LUNAR DISTANCES EXPLAINED
ALONG WITH
A STARTER KIT FOR THE SHORE BOUND NAVIGATOR

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On the forty-third day from land, a long time to be at sea alone, the sky being beautifully clear and the moon being "in distance" with the sun, I threw up my sextant for sights. I found from the result of three observations, after long wrestling with lunar tables, that her longitude by observation agreed within five miles of that by dead reckoning…. I sailed on with self-reliance unshaken, and with my tin clock fast asleep…. The work of the lunarian, though seldom practiced in these days of chronometers, is beautifully edifying, and there is nothing in the realm of navigation that lifts one's heart up more in adoration.
- Joshua Slocum, <u>Sailing Alone Around the World</u>, 1896

From the mid eighteenth century until almost the end of the nineteenth century, measuring the "lunar distance" played a critical role in determining time and its all important derivative, longitude. Even as chronometers became more affordable and widespread, lunars provided a reliable alternative and a rough check on the navigator's timepiece. For many (like Slocum) it remained the preferred method on its own merits. Today, while celestial navigation endures as a wonderful hobby and a back up to electronic navigation, few navigators are familiar with the procedures for determining Greenwich Mean Time (GMT) from a lunar distance.

This no longer need be the case. Modern tables are available that guide the navigator through all the necessary calculations without need for any special knowledge of math or astronomy. This article offers a spreadsheet that allows anyone with a sextant, a watch and a PC to determine GMT via lunar distance from the comfort of their backyard. Anyone longing to deepen their celestial skills will find that lunars are a wonderful way to extend their mastery of the arts of the navigator. Specifically:

- With latitude and longitude for your backyard, you can measure, correct and convert lunars to time without a horizon, making lunars a great way to stay on top of your skills in the off season.
- Observing lunar distances hones the skills of hand and eye as the measurement is 30 times more sensitive to error than a conventional sight. Anyone can make the observation, but even an experienced navigator will find lunars a challenge to observe with precision.
- Understanding *in principle* the trigonometry that allows a lunar distance to be cleared and converted to time deepens one's understanding of the methods that underlie conventional celestial sights, so this article presents them graphically.
- Understanding how lunar distances are corrected sharpens one's appreciation of the various corrections to a sextant observation by applying them in a very different context.

THEORETICAL BASICS

A lunar distance is the angular distance as it appears from earth between the moon and a comparing body (sun, star or planet). The utility of the distance derives from the fact that the moon moves steadily across the dome of the stars at a much faster rate than the sun or planets. In the course of a month, it moves 360° around the backdrop of the sky, 30 arc-minutes or one half a degree per hour on average. Using a Nautical Almanac and some spherical trigonometry, the distance between the moon and a comparing body can be accurately predicted for any moment in time (a triumphant achievement in the 1760's after generations of careful observation). When measured with a sextant, the distance must be "cleared" (corrected for the circumstances of the observer) in order to produce an observed distance that can be compared to the prediction of the Almanac. Thus we can predict the distance to convenient bodies at convenient times, and by comparing these to our observed distance, determine where the moon lies on the "clock of heaven".

