Lunar Methods For 'Longitude Without Time'

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1. INTRODUCTION.[1] In 1966, in a note in the Forum entitled 'Longitude without Time', Francis Chichester[2] described a method of determining longitude, or time (G.M.T.), from observations of the altitude of the Moon; and this gave rise to considerable interest. Recently a number of papers,[3] of varying standards, have been published on the same subject; some, ignoring fundamental principles, have described techniques that are erroneous or misleading. Those principles, which have been well known for centuries, are here restated. In general, techniques of observation, calculation and plotting are not given; they are matters for the choice of the individual.

2. TIME AND POSITION. The difference between the local times (in any time-scale) at the same instant is the difference in longitude, converted from units of time to units of arc at the rate of 15° to one hour. In practice the time-scales used are mean solar time and sidereal time, giving rise respectively to Local Mean Time (L.M.T.) and L.H.A. Aries; mean solar time on the meridian of Greenwich will be referred to here as Greenwich Mean Time (G.M.T.).[4] Without a knowledge of longitude, or of G.M.T., observations of the altitudes of stars give the observer's latitude and L.H.A. Aries at a particular instant. Such a determination will be referred to as a 'space-position'; a knowledge of longitude is required to convert it to a position on the surface of the Earth. A determination of G.M.T. gives the required longitude from the difference between G.H.A. Aries (a function of G.M.T.) and L.H.A. Aries as observed. In lunar methods G.M.T. is determined by comparing observed positions of the Moon with the ephemeris positions tabulated in terms of G.M.T. It is assumed that a time-piece, capable of measuring intervals of time for the purpose of connecting observations, is available.

3. THE MOTION OF THE MOON. The main descriptive features of the Moon's motion relative to the stars concern its revolution about the Earth in an elliptic orbit in a period of about 27·3 days. The Moon's angular speed varies from about 29′ to 38′ an hour, and the perigee of its orbit rotates in a period of about 9 years. The orbit is inclined to the ecliptic at an angle i of about 5°·2, and thus to the equator at an angle I varying between about 18°·2 to 28°·7 according to the position of the node; the node retrogrades in a period of about 18½ years. All three periods are incommensurable, the actual motion is much more complicated and there is no simple method of estimating it among the stars

except by reference to its ephemeris. Also there is no simple connection between its motion, or the position in its orbit, and its phases which depend on the position of the Sun.

4. LUNAR DISTANCES. If the Moon's motion is s minutes of arc an hour, its angular distance from the Sun (or zodiacal star) changes at a rate of more than $0{\cdot}9s$ provided that the distance is greater than about 15° (30° for a star); there is always a choice of bright stars for which the factor is much closer to unity. An observed distance, after reduction for comparison with the geocentric distance, will thus provide a determination of G.M.T. with a precision of nearly E/s hours, where E is the error, in minutes of arc, of the reduced distance.

The computational complexity of reducing the observed distance, which was the main obstacle to the method of lunar distances, has now been removed. If d_0 is the observed distance, corrected only for instrumental errors and semi-diameter, H_M and H_S the approximate altitudes of Moon and Sun (or star), C_M and C_S the altitude corrections for semi-diameter, parallax and refraction, then the 'observed' geocentric distance d is given by:

$$\cos d = \sin (H_M + C_M) \sin (H_S + C_S) + \frac{\cos (H_M + C_M) \cos (H_S + C_S)}{\cos H_M \cos H_S} (\cos d_0 - \sin H_M \sin H_S)$$

This may be evaluated, relatively simply and with adequate precision, on many hand-held calculators.[6]

Geocentric values of d may be calculated for integral hours of G.M.T. (at least two and preferably three) from

$$\cos d = \sin \delta_M \sin \delta_S + \cos \delta_M \cos \delta_S \cos (\text{S.H.A.}_M - \text{S.H.A.}_S)$$

where the declinations and S.H.A. are taken direct from *The Nautical Almanac*. The G.M.T. of observation is then found by inverse interpolation.

