MATHEMATICS

OF

Air and Marine

NAVIGATION

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CHAPTER III

PILOTING

Navigation beyond the sight of land, where observations of the sun and stars are necessary for the determination of latitude and longitude, is called *celestial navigation.* Celestial navigation is to be distinguished from navigation along coasts and in harbors which is termed *piloting.* "Piloting, in the sense given to the word by modern and popular usage, is the art of conducting a vessel in channels and harbors and along coasts, where landmarks and aids to navigation are available for finding the position, and where the depth of water and dangers to navigation are such as to require a constant watch to be kept upon the vessel's course and frequent changes to be made therein" (Bowditch). The greatest dangers to ships usually occur when the ship is near land or is entering or leaving a harbor; and while errors in finding position on the high seas may be corrected by later observations, an error in piloting is frequently disastrous.

In modern piloting the navigator is provided with detailed charts of the coasts, lists of landmarks such as lights and buoys, and precise information concerning the tides, currents, and depth of the water. Soundings are of the utmost importance in piloting. Modern instruments for taking soundings measure the time interval required for a sound wave transmitted from the bottom of the vessel to be echoed back to the vessel. Since sound travels at a constant speed in water, the depth of the water can be automatically determined and plotted by the sounding machine. When the position of an aircraft, or rather the point on the earth directly beneath it, is determined from visible landmarks, methods corresponding to those of piloting may be used. Air navigation, however, is complicated by the rapid change of position and by frequent poor visibility. The aeronautical charts of the United States issued by the U. S. Coast and Geodetic Survey provide the navigator of a plane with detail similar to that of the coast and harbor charts.

Position by Cross Bearings. When it is possible to observe the bearings of two, or more, objects which are indicated on the navigational or aeronautical chart, position is easily found by plotting these bearing lines. The navigator observes the bearing of a lighthouse from his ship, but must plot on the map the bearing of the ship from the lighthouse. In Figure 19 the bearing of point B measured from A is 110°, but the bearing of A from B is 110° + 180° or 290°; similarly the bearing of D measured from C is 240°, but the bearing of C from D is 60°, or 240° − 180°.

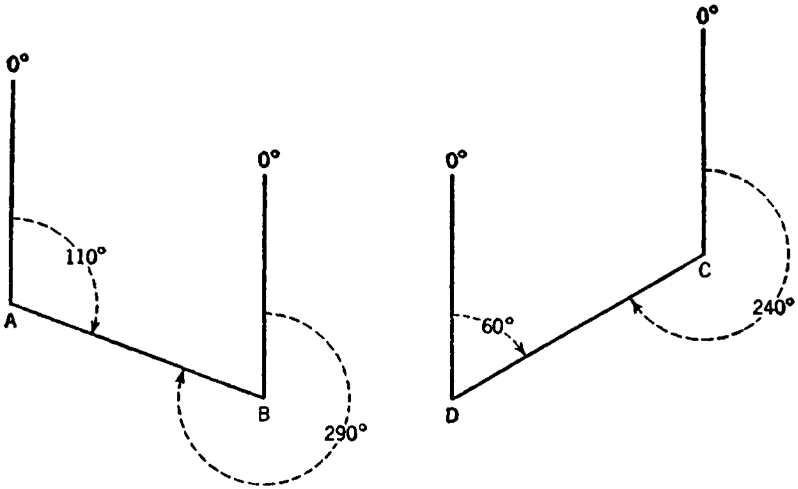

FIGURE 19.

In Figure 19 if A represents a ship and B a lighthouse, then the navigator measures the bearing of 110°, but must plot the *reciprocal* bearing of 290° on his chart. Reciprocal bearings differ by 180°. In problems of this kind the meridians through A and B are considered to be parallel straight lines. Two bearing lines from different objects will intersect in a point, but give no check on the accuracy of the intersection. When possible it is desirable to obtain three bearings and have a check on the accuracy of the intersection, or *fix*.

Example. A plane flying over Long Island Sound observes the bearings of three lighthouses.

Plum Island, 275°
Gardiners Island, 265°
Montauk Point, 140°

Find the position of the plane on the chart.