SEXTANT CORRECTIONS AND THE NAVIGATIONAL TRIANGLE

Clearing the lunar distance is best understood by comparison to the more familiar process of reducing a conventional altitude sight. In both cases, we **measure** with the sextant, **correct** our measurement for various factors, **calculate** an expected measurement based on assumptions, and then **compare** the measured value to the calculated value to improve on our assumptions. Walking through these steps for reducing a conventional moon altitude sight will help establish the concepts needed to understand clearing a lunar distance. In the discussion that follows, we will use slightly different definitions and abbreviations than those used in most publications, but we will need them to explain the unique aspects of clearing a lunar.
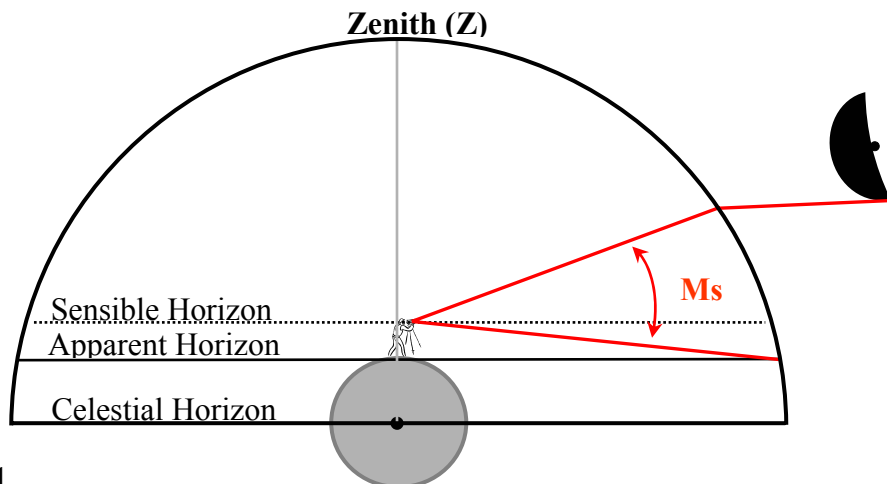
Measure

Figure 1

To reduce moon sight, we use the sextant to bring the moon's limb (in this example the lower limb) to the apparent horizon to measure the **moon's sextant altitude (Ms)** (see Figure. 1). We make our corrections as follows:

Correct:

	Ms	Sextant Altitude
+/-	IC	Index and instrument correction, to correct for known inaccuracies in the sextant
-	Dip	To correct for height of eye above the horizon
+	SD	Semi-diameter, so we measure the center of the body, not the lower limb
=	**Ma**	**Apparent Altitude**

In this discussion, Apparent Altitude means altitude after all corrections EXCEPT for parallax and refraction. This will be crucial to properly correcting our lunar distance. With this definition established, we can proceed

	Ma	Apparent Altitude
-	R	Refraction, to correct for how light bends as it enters the atmosphere
+	P	Parallax
=	**Mo**	**Observed Altitude**, a full corrected observation (see fig. 2)

Parallax is the difference in the moon's position against the backdrop of stars when viewed from two different places. We observe the moon from the earth's surface while the almanac predicts its position as if observed from the earth's center. The parallax correction accounts for the difference. (hold out your finger and alternate between closing your right and left eyes to demonstrate the effect). Parallax is zero for the stars and negligible for the sun and planets because they are almost infinitely distant, but it is critical to measuring the moon's position properly.
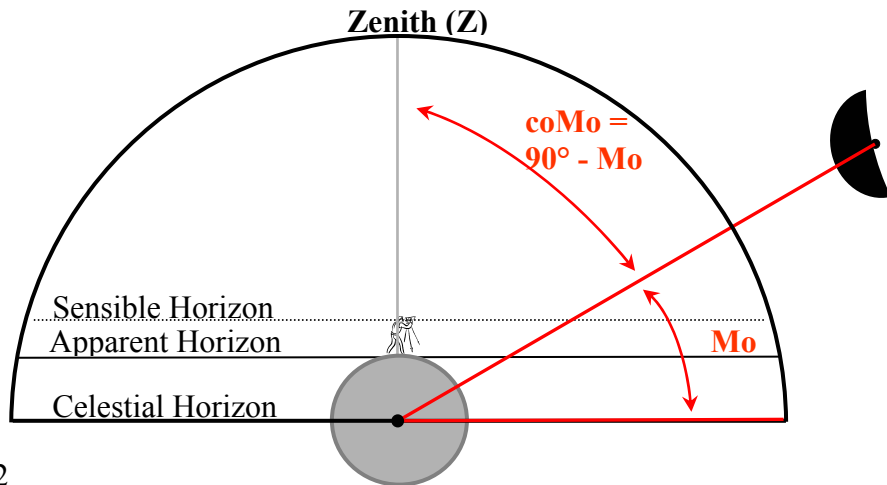

Figure 2

Note that Figure 2 illustrates that the angle from the body to the **observer's zenith (Z)** (directly above the observer) is always **90° - the altitude**. This is the complement or co-altitude (**coMo** in this case). It will be the value that we compute directly in the examples that follow, always knowing that we can covert quickly back to the altitude by subtracting it from 90°. Complements of declination and latitude will be used in the same fashion.

