ACCURACY OF POSITION FINDING USING THREE OR FOUR LINES OF POSITION

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THREE LINES OF POSITION

In certain systems of navigation or position finding, the desired location is determined by finding two or more lines on which the location is situated. The position is then fixed by the intersection of these lines.

If, as is usual, three such lines of position are determined, the errors in measurement cause the lines to form a small residual triangle (Figure 1). It is then assumed that the true position lies somewhere inside this triangle. In most cases the most probable position of the true location will be inside the triangle, but there is nevertheless a three-to-one chance that the true position is outside. Here follows a proof of this theorem.

Let *a, b,* and *c* be the directions of the lines of position. Errors in measurement cause a small parallel displacement of each of these lines. (See Figure 1.)

Let an error cause *a* to fall on the right of the position *P*. (See Figure 2.) There is an equal chance that *b* falls on the right or left of *P*. The direction of *c* shows that *P* can lie inside the triangle only if *b* passes it on the left. The chance for this is one-half. (See Figure 3.)

If we finally apply the line *c*, we see that *P* will fall inside the triangle only if the error causes *c* to lie above point *P* in Figure 3. The probability for this to happen is again one-half. The total probability that *P* lies inside the residual triangle is thus one-quarter. This proof is, of course, independent of the order and choice of direction of the lines *a, b,* and *c*. The only assumption made is that it is equally likely to make an error to one side as to the other of true position *P*. The accuracy of measurement may be different for each of the three lines of position.

FOUR LINES OF POSITION

When a fix is obtained with four lines of position, these lines form an enclosed area, a quadrangle with one receding angle, as seen in Figure 4. The chance that the true position is inside this area is one-half. Here follows a proof.

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Of four directions, such as *a, b, c,* and *d*, one can always choose two in such a manner that the third line will cut off the acute and the fourth the obtuse angle of intersection of the first two. In Figure 5 this means that, for example, we consider first the direction *a* and *c*. The direction *d* will certainly cut off a triangle at the acute angle of intersection of *a* and *c*, where *b* will cut off the obtuse angle. (See Figure 5.) If one considers first the lines *a* and *c*, the position *P* can fall inside the enclosed area no matter whether *P* is in the acute or in the obtuse angle. One of the other lines, either *b* or *d*, must fall on the proper side of *P* in order to enclose the point. The chance for this in either case is one-half.

Extension to more than four lines of intersection is possible too, but it becomes impractical.

THE MOST PROBABLE POSITION

The most probable position can only be determined if the chances for errors are known for each one of the lines of position.

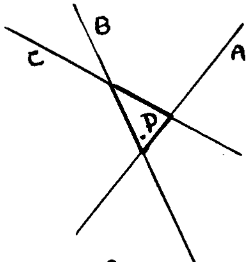

fig. 1

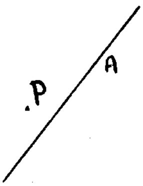

fig. 2

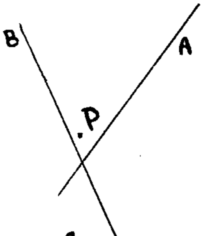

fig. 3

If we assume these to be the same and to follow a Gaussian error distribution, the most probable location is inside the residual triangle at a point for which the distance to each side of the triangle is proportional to that side. It would, however, be misleading to use this point in practice, as it might give the impression of a higher accuracy than is actually obtained. It is simpler and wiser to choose an arbitrary point inside the triangle, perhaps situated somewhat nearer to the shortest side in order to take into account

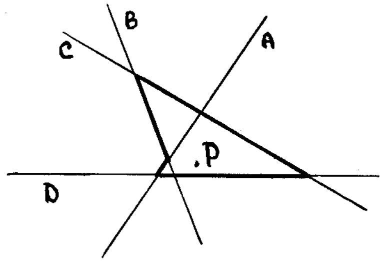

fig. 4

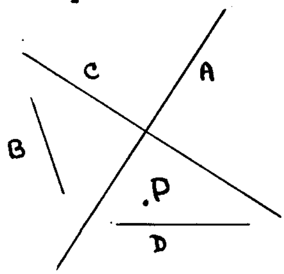

fig. 5

roughly the knowledge about the most probable location.

SUMMARY

If a location is determined by the intersection of three lines of position, there is a chance of only one-quarter that the true position lies inside the residual triangle. If the location is determined by the intersection of four lines of position, the chance is one-half that the true location lies inside the larger quadrangle formed by the four lines.