tedious.

Let's imagine that we have a scale with marks at each unit. And let's imagine that we have a put a point on the scale that doesn't fall exactly on any of the marks. In fact, it falls 7/20 (that is seven twentieths) of the way from one mark toward the next. It doesn't matter for the purposes of the example which two marks it lies between, so we'll say that the point lies 7/20 of the way from the 0 mark toward the 1 mark.

Suppose that A is trying to find out where the point lies on the scale, and we have not told him that it lies at the 7/20 mark. He can't be sure, just by looking at it, whether it really lies at the 7/20 mark, the 3/10 mark, or the 4/10 mark. Here is a trick he can use. A starts at the point and lays off a new scale, with marks at intervals of 1 + 1/20. So the first mark of the A scale—which we'll call 0A—is at 7/20 (only A doesn't know that yet). The second mark, 1A, is at 7/20 + 1 + 1/20 = 1 + 8/20. (It works better for our purposes if we don't reduce our fractions.) The third mark on A's new scale, 2A, is at 7/20 + 2 + 2/20 = 2 + 9/20. We can see where this is going now: 3A is going to fall at 3 + 10/20, 4A at 4 + 11/20, 5A at 5 + 12/20, 6A at 6 + 13/20 —and so on until we get to 13A at 13 + 20/20 = 14. That is to say, the mark 13A on A's new scale lines up exactly with the 14 mark on the original scale.

Now suppose that A had numbered his scale not from 0A up, but from 20A down. That is, he labels his A marks as follows: 20A:0, 19A:1, 18A:2, 17A:3, 16A:4, 15A:5, 14A:6, 13A:7, and so on to 0A:20. But we just found that 13A falls at position 14 on the original scale. That is, the mark labeled 7 on A's scale will align perfectly with the 14 mark on the original scale. Knowing the trick, A has only to look at his scale, note that it is his 7 mark that aligns with one of the original marks, and tell that the point must lie at position 7/20.

A little reflection will show that this works every time, no matter where the point may lie. If it falls at $i/20$, then the marks will line up at mark i on A's scale. It's not difficult to write this out as a simple algebraic equation. If the chosen point lies at $x + i/n$, then if we lay off an "A scale" of n marks from the point spaced at intervals of $1 + 1/n$ we will find that the mark at $n - i$ on the A scale will align exactly at $x + i + 1$.

Suppose that A does not want to be reading his scale as many as 19 divisions away from the point. This presents no problem. Instead of laying off the A scale from the point n steps in one direction, he can lay it off $n/2$ steps in each direction. In effect, he has simply folded the A scale back on itself.

A folded A scale like this is a Type A vernier. It's the sort of vernier that was used from fairly early in the 18th century until around 1780, when people decided that it was not as clear and easy as it could be.

In a B scale, one starts at the point and lays off marks at intervals of $1 - 1/n$ (instead of $1 + 1/n$). If we do the arithmetic (or algebra) over again for this, we find that now it is the 7B mark, rather than the 13A mark, that lines up. So B can number his scale up from

0B instead of down from 20A. It was also decided that folding was not a great idea, because it could be confusing, and that was dropped. So a B scale of the type described is a Type B vernier, which became universal after the latter part of the 18th century.

The reason that I've used verniers of 20 divisions in my examples is that it was all but universal in the vernier era to mark sextant limbs off in degrees and thirds of a degree. Since there are 60 arc minutes in a degree of arc, a third of a degree has 20 arc minutes. So with a vernier scale of 20 divisions one can read directly to one arc minute.

Suppose that the point doesn't lie exactly at some integral multiple of 1/20 in the space between marks. Suppose, let us say, that it lies at 173/200. A little more math will show that in this case there will be no exact lineup between the vernier and limb scale divisions, but that the alignment will be closest at the 17 mark on the vernier, and next closest at the 18 mark. With high enough magnification and some experience, the navigator should be able to be sure at least that the correct reading is somewhere between 17.2 and 17.4 arc minutes. Of course the scales must be divided extremely accurately for this. The actual accuracy that the navigator can achieve in scale reading depends on a number of factors, and is usually poorer than he thinks it is, or that the instrument manufacturer has led him to expect.

All in all, the vernier is a very clever and useful mathematical trick. Pierre Vernier (1580–1637) deserves to be remembered with respect. It would be interesting to know what led him to the discovery.