The true celestial line of position is the **circle of equal altitude** or circle of position: CoP
Any point (B,L) of the CoP satisfied the equation:

sin H = cos B cos Dec cos (GHA+L) + sin B sin Dec

where
For the observer:
- B - Latitude (-S/+N)
- L - Longitude (-W/+E)

For the observed celestial body:
- GHA - Greenwich Hour Angle
- Dec – Declination (-S/+N)
- H – Altitude of the body from the horizon

LHA = GHA + L

For this formulation the intervals are:
-90 ≤ Dec ≤ +90º
0 ≤ GHA ≤ 360º (W to E)
0 ≤ H ≤ 90º
-90 ≤ B ≤ +90º
-180 ≤ L ≤ +180º

The straight lines of position in celestial navigation, (usually called Line of Position: LoP), are approximations to the CoP in the surrounding close to the true position of the observer.
The fix is obtained by the intersection of the LoPs as an approximation of the intersection of the CoPs.

The *determinant* of a Lop is the data necessary for plotting it on a Mercator nautical chart.

There are two kinds LoPs:
1. the secants to the CoP.
2. the tangents to the CoP.

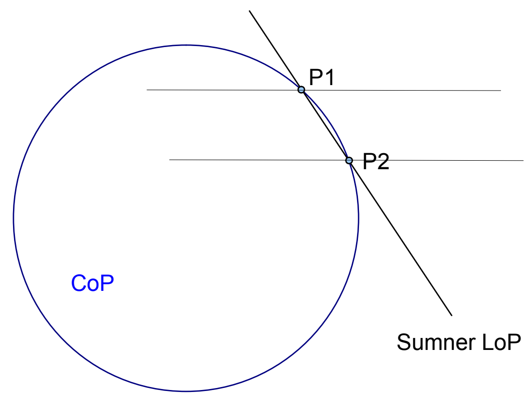
Secant lines have two points in common with the CoP, and tangent lines have only one point.

Secants: the determinant is constituted by two points of the CoP near the estimated position. There are two types:
1. **Sumner** Line or secant by cut with the parallels
2. Secant LoP by cut with the meridians

Tangents: The arc of the CoP near the observer's position is substituted by a rhumb-line tangent to the CoP. Such a line is normal to the azimuth.
There are tree types:
1. Jhomson tangent
2. Borda Tangent
3. **Marcq de Saint-Hilaire** Line

In fact the Latitude at meridian passage is a particular case of the Borda tangent
- Sup: LHA = 0º
- Inf: LHA = 180º

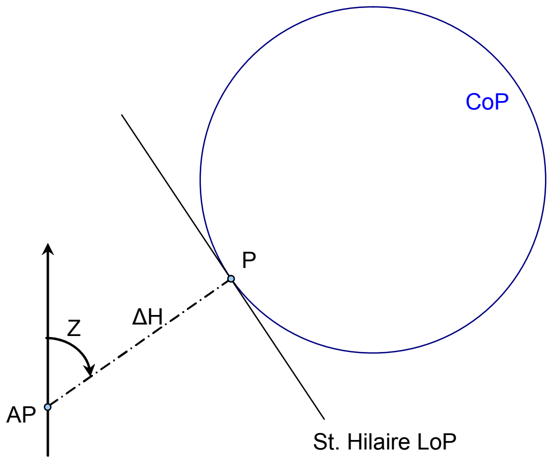
Sumner Line



The method is as follow:
Nautical Almanac (date, UT1, Astro) → (GHA, Dec)
Be → (B1, B2) B2-B1 < 1º
B1 = Be - 5/60
B2 = Be + 5/60
(Ho, B1, Dec) → LHA1
(Ho, B2, Dec) → LHA2
cos LHA = (sin Ho - sin Dec sin B)/(cos Dec cos B)
if(Z < 180º) LHA = 360 – LHA
(GHA, LHA1) → L1
(GHA, LHA2) → L2
L = LHA – GHA

The plot:
P1(B1, L1)
P2(B2, L2)
LoP = P1P2

Marq St. Hilaire - Altitude intercept method



The Marq St. Hilaire – Altitude intercept method in Celestial Navigation, use the tangent line to the circle of equal altitude close to the estimated position of the observer.
The method reduces the problem to that of the intersection of LoPs in a plane in order to obtain the fix.

- It is necessary to know the estimated position (DR). Other position near the DR can be used with no appreciable error
- This method is approximate. The only point in common with the CoP is the tangent one, defined by the intercept (p, Z)

The coordinates of such a point, (assuming p small enough), common to the Cop and the LoP are:

p = (HO-HC)
x = p*SIN(Z)
y = p*COS(Z)

B = Be + y
Bm = (Be + B)/2.0
L = Le + x/COS(Bm)

Any other point in the LoP differs to the matching one in the CoP by the following amount:

Bowditch Table 19 Offsets

- h Altitude [º]
- D Distance along the LoP from intercept [nm]

R = (60*180/PI) / TAN(H)
theta = ASIN(D/R)
Offset = R*(1.0-COS(theta))

The intersection of two LoPs, the fix in St. Hilaire method, is a point that not belongs to the Cops, but is close to the true fix, (defined by the intersection of the CoPs).
Now take the St. Hilaire fix and use it as the new estimated position in order to obtain another fix by this method, (doing this in a paper could help to understanding), the result is a point closest to the true fix.
Usually in the practice only one plot on a Mercator chart is used.

The method:
The determinant is constituted by DR or AP position, the difference in altitude and the azimuth.

LHA = GHA + Le

sin Hc = sin Be sin Dec + cos Be cos Dec cos LHA

cos Z = (sin Dec - sin Hc sin Be)/(cos Hc cos Be)
if(0 < LHA < 180º) Z = 360 – Z

The plot:

RA ⊥ Z

p = 60(Ho-Hc) [nm]

- P = +, Ho > Hc: Towards
- P = -, Ho < Hc: Away

If p is positive the position line is drawn along the azimuth.

If p is negative, the position line is *away* from the assumed position by adding 180° to the azimuth.

20071026
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