Calculate

Having fully corrected our observed altitude, we are ready to **calculate an altitude (Mc)** using the **latitude (lat)** and longitude of our **assumed position (AP)** along with the **Greenwich Hour Angle (GHA)** and **declination (Dec)** of the body at the time of our sight. Whether we use tables or a calculator to do this, we are solving a spherical triangle exactly as we will be doing later with the lunar distance. For this discussion, we will examine the solution in graphic form using the navigational triangle. Our perspective will be that of an observer above the dome of the stars looking down on the universe with the earth far below at the center of the dome.
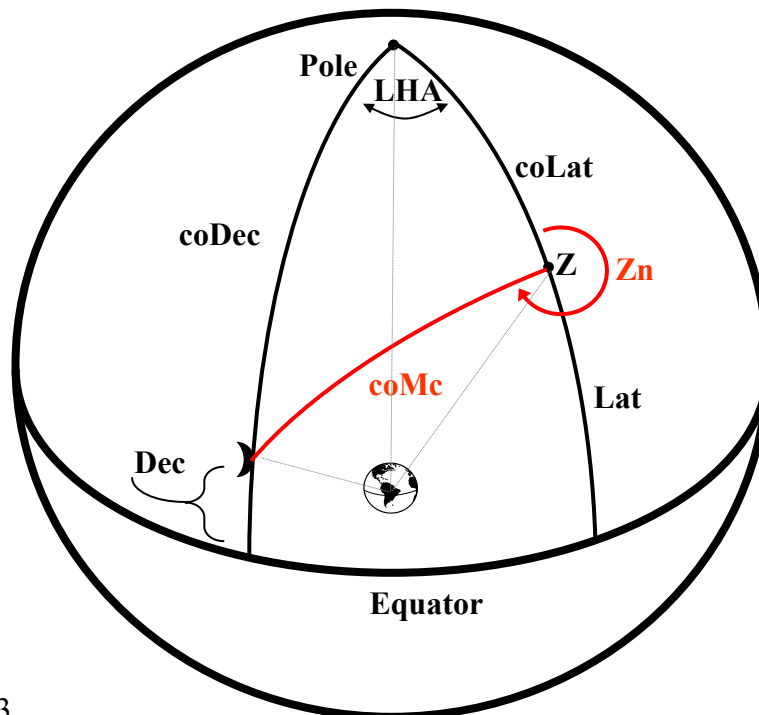

Figure 3

Figure 3 shows the basic navigational triangle on the celestial sphere. That point on the sphere directly above our AP is our zenith (Z). The celestial pole and equator correspond to the pole and equator of the earth. The position of the moon on the sphere at the time of the sight is indicated by the crescent. Trigonometry allows us to calculate any part of the triangle if we know the value of three other parts (see sidebar).

In this case we know:
coLat = 90° - latitude (the complement of latitude of our AP)
coDec = 90° - declination (the complement of declination from the Almanac)
LHA   = GHA - longitude (GHA from the Almanac, longitude of our AP).

Therefore, we can easily calculate:
coMc = the complement of the calculated altitude (from the moon to Z, per Figure 2)
Mc   = 90° - coMc = the **Calculated Altitude** of the moon viewed from our AP
Zn   = **Azimuth** from our AP to the moon.

Compare

Now we can compare Mc and Mo. The difference between the two is the intercept (toward or away) that allows us to move a perpendicular line either toward or away from the moon along the azimuth. We have reduced the moon sight to a line of position.

APPLICATION TO THE LUNAR DISTANCE

To derive GMT from a lunar distance, we will use the same process of measure, correct, calculate and compare, and we will use the spherical triangle both to clear the observed distance and to calculate distances at known times so we can determine GMT.

