Date: 10 Feb 2011 13:01

This solution uses successive converging approximations, and it relies on the results provided by Frank E. Reed's On Line Lunar Computer here-after referred to as "FER's OLLC" ( see : http://www.historicalatlas.com/lunars/lunars v4.html ).

## STARTING DATA

Date : Feb 09 th, 2011
observations took place in the [15:30 UT - 18:30 UT] time frame,
Non moving Observer
Height of eye : 17 ft above sea level, tide was high and (almost) steady.
Temperature : $12^{\circ} \mathrm{C} / 53.6^{\circ} \mathrm{F}$
Pressure : $1017 \mathrm{Mb} / \mathrm{hPa} / 30.03 \mathrm{In} . \mathrm{Hg}$
All heights corrected for only Instrument error (Refraction, SD and Parallax need to be performed)
The horizon was exceptionally clear and sharp/well defined.

## PRELIMINARY STEP :

In order to simplify our matter without losing any accuracy, let's first average the values of the 4 sets of observations. This is a valid call and operation since all sets of observations are sufficiently "tight" in the sense of elapsed time between extreme observations for each set :

| Sun Observations (LL) | Watch time t11 $=00 \mathrm{~h} 02 \mathrm{~m} 56.2 \mathrm{~s} \mathrm{H} 11=11^{\circ} 19 . .40$ |  |
| :---: | :---: | :---: |
| Moon Observations (LL) : | Watch time $\mathrm{t} 12=01 \mathrm{~h} 42 \mathrm{~m} 35.6 \mathrm{~s} \mathrm{H} 12=57^{\circ} 19^{\prime} .20$ | ( $\mathrm{t} 12-\mathrm{t} 11=1 \mathrm{~h} 39 \mathrm{~m} 39.4 \mathrm{~s}$ ) |
| Jupiter Observations and | Watch time $\mathrm{t} 13=01 \mathrm{~h} 49 \mathrm{~m} 16.4 \mathrm{~s} \mathrm{H} 13=31^{\circ} 42^{\prime} .36$ | ( $\mathrm{t} 13-\mathrm{t} 11=1 \mathrm{~h} 46 \mathrm{~m} 20.2 \mathrm{~s}$ ) |
| Lunar Distances (NL) | Watch time t14 $=02 \mathrm{~h} 07 \mathrm{~m} 17.4 \mathrm{~s}$ D14 $=29^{\circ} 38^{\prime} .36$ | $(\mathrm{t} 14-\mathrm{t} 11=2 \mathrm{~h} 04 \mathrm{~m} 21.2 \mathrm{~s})$ |

In order to get a first approximation for the UT time at which the averaged Lunar Distance was performed, let us assume that the Geocentric Distance between Body Centers (which is our very well known "Cleared Distance") is actually equal to the Topocentric Distance between Body Centers, with such Topocentric Distance being approximately equal to the sum "D14 + (Geocentric) Moon Semi Diameter" .

Looking up through a table, we find that the Moon Geocentric SD for 12 UT on Feb 09, 2011 is equal to $14.8^{\prime}$ Therefore we assume that the Geocentric Distance between the Centers of Jupiter and Moon is $29^{\circ} 38^{\prime} .36+14^{\prime} .8=29^{\circ} 53^{\prime} 16$

Looking up a Moon-Jupiter Geocentric Distances Table, we find that on 09 Feb 2011 :
at UT=17h00m00.0s, Moon-Jupiter Distance $=29^{\circ} 47^{\prime} 00$, and
at UT=18h00m00.0s , Moon-Jupiter Distance $=30^{\circ} 15^{\prime} 72$

By interpolation, we find that when the geocentric distance between the Moon and Jupiter is equal to $29^{\circ} 53^{\prime} 16$, then $\mathrm{UT}=17 \mathrm{~h} 12 \mathrm{~m} 52.2 \mathrm{~s}$. We also see that the geocentric distance between Centers increases by +28 ' 72 per hour of elapsed UT.

