between the zenith and the elevated pole, then add together Arc II. and Arc III.; but if the great circle do not so pass, subtract Arc III. from Arc II.; call the sum or difference Q.

(*k*) Take half the sum of the polar distance and zenith distance at the less altitude $\frac{p+z}{2}$; call this A'.

(*l*) Then add together—

Twice log. sine of Q . . . . $\cos.^2 \frac{PXZ}{2}$,
log. sine p. d. at the less altitude $\sin. p$,
and log. sine z. d. at " " $\sin. z$;

divide the sum by 2; this is the log. sine of an arc, which call B'. Find the sum and difference between A' and B', then half the sum of log. sin. $(A' + B')$ and log. sin. $(A' - B')$ will give the log. sine of half the colatitude sin. $\frac{PZ}{2}$. The double of this subtracted from 90° gives the latitude.

(*m*) Having found the latitude, the longitude is deduced, as already shown, from the least altitude.

Ex. 457. 1887, February 28th, in latitude by account 47° 20′ N., longitude 133° 10′ W., the following observations were made :—

App. time ship nearly.	*Chronometer.*	*Observed altitudes.*
9h. 53m. a.m.	6h. 32m. 3s.	☉̲ 27° 55′ 0″
0 56 p.m.	9 35 15	☉̲ 33 32 0

The sun's magnetic bearing at the first observation was S.E. by E. ¼ E., and the run of the ship in the interval was magnetic S. by E. ¾ E., at the rate of 6 miles per hour. The chronometer was slow of G. M. T. on 4th January, 27m. 10s., and gaining daily ·6s. The index error of the sextant was − 2′ 19″, and the height of the eye 23 feet. Required the latitude and longitude of the ship at the time of taking the less altitude.

For Greenwich apparent times nearly.

	First observation.		*Second observation.*
February	27d. 21h. 53m. 0s.		Feb. 28d. 0h. 56m. 0s.
Long. in time	+ 8 52 40		8 52 40
Greenwich app. time	28 6 45 40		Feb. 28 9 48 40

For Z. D., first observation.

Observed alt. L. L.	27° 55′ 0″
Index error	− 2 19
	27 52 41
Dip	− 4 43
	27 47 58
Semidiameter	+ 16 10
	28 4 8
Refrac. and parallax	− 1 38
True alt.	28 2 30
Zenith distance	61 57 30 .. z

For Z. D., second observation.

Observed alt. L. L.	33° 32′ 0″
Index error	− 2 19
	33 29 41
Dip	− 4 43
	33 24 58
Semidiameter	+ 16 10
	33 41 8
Refrac. and parallax	− 1 18
	33 39 50
Correction for run	− 14 10
True alt.	33 25 40
Zenith distance	56 34 20 .. z′

For half polar angle.

G. M. T., first obser.	6h. 58m. 39·83s.
" second "	10 1 51·75
	3 3 11·92
Correction for E. T.	+ 1·43
Polar angle	3 3 13·35
=	45° 48′ 20·25″
∴ $\frac{P}{2}$ =	22 54 10

Var. E. T. 1 hr.	·470s.
Elapsed time	3·0503
Correction	1·434910

For Arc I.

$\frac{P}{2}$	22° 54′ 10″	cos.	9·964338
			2
		cos.²	19·928676
p	97 51 30	sin.	9·995902
p'	97 48 37	sin.	9·995952
A	97 50 4		2)19·920530
B	65 51 45	sin.	9·960265
Sum	163 41 49	sin.	9·448271
Diff.	31 58 19	sin.	9·723869
			2)19·172140
$\frac{D}{2}$	22° 40′ 38″	sin.	9·586070
	2		
D	45 21 16 = Arc I.		

For Arc III.

z'	56° 34′ 20″		
z	61 57 30	cosec.	·054233
D	45 21 16	cosec.	·147845
	2)163 53 6		
S'	81 56 33	sin.	9·995691
$S' - z'$	25 22 13	sin.	9·631917
			2)19·829686
Arc. III.	34° 43′ 11″	cos.	9·914843
Arc. II.	46 37 8		
Q	11 53 57		

For Arc II.

p′	97° 48′ 37·3″		
p	97 51 30·3	cosec.	·004098
D	45 21 16	cosec.	·147845
	2)241 1 24		
S	120 30 42	sin.	9·935268
S − p′	22 42 5	sin.	9·586507
			2)19·673718
Arc II.	46° 37′ 8″	cos.	9·836859

For colatitude.