Measure

For a traditional lunar, we measure the sextant altitude of the moon (Ms), the sextant altitude of the comparing body (Ss) (which in our example is a star), and the sextant distance between the star and the limb of the moon (Ds). We also note the watch time at the moment we measure the distance. In this example we are using the near limb of the moon just as we used the lower limb in the altitude sight. Figure 4 illustrates.
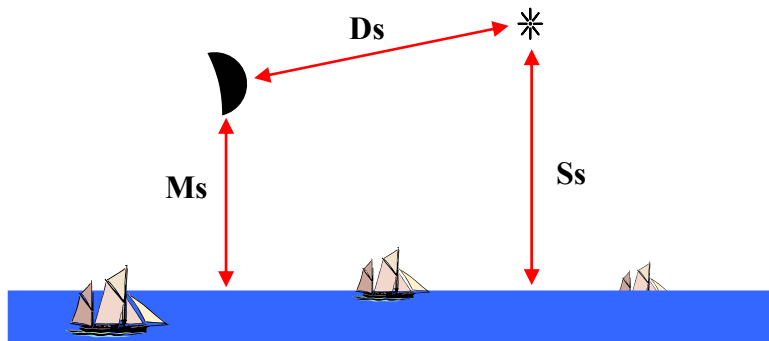

        Figure 4

Note that the altitudes (Ms and Ss) are measured along the observer's vertical plane, perpendicular to the horizon. The lunar distance (Ds) is normally measured along a diagonal plane, or even horizontal depending on the relative positions of the bodies. This makes for some awkward sextant handling, and it complicates the process of correcting the distance.

Correct

Our first step is to make corrections that are applied directly to Ds. Index correction is always applied directly to whatever angles the sextant measures. Semi-diameter also applies directly to Ds since we want to measure the distance from the center of the body, not the outer rim. Dip is not relevant, as we are not using the horizon as a reference, thus:

	Ds	Sextant Distance
+-	IC	Index Correction
+	SD	Semi-diameter (added when using the near limb, subtracted if the far limb)
=	**Da**	**Apparent Distance** (all corrections except refraction and parallax).

Parallax and refraction are a different matter. Both effect the apparent position of the body along the vertical plane of the observer. They can be applied directly to correct an altitude, but they cannot be applied directly against Ds which is measured on a diagonal. Looking at Figure 4, the problem is to determine how much the distance changes if we increase the altitude of the moon and decrease the altitude of the stars (the net effect of parallax and refraction in each case).

A variation on the navigational triangle allows us find an answer. This time, the peak of the sphere is our zenith (Z) rather than the pole, and the great circle below it is the horizon rather than the equator. The trigonometry is unchanged. If we correct the distance and both altitudes for all but parallax and refraction, we have the following triangle:
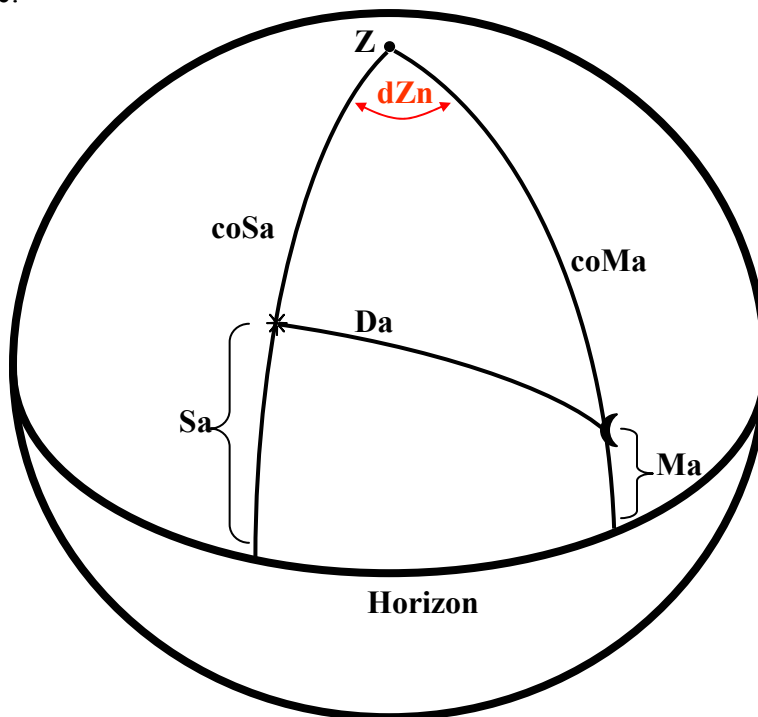