Observation of lunar distances is possible, in the absence of cloud, for most of the time that the Moon is above the horizon except close to new Moon. Although it is desirable also to observe the altitudes of the Moon and Sun (or star), these are required to low precision and may be calculated, though successive approximations may be necessary if the 'space-position' is greatly in error. Otherwise the method is independent of the space-position.

The actual observation consists of aligning the limb of the Moon with a star or the limb of the Sun, a rather unusual observation; but the distance changes by $0'.1$ in about 10 seconds so that timing errors are insignificant. The alignment error is the principal contribution to E and to the error in the deduced G.M.T.

5. LUNAR ALTITUDES. For a given space-position the rate of change of the Moon's altitude with G.M.T., or longitude, can never be greater than s and is often much smaller. The general picture is illustrated in

Fig. 1. P, C, L are respectively the poles of the equator, ecliptic and lunar orbit and M is the position of the Moon. The latitude ϕ is $90° - PZ$. A is the azimuth angle, B the angle PML and C the parallactic angle; $PC = \epsilon = 23\frac{1}{2}°$ is the obliquity of the ecliptic and δ, H are respectively the Moon's declination and altitude. The rate of change of altitude is $-s \sin(B-C)$ where B depends on the relatively fixed positions of C and L, and on the position of the Moon in its orbit, whereas C (positive for western hour angles) depends on the position of the observer's zenith Z. In the figure, in which the declination is of the same name as the latitude, the angle $(B-C)$ increases as Z moves from Z_R (at which the Moon is rising) to a maximum at Z_E (where the Moon is due east); it then decreases to B at Z_M (meridian transit) and to zero at Z_0 on LM; it continues to decrease to Z_W, where it reaches its extreme negative value, and then increases

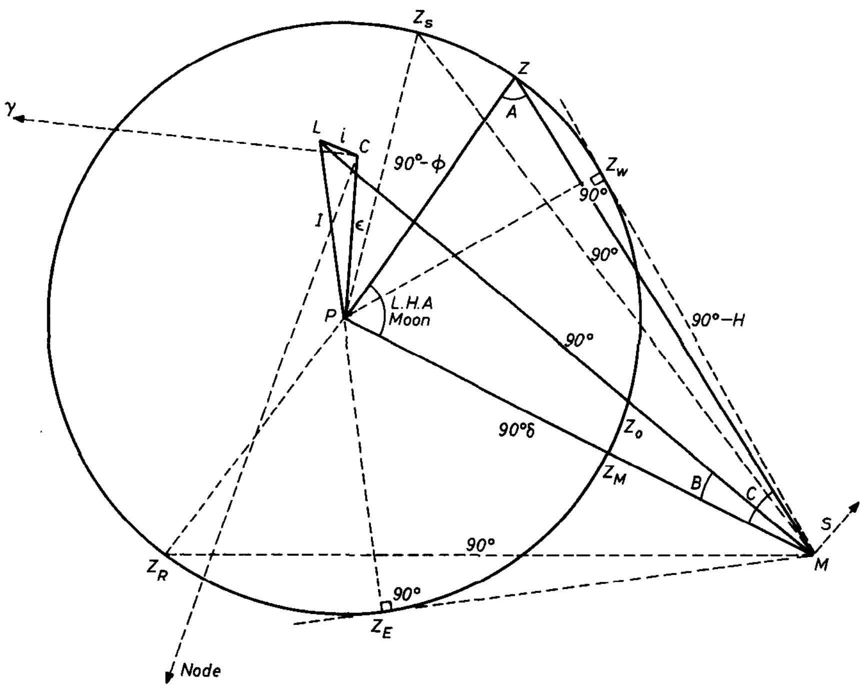

FIG. 1. The diagram shows the relative positions of the poles of the equator, ecliptic and lunar orbit at some date in 1984, when the longitude of the node is about 70° (1978 is not used as the longitude of the node passes through 180°). As shown the longitude of the Moon is about 140°. The various positions of the observer's zenith, Z in latitude ϕ, correspond to various hour angles from rising to setting. The rate of change of altitude with G.M.T., proportional to sin LMZ, is clearly much larger at rising than at setting. If the Moon is near first quarter (as it would be in May) it will rise in daylight, but if near last quarter (in November) it will rise over an horizon illuminated only by itself; the point Z indicates a possible observation. Similar circumstances arise for different longitudes and phases.