NOTE : This PRELIMINARY STEP is ESSENTIAL since it enables us to GET A FIRST (AND SUFFICIENTLY ACCURATE) APPROXIMATION TO THE UT TIME AT WHICH THE (AVERAGE) LUNAR DISTANCE WAS OBSERVED. We should expect this First Approximation to be accurate to about $+/-1$ hour of time, which - under most cases - should be sufficient to start our successive approximations.

## APPROXIMATION 1

## STEP 1a:

Let us then assume that Actual averaged UT for Lunar Distances is equal to the (rough) approximation found hereabove.
We will therefore assume that UT14 $=17 \mathrm{~h} 12 \mathrm{~m} 52.2 \mathrm{~s}$.
Since we need to keep the same intervals between all observations, we need to also assume the following :

| Sun Observations (LL) | UT11 $=15 \mathrm{~h} 08 \mathrm{~m} 31.0 \mathrm{~s} \mathrm{H11}=11^{\circ} 19 . .40$ |  |
| :---: | :---: | :---: |
| Moon Observations (LL) : | UT12 $=16 \mathrm{~h} 48 \mathrm{~m} 10.4 \mathrm{~s} \mathrm{H12=57}^{\circ} 19.20$ | (UT12- UT11 = 1h39m39.4s) |
| Jupiter Observations and | UT13 $=16 \mathrm{~h} 4 \mathrm{~m} 51.2 \mathrm{~s} \mathrm{H13}=31^{\circ} 42^{\prime} .36$ | (UT13- UT11 = 1h46m20.2s) |
| Lunar Distances (NL) | UT14 $=17 \mathrm{~h} 12 \mathrm{~m} 52.2 \mathrm{~s}$ D $14=29^{\circ} 38^{\prime} .36$ | (UT14-UT11 = 2h04m21.2s) |

## STEP 1b :

From the data derived in STEP 1a hereabove, let us derive an Observer's Position from the 3 observed (averaged) heights and updated UT's. We get for Initial Position 1: UT14 = 17h12m52.2s N $46^{\circ} 17^{\prime} .2$ E $011^{\circ} 42^{\prime} .0$

NOTE : No need here, as well as for subsequent steps $2 b, 3 b, 4 b, \ldots$ to know in advance any approximate/DR position in order to derive the position given here-above. As long as we have 3 sights available and adequately spaced - which is the case here - recent software such as the "Vectors Method" by Andrés Ruiz, or the "Navigation Spreadsheets" by Peter Hakel easily derive such positions. I did not mention all other "Navigators" - and there are quite a few nowadays and certainly more and more - who independently and on their own derived and implemented such "magic" algorithms into powerful computing software.

## STEP 1c :

From the position obtained in STEP 1b hereabove, (UT14 $=17 \mathrm{~h} 12 \mathrm{~m} 52.2 \mathrm{~s} \mathrm{~N} 46^{\circ} 17^{\prime} .2 \mathrm{E} 011^{\circ} 42^{\prime} 0$ ) and with D14=29 $38^{\prime} .36$ (Near Limb) let us enter FER's OLLC. We get :

```
Error in Lunar: 26'
    Approximate Error in Longitude: 13'01.0' (which - more accurately - means that the "Error in
    Lunar" is actually 26'033333 ...)
```


## STEP 1d :

Let us transform this 26 '. 03 Error in Lunar value hereabove into a correction for time, with a Centers Distance hourly variation known to be equal to 28.172 (a result given in the last part of the PRELIMINARY STEP hereabove).
We then find a correction of +0 h 54 m 23.2 s to UT

## APPROXIMATION 2

## STEP 2a :

Let us then assume that Actual averaged UT for Lunar Distances is equal to UT14 + 0h54m23.2s (from our correction in STEP 1d). We will therefore assume that UT24 $=18 \mathrm{~h} 07 \mathrm{~m} 15.4 \mathrm{~s}$ (vs. previous value UT14=17h12m52.2s).
Since we need to keep the same intervals between all observations, we need to also assume the following :