Figure 5

In this case we know:
coSa   = 90° - Sa (complement of the apparent altitude of the star)
coMa = 90° - Ma (complement of the apparent altitude of the moon)
Da   = Apparent distance

Therefore, we can easily calculate:
dZn   = the **Difference in Azimuths** from the observer to each body.

Now we can correct Sa and Ma for parallax and refraction and get So and Mo. This simply moves the position of the bodies along the vertical lines from the horizon to the observer's zenith as shown in Figure 6:
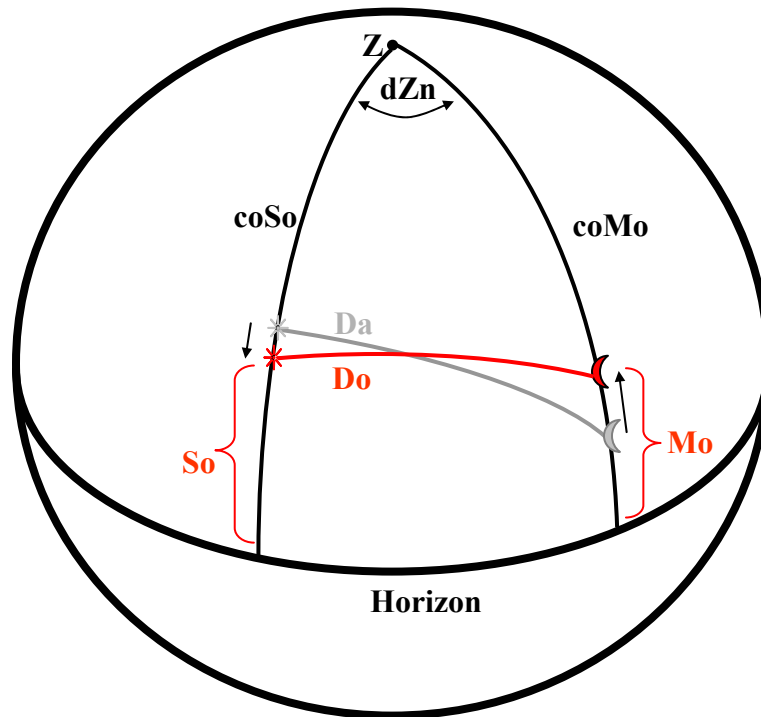

Figure 6

The adjustment of the altitudes for parallax and refraction does not change the angle dZn (look again at Figure 4 to see that moving a body up or down the vertical does not change its bearing from the observer).

Therefore, we now know:
dZn   = the same value we calculated previously (the azimuths have not changed)
coSo   = the complement of So (which is Sa corrected for parallax and refraction)
coMo   = the complement of Mo (which is Ma corrected for parallax and refraction)

Now we can easily calculate:
Do   = the **Observed Distance**, fully corrected, including parallax and refraction.

With this calculation, we have cleared the lunar distance and have an observed value that is ready to be compared to a set of calculated expectations.