until the Moon sets at Z_S. The Moon will not, of course, be above the horizon on the prime vertical for declinations of opposite name; and many details will require to be changed for low and high altitudes and for various positions of the Moon in its orbit. For latitudes less than I, $(B-C)$ can reach 90° but otherwise its maximum numerical value is $90° + I - \phi$ which can only occur when the Moon, with zero declination, is rising or setting on the prime vertical.

The navigator has no choice of the positions of L and M but may choose Z (that is the local time) within the limitations imposed by the need that both the Moon and the horizon are visible. Guidance as to his optimum choice (to maximise sin $(B-C)$) can readily be obtained from the tabulated times of moonrise and moonset. The difference between successive times, less the sidereal day of $23^h 56^m$, is approximately $5/3\ s \sec\phi \operatorname{cosec} A \sin (B-C)$ minutes of time so that the values at rising and setting are in the ratio of sin $(B-C)$[7]: the optimum position, if available, is on or near the prime vertical associated with the larger. Limitations on availability, which depend on phase, are not considered here.

The G.M.T., at the time of the observation, is obtained by inverse interpolation in a series of altitudes, calculated for the space-position and integral hours of G.M.T., to the corrected observed altitude. The error is $(E/s) \operatorname{cosec} (B-C)$ hours where the total error E, in minutes of arc, is now compounded of:

(a) the errors in the observed altitude, including those due to the uncertainty of the horizon and dip;
(b) the errors in the calculated altitude arising from the error in the space-position.

The actual alignment error is, in spite of the familiarity, unlikely to be smaller than that for a lunar distance, since the altitude may change by 0'.25 in 1 second and timing to a fraction of a second is necessary. The other, possibly larger, errors do not enter into the lunar distance. One interesting point to be noted is that, for latitudes greater than about 30°, the mean value[8] of the extremes of $(B-C)$ is about $90-\phi$ so that the average minimum error in departure is proportional to E/s; whereas, in the method of lunar distances, it is the actual error in longitude that is proportional to E/s.

The method of lunar distances is clearly superior in principle, in precision and in availability; the additional computation required to 'reduce' the observed distance is no longer a deterrent. The only factor in favour of the altitude method is the familiarity of the observation and the associated calculations.

The above factor is undoubtedly the reason why so many techniques, starting with that proposed by Francis Chichester, have been suggested for deducing longitude direct from an observation of altitude, using plotting procedures or differences of hour angle. None can possibly increase the precision obtainable by comparing the observed quantity (the altitude) with the calculated values; at best they can present the problem, particularly the procedure for inverse interpolation, graphically —with or without loss of precision; all tend to obscure the dependence on various sources of error; and, at worst, they can be misleading or wrong.

6. HISTORICAL NOTE. The altitude method has been regarded as significantly inferior to that of lunar distances for at least 300 years. Until recent years it has tended to be ignored or, at least, passed over with a cursory critical comment. Three references only are quoted.

(a) In 1674 the Sieur de St. Peirre announced to King Charles II that he had discovered a lunar method of finding longitude; from that event arose the foundation of the Royal Observatory at Greenwich. In principle the method was precisely that recently described by J. W. Luce[9] as 'A new lunar method'. John Flamsteed, to become the first Astronomer Royal, soundly denounced the method (mainly because the lunar theory was inadequate) but nevertheless supplied the data demanded by St. Pierre as a test of the method. St. Pierre complained that the data were 'feigned and obscured', but recently research has demonstrated conclusively that they were based on actual observations made by Flamsteed himself.[10]

(b) At the meeting[11] of 'The Commissioners appointed by Acts of Parliament for the discovery of Longitude at Sea' held at the Admiralty on Saturday, 5 June 1802, it was 'Resolved

> Unanimously that this Board will not in future take into their consideration any methods of ascertaining the Longitude founded on the Moon's Altitude, or on the Polarity of the Magnetic Needle, and that Mr. Gilpin, the Secretary, be directed to give this answer to all persons who shall in future send projects for discovering the Longitude founded on either of these principles.'

