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HISTORICAL VALUES OF THE EARTH'S CLOCK ERROR ΔT AND THE CALCULATION OF ECLIPSES

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Introduction

Numerous observations of the Moon, Sun and planets are recorded in ancient and medieval history. These observations — which include many eclipses and lunar and planetary conjunctions — frequently attract the interest of historians of astronomy. If the positions of the Moon and Sun (and to a lesser extent the planets) in the historical past are to be computed with high precision, it is usually necessary to make satisfactory allowance for the effect of variations in the Earth's rate of rotation, or, equivalently, the length of the day (LOD).

Long-term variations in the LOD are mainly produced by lunar and solar tides, but other causes — such as the continuing rise of land that was glaciated during the last ice-age — are also significant. Although actual changes in the LOD amount to only a few hundredths of a second over several thousand years, the cumulative effect (known as ΔT) of these minute changes can be very large. For instance, the estimated value of ΔT at the epoch 1000 B.C. is as much as 7 hours. During this interval, the Moon can change position by nearly 4°.

It is therefore a matter of concern that at present there appears to be a degree of confusion and misapprehension among historians of astronomy over the choice of values of ΔT that should be used in making retrospective computations of lunar and solar positions. Accurate knowledge of the value of ΔT is often crucial in assessing the local circumstances of solar eclipses. Neglect of variations in the Earth's spin rate would materially affect the calculated positions of where these phenomena could be seen on the Earth's surface.

In this paper we shall try to elucidate the necessary procedures and — in particular — draw attention to several important points regarding ΔT in the calculation of solar eclipses. We shall place special emphasis on three specific issues: the adopted time-scale, the importance of tidal friction in the ephemeris of the Moon, and the enumeration of ΔT at various epochs in the past.

Time-scales

For most practical purposes today, the time system known as Universal Time (UT) — formerly known as GMT — is adopted; this is defined by the (variable) rotation of the Earth. However, UT is not suitable for computation of the positions of the Moon, Sun and planets using gravitational theories of their motion. Such theories do not allow for variations in the rate of rotation of the Earth on its axis. An ideally

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uniform time-scale (known as Terrestrial Time: TT) is implicit in these theories. By definition, ΔT is equal to the difference TT – UT. In defining TT, astronomers have adopted the standard length of day (LOD) as defined by the average rate of rotation of the Earth over the period from A.D. 1750 to 1892; the effective mean epoch is thus near A.D. 1820.[1] Unless adequate allowance is made for changes in the Earth's spin rate, the computed positions of the Moon and Sun (and also the planets) for some particular time in the past will be correspondingly in error.

Ephemerides and Tidal Friction

Both lunar and solar tides raised in the oceans and — to a lesser extent — in the solid body of the Earth gradually decrease the Earth's rate of spin, and thus increase the LOD. The reciprocal effect of the lunar tides produces a significant acceleration (actually negative) of the lunar motion. Because of the complexity of tidal friction, an accurate value for this lunar orbital acceleration ($\dot{n}$) can be derived only empirically. Although it materially affects all computations of the Moon's position in the past, the lunar acceleration is especially important in the computation of eclipses of the Sun and Moon.

Reliable measurements of the lunar tidal acceleration have been available only during the last twenty years or so. The most accurate results for $\dot{n}$ are obtained from lunar laser ranging observations, and these indicate a value very close to –26 arcseconds per century per century (″/cy^2). For example, Williams and Dickey[2] have recently derived –25.7″/cy^2. This is the essentially the value used in major Jet Propulsion Laboratory (JPL) Development Ephemerides such as LE51 and the LE400 series. For several decades, the most commonly used value for $\dot{n}$ had been –22.44″/cy^2, which was derived from the work of Jones.[3] For example, this value was used in the *Astronomical almanac* until 1984. However, use of such a low figure for $\dot{n}$ leads to errors in the lunar position of as much as 0.5° (roughly the Moon's apparent diameter) around 1000 B.C. In all computations of lunar co-ordinates, we recommend use of $\dot{n}$ = –26.0″/cy^2. Our own results for ΔT are all derived using this figure. Global sea-level studies suggest that tidal friction has not changed significantly over the last three milllennia or so.[4]