Q	11° 59′ 57″	cos.	9·990566
			2
			19·981132
p	97 51 30	sin.	9·995902
s	61 57 30	sin.	9·945767
A′	79 54 30		2)19·922801
B′	66 11 58·6	sin.	9·961400
Sum	146 6 28·6	sin.	9·746346
Diff.	13 42 31·4	sin.	9·374723
			2)19·121069
$\frac{\text{Colat.}}{2}$	21° 19′ 1″	sin.	9·560534
Colat.	42 38 2		
Lat.	47 21 58 N.		

The student should notice that Arc I. and half the colatitude are found by a similar process, as are also Arcs II. and III. If a blank form be first written down, the work will be much facilitated, because many logarithms are taken from the same opening of the tables. Having now found the correct latitude, the longitude is found in the usual manner.

For longitude at first observation.

a	28° 2′ 30″		
l	47 21 58	sec.	·169202
p	97 51 30	cosec.	·004098
	2)173 15 58		
S	86 37 59	cos.	8·768864
s − a	58 35 29	sin.	9·931189
			2)18·873353
$\frac{\lambda}{2}$	15 51 43·6	sin.	9·436676
	2		
λ	31 43 27·2		
	4		
	60)126 53 48·8		
Eastern hour angle	2h. 6m. 53·81s.		
	24		
App. time ship, Feb.	27d. 21h. 53m. 6·19s.		
Equa. time	+ 12 41·77		
M. T. ship, Feb.	27 22 5 47·96		
M. T. Greenwich, Feb.	28 6 58 39·83		
Long. in time	8 52 51·87		
	60		
	4)532 51·87		
Longitude	133° 12′ 58″ W.		

This is the longitude at the place where the first observation was made, because we have used the altitude as taken at that station.

EXERCISE XIX.

Ex. 458. 1887. On December 24th, in latitude by account 47° 30′ N., longitude 8° 30′ W., the following observations were made:—

Approx. time ship.	*Time by chronometer.*	*Observed altitudes.*
11h. 15m. a.m.	0h. 16m. 6s.	$\overline{\odot}$ 18° 21′ 20″
2 30 p.m.	3 31 4	$\overline{\odot}$ 11 43 45

The sun's compass-bearing at the time of taking the first observation was S. by W., and the run of the ship in the interval between the observations was 30 miles on a S.W. by W. ¾ W. course. The chronometer was fast 27m. 19s. of G. M. T. on 17th of November, and losing daily ·7s. The index correction of the sextant was − 1′ 17″, and the height of the eye above the sea was 20 feet. Required the latitude and longitude of the place at the time of taking the second observation.

Ex. 459. 1887, September 1st, at ship by reckoning in latitude 49° N., longitude 180° E., the following observations were made:—

Approx. time ship.	*Time by chronometer.*	*Observed altitudes.*
0h. 33m. p.m.	0h. 42m. 57s.	$\underline{\odot}$ 48° 51′ 0″
4 41 p.m.	4 50 20	$\overline{\odot}$ 19 30 20

The sun's bearing at the first observation was S. by W., and the run of the ship in the interval was E.N.E., both by compass, at the rate of 6¼ knots per hour. The chronometer was fast 5m. 13s. on 12th May for G. M. T., and gaining daily 2s. The height of the eye above the sea was 17 feet, and the index error of the sextant was + 1′ 10″. Find the ship's position at the second observation.

Ex. 460. 1887, March 20th, in latitude by account 26° 20′ S., longitude 115° 25′ W., the following observations were made:—

Approx. ship app. time.	*Time by chro.*	*Observed altitudes.*
11h. 18m. a.m.	6h. 51m. 22s.	$\underline{\odot}$ 62° 7′ 20″
3 40 p.m.	11 5 10	$\underline{\odot}$ 30 31 40

The index correction of the sextant was + 3′ 24″, and height of eye above the sea 22 feet. The chronometer was slow of G. M .T. on 23rd February, 23m. 39s., and losing daily 1·4s. The sun's bearing at the first observation by compass was