The plane bears

95° from Plum Island
85° from Gardiners Island
320° from Montauk Point

The three bearings do not determine a point but a small triangle, the so-called "cocked hat," or triangle of error. The accuracy of the fix depends of course on the size of this triangle.

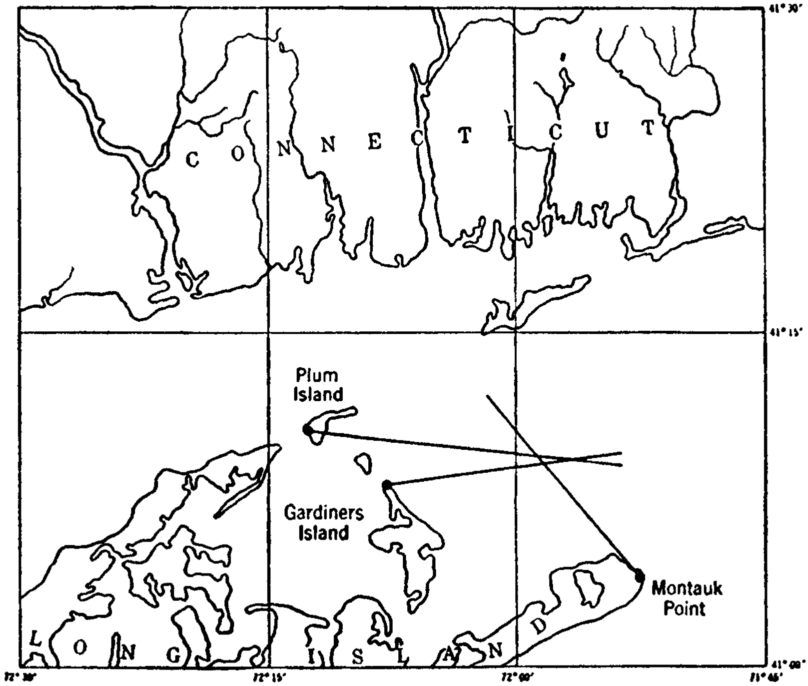

FIGURE 20.

Problems

1. Find the reciprocal bearings for the following bearings. Construct in each case a figure like those of Figure 19. (Do not write in your text.)

Bearing of A from B	*Bearing of B from A*
350°	------
175°	------
87°	------
247°	------
190°	------
10°	------
298°	------
90°	------
37°	------

2. Construct a triangle ABC with AB = 3.5 miles, BC = 4.0 miles, and CA = 5.4 miles. Make the direction of AB, 220°. AC is east of B. A navigator observes these bearings from his ship; A, 295°; B, 270°; C, 250°. Find the position of his ship on your figure.

3. AB is 3.8 miles long and has a direction of 130°, BC is 2.8 miles, and CA 5.6 miles. AC is west of B. A navigator observes the bearing of A to be 35°, B 60°, and C 80°. Find the position of his ship.

Radio Navigation. Various radio devices have been developed which are of the greatest importance in modern air and marine navigation. There are three means of finding position by radio: the radio beam, bearings of the plane or ship from the radio station, and bearings of the radio stations from the plane or ship. These methods are used to determine position in much the same way as cross bearings.

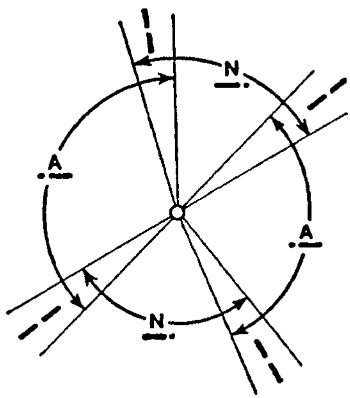

FIGURE 21.

Along the main airlines in the United States there are stations at frequent intervals which broadcast signals for the guidance of pilots. These signals consist simply of the letters A (• —) and N (— •) in Morse code sent out in opposite directions, so that each letter is audible in two opposite quadrants or sectors. Along the boundaries of adjacent sectors the two signals merge in a continuous signal (— —); these equisignal zones are the *radio beams*. A pilot approaching the transmitting station will receive one letter more pronounced if he is to the left of the beam and the other more pronounced if he is to the right of the beam. The letters A and N are interrupted frequently for other signals which identify the direction of the beam.