Derivation of ΔT

Although the possibility that the Earth's rate of rotation can vary was first suggested as long ago as the mid-eighteenth century, it was not until 1939 that this inference was fully substantiated. In that year, Jones[5] demonstrated that — relative to the then standard UT timescale — the observed fluctuations in the motions of the Moon, Sun and inner planets were in the ratio of the mean motions; hence they were purely apparent and owed their origin to variations in the LOD. Clemence[6] used the observational results of Jones to derive a parabolic expression representing the approximate long-term behaviour of ΔT. The coefficient of the quadratic term was +30 sec/cy^2.

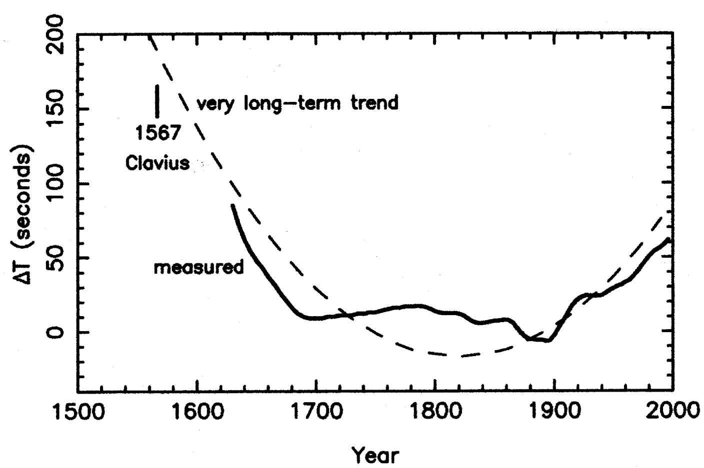

FIG. 1. The measured values of ΔT derived from telescopic observations after A.D. 1600. The possible range of values of ΔT from Clavius's observations of the solar eclipse in Rome is shown at 1567. The very long-term trend is part of the parabola fitted to the data in Figures 2 and 3.

Constant and linear terms in the parabolic expression were introduced to adjust for the reference epoch and the unit of time in Jones's reduction of the observations.

Values of ΔT over most of the telescopic period — between about A.D. 1620 and 1955 — are usually determined from a comparison of the observed and computed positions of the Moon. Most of these observations — numbering many thousands — are in the form of lunar occultations of stars. The rapid apparent motion of the Moon affords high angular resolution in position, and hence in the time difference ΔT. Figure 1 shows the smooth curve fitted through the observations in the period A.D. 1620 to 1955.5; this diagram is taken from Stephenson and Morrison.[7] The fairly narrow range of possible solutions for ΔT derived from a remarkable observation of an almost total solar eclipse by Christopher Clavius at Rome in A.D. 1567 is also shown, as a short vertical bar.[8]

A new time-system known as Atomic Time (TAI) became available in mid-1955 and values of ΔT since that date are independent of the lunar ephemeris. Instead these results are obtained by direct comparison between TAI and UT1 (a measure of Universal Time freed from polar motion), using the expression ΔT = TAI – UT1 + 32.184 sec.

Prior to the telescopic era, eclipses provide the most effective data for determining results for ΔT. The long-term parabolic trend, shown as a dashed line in Figure 1, is deduced from an extensive series of observations of ancient and medieval eclipses.

Telescopic observations reveal irregular fluctuations in ΔT about the mean trend amounting to ±20 sec. These fluctuations, which arise from the so-called "decade variations" in the terrestrial rate of rotation, are caused by a variety of geophysical mechanisms, such as the interaction between the core and lower mantle of the Earth. In all probability, ΔT fluctuations of similar magnitude and time-scale occurred throughout the whole of the historical period, but they cannot be resolved from the relatively crude observational data that are available before about A.D. 1600. In fact, even using telescopic measurements, the resolution is poor until well after A.D. 1700.

