GWB
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Linear Least Squares Fit

Fit to $y = f(x) = ax + b$

$$\chi^2 = \sum_i \frac{(y_i - a x_i - b)^2}{\sigma_i^2}$$

$$\frac{\partial \chi^2}{\partial a} = -2 \sum_i x_i \frac{(y_i - a x_i - b)}{\sigma_i^2} = 0$$

$$\Rightarrow \quad a\overline{x^2} + b\bar{x} - \overline{xy} = 0 \qquad \left(\bar{x} = \sum_i \frac{x_i}{\sigma_i^2} \Big/ \sum_i \frac{1}{\sigma_i^2}\right)$$

$$\frac{\partial \chi^2}{\partial b} = -2 \sum_i (y_i - a x_i - b) = 0$$

$$\Rightarrow \quad a\bar{x} + b - \bar{y} = 0$$

$$a = \frac{\overline{xy} - \bar{x}\,\bar{y}}{\overline{x^2} - \bar{x}^2}, \qquad b = \frac{\overline{x^2}\,\bar{y} - \bar{x}\,\overline{xy}}{\overline{x^2} - \bar{x}^2}$$

and

$$f(\bar{x}) = a\bar{x} + b = \frac{\cancel{\bar{x}\,\overline{xy}} - \bar{x}^2\,\bar{y} + \overline{x^2}\,\bar{y} - \cancel{\bar{x}\,\overline{xy}}}{\overline{x^2} - \bar{x}^2}$$

$$f(\bar{x}) = \bar{y}$$

So finding $\bar{x}$ and $\bar{y}$ is as good as χ^2 fit
(as long as you don't need slope a)

[Note if slope a is known, $b = \bar{y} - a\bar{x}$
and $f(\bar{x}) = a\bar{x} + \bar{y} - a\bar{x} = \bar{y}$ as before.]