Solar Declination

Precision can be important in navigation, depending on our requirements. If we're interested in the direction "south", this requires low precision. If we want to know our latitude and longitude to within a few miles, this requires a much higher precision. Depending on your needs, memory may or may not be sufficient. You may need to resort to navigational tables or computer programs, and we leave the realm of primitive navigation. On the other hand some memorization can allow a person to navigate reasonably well. The earth's orbit is an ellipse. We will be slightly closer to the sun and moving faster during January then we will be during July, but to a good first approximation, we can think of the earth's orbit as a circle.

The maximum and minimum declination of the sun, expressed in decimal units is easy to remember: $+23.45^o$ and -23.45^o. Here, the sequence 2-3-4-5 are just the integers. The simplest approximation to the sun's declination as a function of day is a sinusoid. A sinusoid is the simplest expression of the shape of a wave – an ocean swell, a sound wave. The maximum extent is the *amplitude*. A sinusoid can be expressed as a function of time, in the form:

$$d = A\sin\left(\frac{day}{period}\right)$$

Here d is the declination we seek, *A is the amplitude*, which is 23.45^o . The sine function is something that asks for an angle. You can think of the angle as the number of degrees that have passed in the earth's orbit from some starting point. As with the equinoctial colure, the starting point here is the Vernal Equinox – March 21st. One minor nuisance is the fact that there are 360^o in a circle and 365.25 days in a year, so a rendering of this sine function would be:

$$d = 23.45^o * \sin(\left(\frac{360}{365.25}\right) * (Day))$$

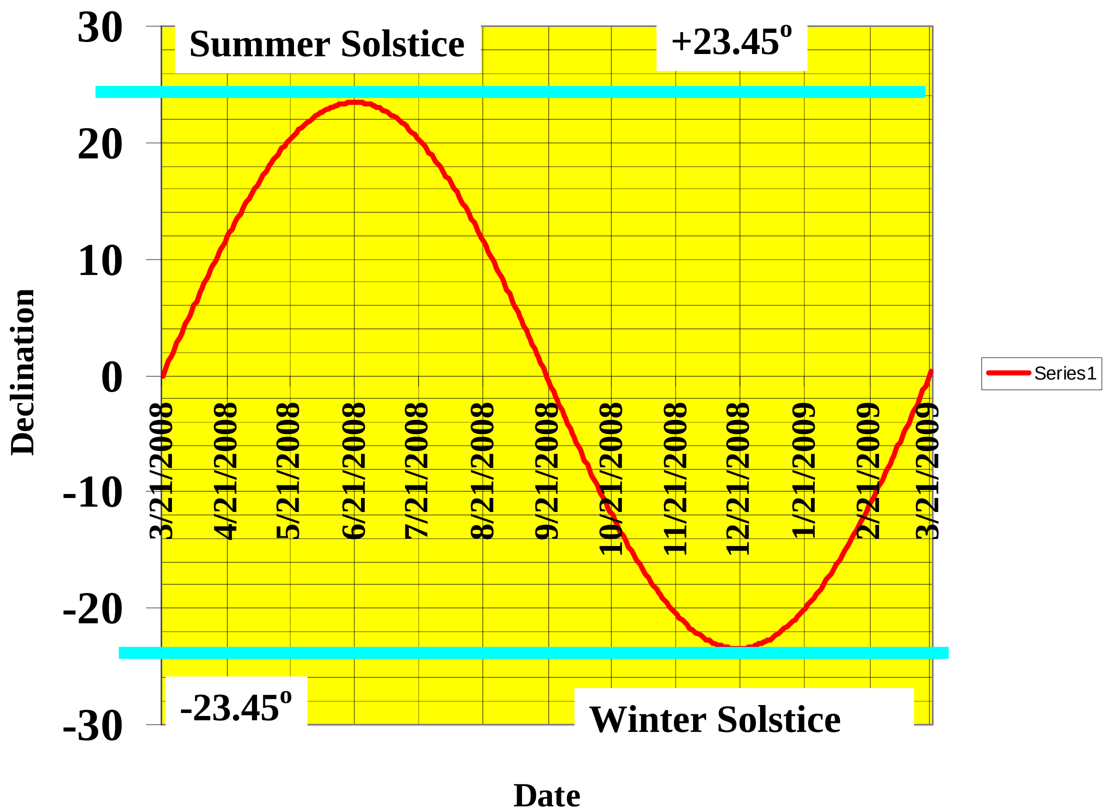

Figure 1 Solar declination as a function of date, using the sine function.

In Emergency Navigation, David Burch[i] suggests a way that the primitive navigator can calculate the declination of the sun at any time by drawing the arc of a circle, and graphically reproducing a sine function. If the surface where the arc is being traced is large enough, he claims that one can reproduce solar declination to an accuracy of approximately 10 arc-minutes. In fact, there are a number of ways of reproducing a sine function on a desert island or in an emergency without tables or a calculator, including using a polynomial approximation. There is a warning however, a *NOT SO FAST!* you needs to be aware of. The earth's orbit is not a circle, but an ellipse. In fact, all planetary orbits are ellipses. This was one of the key discoveries of Kepler, and was the result of precise measurements of the orbits of the planets by Tycho Brahe. It is a consequence of the inverse square law of Newton's formulation of gravity.