Hence, regardless of the choice of expression used to extrapolate ΔT in the pre-telescopic period — or in the future — there will be an inherent uncertainty of about ±20 sec due to the indeterminate behaviour of the decade fluctuations. It should be emphasized, however, that for the very near future, the uncertainty will be much smaller than this because over a period of a few years mathematical modelling can be used to project the value of ΔT forward with more confidence.

As noted above, before the advent of the telescope, recorded observations of both solar and lunar eclipses form the most reliable source of data. Few suitable observations of other celestial phenomena are preserved from this early period. The available eclipse observations originate from only four cultures: Babylon, East Asia, Europe, and the Arab world. Figures 2 and 3 show our results for ΔT as deduced from about 400 eclipse records in the period from 700 B.C. to A.D. 1600. These results are taken from Stephenson and Morrison,[9] as updated in Morrison and Stephenson.[10] Some less critical observations in these two papers have been omitted in Figure 2 for greater clarity of presentation.

It should be emphasized that the data plotted in Figures 2 and 3 are independent of one another. Figure 2 is based entirely on untimed observations of total or near-

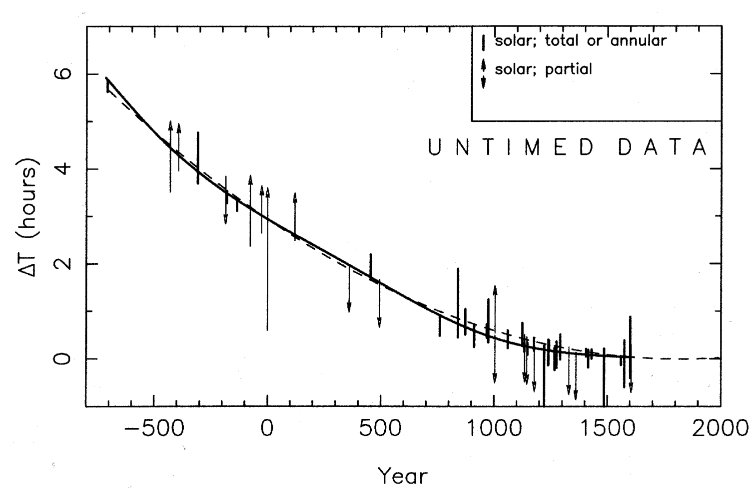

FIG. 2. The results for ΔT derived from untimed observations of total or near-total solar eclipses. The vertical bars show the ranges of values of ΔT that satisfy each observation. The boundary of each range is sharp and definite, except in the case of those with an arrowhead, which indicates that the value could lie somewhere in that direction. The curve fitted by cubic splines is an attempt to satisfy the greatest number of reliable observations. The dashed curve is the best-fitting parabola to all the data.

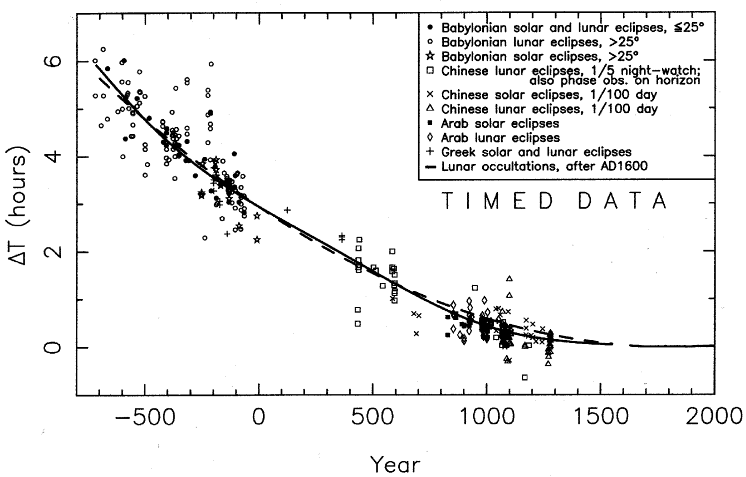

FIG. 3. The results for ΔT derived from timed observations of total and lunar eclipses. Each is a discrete point with an associated uncertainty dependent on the resolution of the timing method. The vertical scatter of the points at each epoch gives a measure of that uncertainty. The curves are the same as in Figure 2. The curve after +1600 is the same as that in Figure 1.