Calculate

Calculation of the distance that we should observe at any point in time is another straightforward application of the navigational triangle. This time, we again use the pole and the equator as reference points, and the GHA and declination of both the moon and our star to determine the known parts of the triangle shown in Figure 7.
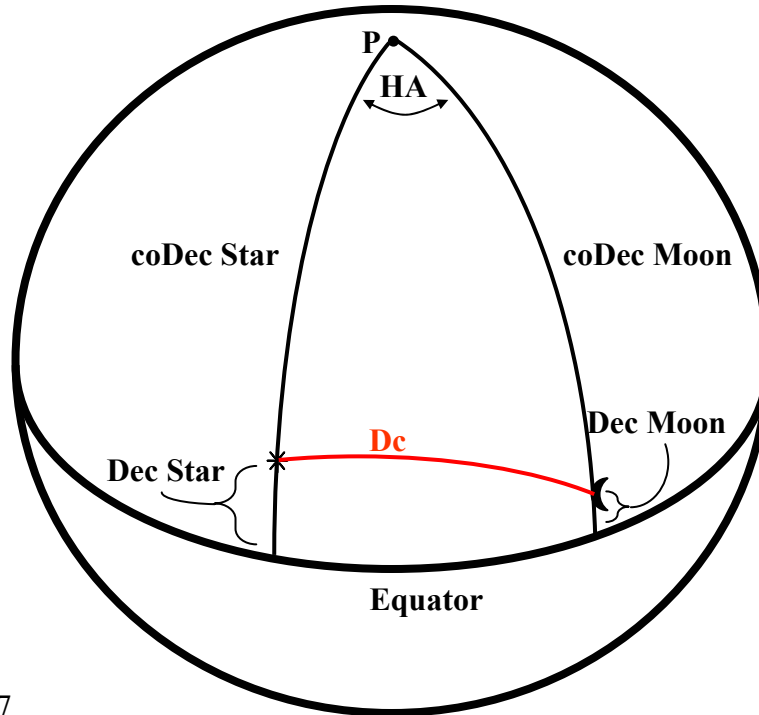

Figure 7

In this case, we know:
HA   = the hour angle between the bodies (GHA Star – GHA Moon)
coDec Star   = the complement of the star's declination (90° - Dec Star)
coDec Moon   = the complement of the moon's declination (90° - Dec Moon)

Now we can easily calculate:
Dc   = the Calculated Distance.

Compare

Calculated distances can readily be obtained for both the hour before and the hour after the approximate time of the observed distance. The lunar distance of a well selected comparing body will change at a constant rate over a period up to three hours. This allows us to interpolate between hourly values to derive GMT at the moment of our observed distance.

For example:
Dc at 02:00 GMT   = 45° 0'
Dc at 03:00 GMT   = 45° 30'

If Do   = 45° 10', we know our observation was made one third of the way through the hour, so:
Time of Observation = 02:20 GMT.

We now have a GMT that fits the observed distance to compare to the watch time we recorded as we made the observation.

TRYING IT OUT AT HOME

If you have made it this far through the theory, it is time to have some fun putting it into practice. Provided below is a step-by-step procedure for taking your first lunar from the backyard (or any convenient point of observation) and finding out immediately how well you did.

For a backyard lunar, we will make some changes that simplify the sextant work. Most people's backyards do not include a clear sea horizon against which to take the altitudes we need to clear the lunar distance (remember Figure 4). But if we have the latitude and longitude of the point of observation and the GMT of the lunar, we can calculate the altitudes of the moon and comparing body, which is just as good for clearing the distance (see Figure 3).

Since the late 1700's, there have been techniques by which position and GMT could be estimated in a manner which allowed the navigator to calculate the altitudes needed to clear the lunar distance. This is a powerful advantage when the horizon is obscured by darkness or fog, or for land based navigators (see sidebar). Calculating altitudes based on estimated GMT and position are outside the scope of this article. In this practice exercise, we simplify the process by using modern means to determine GMT and position in advance. This provides the opportunity to compare our GMT per lunar to an actual value and get instant feedback on how accurately we observed the distance.

So for this exercise all you need is a sextant, your position, a watch set to GMT, some Almanac data for the period of observation, and a convenient way to make all the calculations. Most handheld GPS units provide accurate latitude and longitude along with precise Universal Time (close enough to GMT for us). If you don't have one, find a good map or chart, or go to www.topozone.com for U.S. locations, and pull off the coordinates for your spot. Exact time is available online at www.time.gov. If you do not have a current Nautical Almanac, you can get the data you need at www.tecepe.com.br/nav/.