As a result of the eccentricity of the earth's orbit, it will speed up and slow down relative to some average speed. The extent to which a sine function works at all demonstrates how circular the Earth's orbit really is. When the solar declination is changing rapidly, around the equinoxes, this doesn't work to the precision of 10 arc-minutes, however. In point of fact, I tried David's approximation one October and found that I was off in my latitude by more than a degree, despite the accuracy of my home-built quadrant, which was about 20 arc minutes. My error in latitude was equivalent to 100 miles, while my quadrant was capable of establishing my position to something on the order of 20 miles. In comparing the sine approximation to the actual declination, as derived from NOAA's website (National Oceanic and Atmospheric Administration), one can examine the differences and even derive a correction factor that's accurate to about 10 arc-minutes.

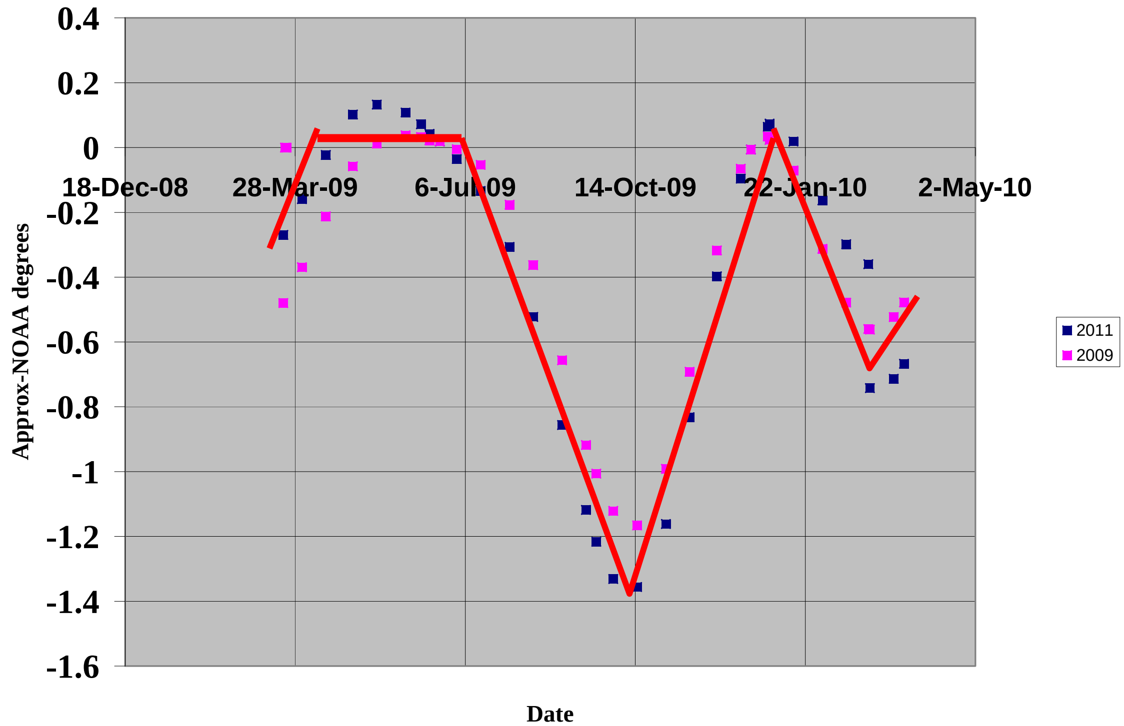

Figure 2 Deviation of solar declination from a perfect sine function. The red line shows the approximation for the correction described in the text.

In principle, one could, in fact, use the deviation of the solar declination from a perfect sinusoid as a means of roughly figuring the eccentricity of the Earth's orbit. As one can see from the figure above, the solar declination is not identical for every year, but changes from year-to-year. There is a simple mnemonic that one can use to approximate this deviation and get a solar declination that is based on David's scheme with this correction factor layered on top. You need to remember that all the deviations are negative – that is to say, you need to add a positive angle sometimes, mainly around the equinoxes, in order to gain more precision. You need to remember the number 14, the months October, March, and the period of April Fools to Independence Day (this is a mnemonic…it doesn't have to make total sense!).

1.) The correction factor is zero at the solstices – December 21st and June 21st.
2.) Around the summer solstice, the correction factor is zero between April 1st and July 4th.
3.) On October 14th, the correction factor is 1.4 degrees.
4.) On March 14th, the correction factor is 0.7 degrees (1/2 the correction factor on October 14th).

5.) You then chart these points, and connect them as lines as in the figure. This results in a precision of about 0.2 degrees (12 arc-minutes), which is about as good as a person can do, given that the solar declination will vary by approximately this amount from year to year.

[i] David Burch, Emergency Navigation, p. 173.