total solar eclipses (numbering about one hundred in all), for which, apart from a qualitative description of the maximal phase, the date and position of the observer are necessary and sufficient conditions. Most of the observers of these events had no particular astronomical interest; the individual records are found mainly in chronicles. Figure 3 is exclusively based on some 300 discrete timings by early astronomers of the various phases of both solar and lunar eclipses; these reports are found mainly in astronomical treatises. The close agreement between these two independent sets of results in Figures 2 and 3 gives us confidence in arriving at a description of the behaviour of ΔT in the historical past.

Figure 3 extends to A.D. 2000. The data for the period after A.D. 1600 are the same as in Figure 1. On the reduced scale of Figure 3, the curve appears almost flat and smooth. This was used as the criterion for the degree of smoothing employed in fitting a series of curves (cubic splines) to the combined data sets in Figures 2 and 3 (details are given by Stephenson and Morrison[11]). Results for ΔT at discrete intervals, together with their estimated uncertainties, are given in Table 1. Pre-telescopic values as far back as the year –700 are taken at intervals of a century along the cubic spline curve in Figures 2 and 3. Intermediate figures for ΔT can be found with adequate precision by simple interpolation. More precise values for the telescopic period, at 10-year intervals, are taken from Stephenson and Morrison.[12]

It should be noted that in Table 1 negative and positive years are used rather than B.C. and A.D. Following standard convention, years with negative numbers differ by 1 from B.C. years: for example, –100 is equivalent to 101 B.C. This difference results

TABLE 1. Values of ΔT taken from the curve shown in Figures 2 and 3. The standard errors are estimated as follows: from –1000 to +1200, 0.8[(year –1820)/100]^2; from +1300 to +1600, 20; after +1600, from the scatter of the points in Fig. 2(b) in Stephenson and Morrison, *op. cit.* (ref. 7).

Year	ΔT(s)	σ(s)	Year	ΔT(s)	σ(s)	Year	ΔT(s)	σ(s)
–1000	+25400*	640	+1000	+1570	55	+1800	+14	1
–900	+23700*	590	1100	1090	40	1810	13	1
–800	+22000*	550	1200	740	30	1820	12	1
–700	+21000	500	1300	490	20	1830	8	<1
–600	+19040	460	1400	320	20	1840	6	
–500	17190	430	1500	200	20	1850	7	
–400	15530	390	1600	120	20	1860	8	
–300	14080	360	1700	9	5	1870	+2	
–200	12790	330				1880	–5	
–100	11640	290	1710	10	3	1890	–6	
0	10580	260	1720	11	3	1900	–3	
+100	9600	240	1730	11	3	1910	+10	
+200	8640	210	1740	12	2	1920	21	
+300	7680	180	1750	13	2	1930	24	
+400	6700	160	1760	15	2	1940	24	
+500	5710	140	1770	16	2	1950	29	
+600	4740	120	1780	17	1	1960	33	
+700	3810	100	1790	17	1	1970	40	
+800	2960	80	+1800	+14	1	1980	51	
+900	2200	70				1990	57	
+1000	+1570	55				+2000	+65	

*By parabolic extrapolation using +32[(year –1820)/100]^2.

from the lack of a year zero on the B.C./A.D. system.