The *radio direction finder* is an instrument which determines the direction from which a radio message is received. By means of the radio direction finder a station may determine the bearing of a plane from the station and then transmit this bearing back to the plane. Two stations can give the plane intersecting bearings which determine the position of the plane. In preparation for their trans-Pacific flights the engineers of Pan American Airways developed radio direction finders with a range of two thousand miles. Radio direction finders have also been developed for the plane to carry, so that it may find its own bearing from a ship or land station. Radio direction finders of this type have been developed with a range of one thousand miles.

Radio bearing lines may be plotted on a chart and used to find a *fix* in much the same way as the bearings obtained from visible objects. Radio waves follow a great circle, and for long distances the great circle will not have the same direction as the rhumb line (see Figure 12). Before a radio bearing is plotted as a straight line on a Mercator chart, the bearing must be corrected by special tables.

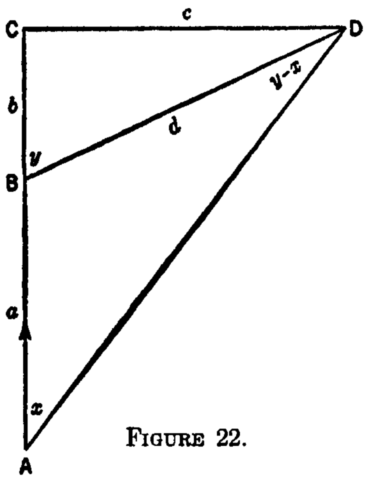

FIGURE 22.

Position by Two Bearings of a Single Object. The position of a ship may be determined by two bearings of a single object obtained at different times. In Figure 22, A and B are two positions of a ship following the course AB, and D is the object sighted. The angles x and y, called *angles on the bow*, are obtained from the bearings of D, measured at A and B. AB is known, since it is the distance run between the times of the two observations. Figure 22 is simply a triangle ABD with altitude c, and plane trigonometry may be used to find d, the distance from the object at the time the second bearing is taken, or to find c the perpendicular. The distance CD (c) is called the *distance at which the object is passed abeam.* In practice the use of trigonometry is avoided by special tables.

There are numerous special cases of Figure 22. If angle $y = 2x$, then BD = AB, or the distance run equals the distance to the object at the second observation. This is called *doubling the angle on the bow.* In this case any convenient value of x may be taken and the angle observed until it has doubled.

There is a special method of finding DC, the distance abeam. This consists of selecting the two angles x and y so that $\cot x - \cot y = 1$. For example, if $x = 30°$, then $\cot x = 1.7321$ and $\cot y = .7321$, using a table $y = 53° 48'$. The navigator is supplied with special tables which give corresponding values of x and y for this special case.

Problems

1. Prove that $d = \frac{a \sin x}{\sin (y - x)}$ and that $c = \frac{a \sin x \sin y}{\sin (y - x)}$.
2. Prove that BD = AB if $y = 2x$.
3. Prove that if $\cot x - \cot y = 1$, then $a = c$.
4. Prove that $\frac{\sin x \sin y}{\sin (y - x)} = \frac{1}{\cot x - \cot y}$.

5. Find values of x and y, such that $\cot x - \cot y = 1$ by completing the following table. Find the other angles to the nearest degree only. Hint: Unless you have access to a table of natural trigonometric functions, you will have to use both the logarithms of numbers (pages ii–xix) and the log cotangents (pages xx–xlii) to solve this problem.