A spreadsheet developed by the author is available at www.LunarDistance.com which takes care of the math, including calculating the altitudes when latitude and longitude are known. There is a version that will run on Microsoft Works Spreadsheet, which comes

standard on many new PC's, and anyone who runs Lotus or Excel will find a compatible version available. The spreadsheet allows you to record all the information collected in the instructions that follow. It then calculates GMT and compares it to the watch time you used for the observation. An elegant manual alternative to the spreadsheet is Bruce Stark's book *Tables for Clearing the Lunar Distance and Finding G.M.T. by Sextant Observation* (information on where to find it is in a sidebar).

With tools now in hand, we need to select a comparing body. For a beginner, it is best to start with a distance in the 15° to 60° range. Longer distances are more difficult to observer, shorter ones prone to certain errors in interpolating for GMT. The sun is ideal a day or two before and after the quarter moon. Alternatively, the night moon allows a selection of various stars. Choose one that lies along the path of the moon's orbit which can be judged by extending a line perpendicular to the line drawn between the horns of the moon. Alternatively, choose a star that lies along the path of the ecliptic shown on pp. 266-267 of the Almanac. Choosing a well aligned start ensures that the change in distance during the period of observation will be relatively constant, making our lunar clock more accurate (the shorter the distance, the more important this becomes).

Be prepared to take a series of 4 or more sights. Note that for the sun, you will be measuring from the limb of the sun to the NEAR limb of the moon. For stars, you may be measuring to either the NEAR or FAR limb of the moon. Make note of which limb you are using, as it will determine whether to add or subtract the semi-diameter(s) when we correct the distance. You will also need to set your watch to GMT (or determine the correction) and record the GMT of each observation.

Consider a comfortable chair as a base for your observations, or find a way to steady your sextant with a tripod or a post. You will be holding your sextant horizontally or on the diagonal, so experiment with the most comfortable way to position your hands and body. You will need to bring one body to the other in the same way you would bring a body to the horizon for an altitude, so start with the index arm set to 0 and try bringing the brighter body to over to the dimmer (eg. sun to moon, or moon to star). You are not alone if you lose it once or twice along the way, and fumbling with filters does not help. Try taking the scope off the sextant if you are having trouble. When you get it close enough to have both bodies in the same field of view, take a break and relax before zeroing in for a precise observation.

Making the actual contact between the moon's limb and the comparing body is the moment of truth. Only experience brings expertise. A 1.0' error in an altitude gives a 1 nm error in the line of position. A 1.0' error in a lunar gives a 30' error in longitude or about 30 nm at the equator. It can be humbling in the early going. I take as many observations as I can in a 10 or 15-minute period in hopes that my inconsistencies will balance out.

With a set of observations in hand, you can average the distances and times, graph them and chose a point along the line of best fit, or chose the distance in which you have the greatest confidence because it just felt right.

However you choose your observation, we enter into the calculations with a measured distance (Ds), and for our practice exercise, a GMT for that distance. Whether you use the spreadsheet or Bruce Stark's tables, you will need to assemble:

- Latitude and Longitude of the observation
- Index Correction for the sextant
- For the whole GMT hour before and after the observation (e.g. 14:00 and 15:00), the following data from the main pages of the Nautical Almanac (or www.tecepe.com.br/nav/).
    - GHA of the Moon
    - Declination of the Moon
    - Horizontal Parallax of the Moon
    - GHA of the comparing body (GHA Aries if it is a star)
    - SHA of the comparing body if it is a star
    - Declination of the comparing body
    - Semi diameter of the comparing body if it is the sun

Note that collecting and inputting the Almanac data is the most time consuming part of using the spreadsheet. If you time all your observations to fall between the same hours, it is an easy matter to solve all your observations by simply entering each distance and time in sequence for a solution using the same set of Almanac data.

The spreadsheet guides you through the data input and calculates GMT per lunar and compares it to the watch time of the observation. Presuming your watch was set to GMT, any discrepancy is attributable to error in the observation, and this presumed error is calculated and presented as arc-minutes of distance.