Before about 700 B.C., few — if any — reliable eclipse observations are preserved. Hence values of ΔT in more ancient times must be obtained by extrapolation. For this purpose, a parabolic fit to the ancient and medieval data leads to a useful expression for ΔT, although this is less accurate than the cubic spline fit. From Morrison and Stephenson,[13] the long-term mean parabolic trend has the equation

$$\Delta T = -20 + 32t^2 \text{ sec},$$

where t is measured in (Julian) centuries from the reference epoch A.D. 1820. In an earlier paper,[14] we derived the coefficient of t^2 as +31 sec/cy^2. However, from a consideration of more data, this was revised to +32 sec/cy^2 in our later paper.[15] The long-term trend based on this higher coefficient is also plotted in Figure 1. As noted above, in our various computations we have systematically used a value of –26.0″/cy^2 for the tidal acceleration of the Moon. If a figure for $\dot{n}$ differing from this by 1″/cy^2 (–27.0″/cy^2) were to be adopted, it would lead to a corresponding change in ΔT of approximately +0.9t^2 sec. For instance, if we had adopted Jones's value[16] for $\dot{n}$ of –22.44″/cy^2, we would have obtained an expression for ΔT of very close to +29t^2 sec.

It should be pointed out that this close correlation between changes in $\dot{n}$ and changes in ΔT is not exact, because the lunar orbit is appreciably inclined to the Earth's equator. Changes to $\dot{n}$ have the effect of moving the Moon along its orbit, while changes to ΔT have the effect of rotating the Earth parallel to its equator. Nevertheless, given the uncertainties in the tidal acceleration of the Moon (around $\pm 0.5''/\text{cy}^2$) and in our derived expression for ΔT, this inexactness is of little practical importance, even in calculating the local circumstances of solar eclipses.

In Table 1, the standard errors (σ) in the values of ΔT between the years –1000 and +1200 are estimated from the following equation:

$$\sigma = 0.8t^2 \text{ sec},$$

where, as above, t is measured in (Julian) centuries from the epoch 1820. During the interval from +1300 to +1600, the likely uncertainty in ΔT is around 20 sec. Finally, after +1600, errors are derived from the scatter of points in Figure 2(b) of the paper by Stephenson and Morrison.[17]

Eclipse Calculations

As discussed above, we advocate use of the values of ΔT listed in Table 1 in conjunction with a lunar acceleration $\dot{n}$ of $-26.0''/\text{cy}^2$ for all computations of past solar and lunar eclipses. However, for any other choice of $\dot{n}$, provided the adopted expression for ΔT is consistent with the lunar acceleration, the results of such computations should not be materially different.

As is well known, the computation of lunar eclipses is a fairly simple matter. In particular, the magnitude (maximum degree of obscuration of the Moon) is completely independent of ΔT. Provided the Moon is above the horizon, no matter where an observer is located on the Earth's surface the appearance of the eclipsed Moon at any particular instant is practically the same. On the other hand, any error in the value of ΔT is directly reflected in the uncertainty in the computed time of occurrence of the various eclipse phases. For instance, if we refer to Table 1, we find that the estimated uncertainty in the calculated time of each of the individual stages of a lunar eclipse around 1000 B.C. is about ±11 minutes.

For solar eclipses, computation is much more complex, since a lot depends on the changing geometry as the Moon's shadow moves across the Earth's surface. In particular, if the track of totality or annularity happens to travel roughly parallel to the terrestrial equator, even quite significant changes in ΔT will scarcely affect the eclipse magnitude (although, of course the time of occurrence will be materially altered). Alternatively, if the angle that the track makes with the equator is large, the computed magnitude may vary considerably with the choice of ΔT. In either case errors in the computed times of the various phases are not directly related to uncertainties in ΔT.

Nevertheless, the use of 'eclipse canons' based on obsolete parameters need not necessarily be very inaccurate. This remark is especially true for canons covering only a few centuries around the present day. For instance, the solar eclipse canon of Meeus *et al.*,[18] which covers the the entire Earth's surface over the interval from A.D. 1900 to 2509, uses a value for $\dot{n}$ of $-22.4''/\text{cy}^2$ for the tidal acceleration of the Moon together with the expression $\Delta T = +30t^2$ sec. As shown in the previous section, the optimum expression for ΔT to be used with this lunar acceleration is $29t^2$ sec. However, even by the latest date in the canon of Meeus *et al.* (A.D. 2509), the difference of $-1t^2$ amounts to a discrepancy of only about 46 sec. This does not seriously affect the computed paths of eclipses even at such a relatively remote epoch in the future.