Cot x	*x*	*y*	*Cot y*
2.475	22°	----	----
----	----	41°	1.150
1.963	27°	----	----
----	----	51°	0.810
1.600	32°	----	----
----	----	79°	0.194

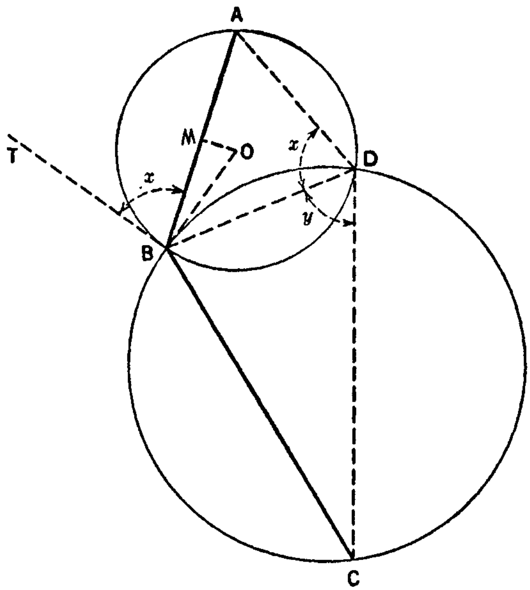

FIGURE 23.

The Three-Point Problem. An important method of fixing position is to measure the angles which the ship makes with three known points. In Figure 23, let A, B, and C be three visible points and D the ship. D may be determined on the chart by the angles x and y. Since the solution depends only on x and y and not on bearings, the angles may be measured with a sextant and compass errors eliminated. Errors caused by estimated distances are also eliminated.

The simplest solution consists merely in laying out the angles on a sheet of tracing paper and moving the paper over the chart until the sides of the angles pass through A, B, and C. D may be located in a similar manner by using a three-arm protractor. Since D lies on the intersection of two circles which have AB and BC as chords, the problem may also be solved by a construction of geometry: "To construct, on a line segment [AB] as a chord, a segment of a circle in which a given angle [x] may be inscribed." If this construction is repeated with the line BC and angle y, D is found by the intersection of the two circles.

To determine O, the center of the circle through A and B, construct:

angle ABT = angle x
BO perpendicular to BT
MO the perpendicular bisector of AB

The position of D is indeterminate from the given data when the two circles coincide; and for an accurate fix it is necessary to avoid the case where the centers are close together. In general, the position of D is determined more accurately when the angles x and y are large and the distances from D to A, B, and C are short.

The three-point problem was first used by the Dutch mathematician, Willebrord Snell, who published a trigonometrical solution in 1617. It has many applications in marine surveying and military engineering as well as in navigation. Many solutions other than the geometrical one have been devised.

Problems

1. Use the construction of the three-point problem to locate a ship D from three lights, A, B, and C. AB = 2.1 miles and bears due south, BC = 3.2 miles and bears 164°. Angle ADB = 33°, angle BDC = 68°. The ship is on the east side of AB.

2. Give the proof for the geometric construction in Figure 23.

3. Construct the circle through AB in Figure 23 by making an isosceles triangle on AB as a base. Make each base angle of the isosceles triangle equal to $90° - x/2$.

4. Construct the circle through AB in Figure 23 by constructing a triangle on AB as a base with the sum of the base angles equal to $180° - x$.

The Danger Angle. Another application of elementary geometry is used in sailing along a coast where there are hidden reefs or other dangers. In Figure 24, A and B are two lighthouses and C a hidden danger to be avoided. The navigator constructs a circle on the chart which passes through A and B and safely includes C. He then measures the angle ADB inscribed in the larger arc AB. In order to avoid C, the ship must be steered so that the angle ASB, which it makes with A and B, is less than angle ADB.

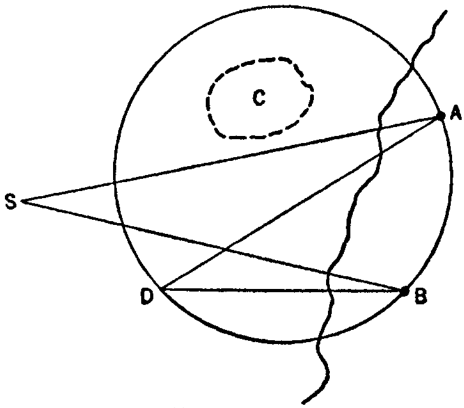

FIGURE 24.

Problems

1. Prove that (*a*) the ship is inside the circle when angle S is greater than angle D; (*b*) the ship is on the circle when angle S equals angle D; (*c*) the ship is outside the circle when S is less than angle D.

2. Construct a figure to show how a double danger angle might be used to steer a safe course between two dangers.

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