There were present at the meeting: Earl St. Vincent (First Lord), Sir Andrew Hammond, Bt. (Comptroller), Rt. Hon. Sir Joseph Banks, Bt. (P.R.S.), together with the Astronomer Royal (Maskelyne) and four professors: Hornsby and Robertson (Oxford) and Milner and Lax (Cambridge). At least twenty proposals, using the Moon's observed altitude, were subsequently submitted before the Board was dissolved in 1828.

(c) Francis Chichester[2] states that he used Raper's *Navigation* (1840 edition)[12] for guidance as to 'how to work out a lunar'. If he had read on he would have found the following statements in relation to the altitude method.

p. 240. 'It is evident, therefore, since the place of the sea horizon is often doubtful from 1′ to 3′, that the result of a simple lunar altitude must be in general greatly inferior to that of a lunar distance, in which a good observer rarely makes an error exceeding half a minute.'

p. 243. '. . . it cannot be prudent, notwithstanding the occasional success of observations of this kind, to depend upon the result as nearer than ¼ of a degree.'

NOTES AND REFERENCES

1 This note was written at the request of the Editor.

2 This *Journal*, 19, 106.

3 In particular, the following:

Ortlepp, B. (1969). Longitude without time, *Navigation*, Vol. 16, 29.

Ortlepp, B. (1969). Longitude without time, *Nautical Magazine*, Vol. 210, 276.

Wright, F. W. (1971). Examples of Moon sights to obtain time and longitude, *Navigation*, Vol. 18, 292;

Kerst, D. W. (1975). Longitude without time, *Navigation*, Vol. 22, 283.

Luce, J. W. (1977). Longitude without time, *Navigation*, Vol. 24, 112.

Ortlepp, B. (1977). Improved plotting solution to longitude without time, *Nautical Magazine*, Vol. 218, 334.

4 The recommended terminology (according to a Resolution of the International Astronomical Union adopted at Grenoble in September 1976—*Transactions* of the I.A.U., Vol. XVI B, 218) is now Universal Time, with abbreviation U.T.1 in order to distinguish it from Co-ordinated Universal Time (U.T.C.) which may also be included in the general term Universal Time (U.T.). G.M.T. is here used, instead of U.T.1, in order to emphasize its relationships with L.M.T. and G.H.A.

5 But, since the Moon's ecliptic latitude is always small, its declination at new Moon differs little from that of the Sun. The full Moon thus 'rides high' in winter months.

6 It can be performed on all 'mathematical' models, but the amount of intermediate recording required depends on the sophistication (storage, programmability, &c.) of the calculator.

7 Interested readers might note how these differences vary through a lunar month; the longitude of the node of the Moon's orbit is 180°, and thus the inclination, I, to the equator is a minimum in July 1978.

8 Deliberately vague: $(B-C)$ varies between the extremes $(90° - \phi) + I \cos \theta$ and $-(90° - \phi) + I \cos \theta$, but the optimum value cannot always be chosen.

9 The method, as described by J. W. Luce (*loc. cit.*, see note 3), appears in fact to be 'new' since it is erroneous; he ignores the Moon's motion in declination.

10 See the following papers presented at the Royal Observatory Tercentenary Symposium held at the National Maritime Museum in July 1975:

Forbes, E. G. (1976). The origins of the Greenwich Observatory, *Vistas in Astronomy*, Vol. 20, 39.

Sadler, D. H. (1976). Lunar distances and the Nautical Almanac, *loc. cit.*, 113.

11 Board of Longitude Papers, Vol. 7 (confirmed Minutes 1802–1823) page 10. At the Royal Greenwich Observatory.

12 Raper, H. (1840). *The Practice of Navigation*, first edition. There was a twentieth edition in 1914!