If you take pleasure in the craft of celestial navigation, lunars open up a new realm that can be enjoyed day or night without leaving home. Additional resources are listed in the sidebar for those who wish to explore further. Like Slocum, throw up your sextant for sights and tap into a 240 year tradition.

Sidebar 1: Spherical Trigonometry

The Spherical Law of Cosines provides the formula by which all celestial navigation problems in this article are solved:

cos (c) = cos (a) * cos (b) + sin (a) * sin (b) * cos (C')

where a, b, and c are legs of the spherical triangle, and C' is the angle opposite c.

Restating the formula to solve the triangle in Figure 3, we get:

cos (coMc) = cos (coLat) * cos (coDec) + sin (coLat) * sin (coDec) * cos(LHA).

Restating to solve the triangle in Figure 5:

cos (Da) = cos (coSa) * cos (coMa) + sin (coSa) * sin (coMa) * cos(dZn)

or

cos (dZn) = (cos (Da) – cos (coSa) * cos (coMa)) / sin (coSa) * sin (coMa).

By simply substituting variables, all the other problems discussed can be solved.

Sidebar 2: Additional Resources

The author thanks members of the Navigation-L online discussion group for their expertise and guidance as he researched lunars. The archives of the group's technical discussions of this and other traditional navigation topics are at http://www.irbs.com/lists/.

You can find a more rigorous discussion on lunars in four postings to the Nav-L list by George Huxtable, who was particularly helpful in educating the author about lunars and their subtleties:
http://www.irbs.com/lists/navigation/0201/0177.html
http://www.irbs.com/lists/navigation/0201/0178.html
http://www.irbs.com/lists/navigation/0202/0149.html
http://www.irbs.com/lists/navigation/0301/0066.html

Bruce Stark's book *Tables for Clearing the Lunar Distance and Finding G.M.T. by Sextant Observation* allows you to make all the calculations for a lunar at sea or in the backyard, including clear work forms and instructions. It is available at http://www.celestaire.com/ under their section on sight reduction and is a must for anyone serious about lunars.

Development during the 18th century of the lunar distance method is well documented in Dava Sobel's popular book *Longitude*. While obviously strategic for naval powers of the day, land based explorers also adopted the technique. Lewis and Clark in the American west, and David Thompson in western Canada, all took lunar distances as they mapped the continent. Fascinating articles are available at the following links, both of which document the techniques by which estimated position and Greenwich Time can be used to calculate the altitudes needed to clear the distance.
http://www.aps-pub.com/proceedings/jun00/Preston.pdf (Lewis and Clark) and
http://www.northwestjournal.ca/dtnav.html (David Thompson).

Sidebar 3: Historical Context

In the late 18th and early 19th centuries, the line of position method for determining position from multiple celestial sights had not yet been developed. Therefore, the lunar distance was not used to correct the chronometer in preparation for evening stars. It was used in as part of a sequence of sights to solve directly for latitude and longitude.

Latitude was easily determined by a sun sight at local apparent noon (LAN). Reducing a LAN sight to latitude requires only the sun's declination which changes very slowly, so the date and a rough estimate of Greenwich time was sufficient for an accurate solution.

Longitude was found by first determining Local Apparent Time (LAT) using a morning or afternoon sun sight. Using the results of his LAN sight, the navigator with a sun sight knew three legs of a navigational triangle: co-altitude and co-declination of the sun, and co-latitude of his position. He could then solve for LHA, the local hour angle between the observer's meridian and the sun. This angle converted to time (at 1 hour per 15°) represented the time elapsed since LAN (when the sun was on the observer's meridian). This defined LAT at the observer's longitude.

Then Greenwich Apparent Time (GAT) was determined by lunar distance (or by using its competition, the chronometer). Almanacs of the day contained tables showing the predicted lunar distance for every three hour interval of GAT, allowing the navigator to determine the GAT of his lunar by interpolation. The difference in time between LAT and GAT was converted to longitude (at 15° per hour). At the end of a day of numerous observations and laborious calculations, the navigator had a position.