Oppolzer's Canon

We have made a special study of the accuracy of the celebrated canon of eclipses by Oppolzer,[19] which was reprinted in 1960. This compilation provides tabular information on (almost) every solar and lunar eclipse visible between 1207 B.C. and A.D. 2163. In addition, Oppolzer produced charts (on a polar projection) showing the central lines of all total and annular solar eclipses visible between the north pole and latitude 30° south during this same interval. When this work was compiled, changes in the Earth's rotation were not considered. Instead, an attempt was made to derive the acceleration of the Moon (on UT) and (erroneously) the motion of the lunar node from series of past eclipse observations.

Oppolzer's solar eclipse charts present their own peculiar problems. Because of the vast amount of labour involved in computing the central lines of some 5,000 solar eclipses, Oppolzer made a number of approximations in order greatly to simplify his task. For each central line that he depicted, he computed only three positions on the Earth's surface: sunrise, noon and sunset (making no allowance for refraction). He then drew the arc of a circle through these three points on the appropriate chart.

Direct comparison between the modern tabulated central line data of Oppolzer and Meeus shows that Oppolzer's sunrise and sunset positions (tabulated to the nearest degree) are of adequate accuracy (to about 0.3° in both latitude and longitude, and thus merely the result of rounding errors). We have also compared a selection of Oppolzer's local mean noon positions and found errors of similar magnitude (typically 0.4° in both latitude and longitude). If all positional discrepancies were as small as this, Oppolzer's central line charts would be adequate for most general purposes. However, as can be readily shown by comparing Oppolzer's charts for present-day eclipses with the precise charts depicted by Meeus *et al.*[20] or as published annually in the *Astronomical almanac*, Oppolzer's errors around the mid-morning or mid-afternoon positions are often very large. His central lines frequently deviate from their true positions by at least 500km and errors occasionally exceed 1,000km. Hence, except near discrete points, Oppolzer's charts give only a very crude guide to the tracks of totality and annularity for even modern solar eclipses.

For medieval and ancient solar eclipses, a further error is introduced as a result of Oppolzer's erroneous choice of lunar orbital parameters. In consequence, errors in the longitudes of his sunrise, noon and sunset positions often exceed 5° — although the corresponding latitudes are usually accurate to within about 1°. In summary, Oppolzer's *Canon* is of severely limited usefulness for the investigation of both modern and ancient/medieval solar eclipses.

Conclusion

For the most precise calculations of the local circumstances of solar eclipses, we recommend that researchers use values of ΔT taken by linear interpolation from Table 1, combined with a modern ephemeris of the Moon which has a value for the tidal acceleration of the Moon close to $-26''cy^2$. For extrapolation beyond the limits of Table 1 we recommend

$$\Delta T = -20 + 32[(\text{year} - 1820)/100]^2 \text{ sec.}$$

Some empirical adjustment will be required to merge this long-term approximation with extrapolation forward from the precise values formed from ΔT = TAI – UT1 + 32.184 sec.

It is our hope that the numerical data presented in Table 1 will prove helpful to historians of astronomy. The values of ΔT since about A.D. 1600 that are listed in this table are based on the analysis of vast numbers (several tens of thousands) of measurements, mainly of occultations of stars by the Moon. In contrast, the results over the previous 2300 years are derived from only about 400 eclipse observations: an average of about two per decade. Further, as is clear from Figures 2 and 3, the temporal distribution of pre-telescopic observations is far from uniform. However, unless further significant archives of ancient and medieval eclipse records come to light, the prospect of materially refining the pre-telescopic results for ΔT seems remote. This conclusion seems especially true for the most ancient period, prior to 700 B.C. In our experience, extreme caution needs to be exercised when investigating allusions to eclipses and other celestial phenomena at more remote epochs.

Nevertheless, we have a considerable degree of confidence in the ΔT figures cited in Table 1 as far back as 700 B.C. These are derived from at least roughly contemporaneous observations. However, the earlier ΔT values (back to 1000 B.C.) are, of necessity, based on extrapolation. We have thus felt it unwise to extend the table beyond 1000 B.C.

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