| Sun Observations (LL) | UT21 $=16 \mathrm{~h} 02 \mathrm{~m} 54.2 \mathrm{~s} \mathrm{H} 11=11^{\circ} 19^{\prime} .40$ |  |
| :---: | :---: | :---: |
| Moon Observations (LL) : | UT22 $=17 \mathrm{~h} 42 \mathrm{~m} 33.6 \mathrm{~s} \mathrm{H} 12=57^{\circ} 19^{\prime} .20$ | (UT22- UT21 = 1h39m39.4s) |
| Jupiter Observations and | UT23 $=17 \mathrm{~h} 49 \mathrm{~m} 14.4 \mathrm{~s} \mathrm{H} 13=31^{\circ} 42^{\prime} .36$ | (UT23-UT21 = 1h46m20.2s) |
| Lunar Distances (NL) | UT24 $=18 \mathrm{~h} 07 \mathrm{~m} 15.4 \mathrm{~s}$ D14 $=29^{\circ} 38^{\prime} .36$ | (UT24-UT21 $=2 \mathrm{~h} 04 \mathrm{~m} 21.2 \mathrm{~s}$ ) |

## STEP 2b :

From the data derived in STEP 2a, let us derive an Observer's Position from the 3 observed (averaged) heights and updated UT's. We get for Updated Position 2 : UT24 $=18 \mathrm{~h} 07 \mathrm{~m} 15.4 \mathrm{~s}$ N $46^{\circ} 43^{\prime} .1 \mathrm{~W} 002^{\circ} 23^{\prime} .6$ (see Note concluding STEP 1b)

STEP 2c :
From the position obtained hereabove, (UT24 $=18 \mathrm{~h} 07 \mathrm{~m} 15.4 \mathrm{~s} \mathrm{~N} 46^{\circ} 43^{\prime} .1 \mathrm{~W} \mathrm{002}{ }^{\circ} 23 . \mathrm{'}^{\prime} 6$ ) and with D14=29 $38^{\prime} .36$ (Near Limb) let us enter FER's OLLC, we get :

Error in Lunar: 0.1'
Approximate Error in Longitude: $0^{\circ} 03.2^{\prime}$ (which - more accurately - means that the "Error in Lunar" is actually 0'064)

## STEP 2d :

Let us transform this $0^{\prime} .064$ Error in Lunar value hereabove into a correction for time, with a Centers Distance hourly variation equal to 28.172 (a result given in the last part of the PRELIMINARY STEP hereabove).
We then find a correction of $+0 \mathrm{~h} 00 \mathrm{m08.0s}$ to UT

## APPROXIMATION 3

## STEP 3a :

Let us then assume that Actual averaged UT for Lunar Distances is equal to UT24 + 0h00m08.0s (from our correction in STEP 2d). We will therefore assume that UT34 $=18 \mathrm{~h} 07 \mathrm{~m} 23.4 \mathrm{~s}$ (vs. previous value UT24=18h07m15.4s).
Since we need to keep the same intervals between all observations, we need to also assume the following :

| Sun Observations (LL) | UT31 $=16 \mathrm{~h} 03 \mathrm{~m} 02.2 \mathrm{~s} \mathrm{H11}=11^{\circ} 19.40$ |  |
| :---: | :---: | :---: |
| Moon Observations (LL) : | UT32 $=17 \mathrm{~h} 42 \mathrm{~m} 41.6 \mathrm{~s}$ H12=57¹9'. 20 | (UT32- UT31 = 1h39m39.4s) |
| Jupiter Observations | UT33 $=17 \mathrm{~h} 49 \mathrm{~m} 22.4 \mathrm{~s} \mathrm{H} 13=31^{\circ} 42^{\prime} .36$ | (UT33-UT31 = 1h46m20.2s) |
| and |  |  |
| Lunar Distances (NL) | UT34 $=18 \mathrm{~h} 07 \mathrm{~m} 23.4 \mathrm{~s}$ D14 $=29^{\circ} 38^{\prime} .36$ | (UT34-UT31 = 2h04m21.2s) |

## STEP 3b :

From the data derived in STEP 3a, let us derive an Observer's Position from the 3 observed (averaged) heights and updated UT's. We get for Updated Position 3 : UT34 $=18 \mathrm{~h} 07 \mathrm{~m} 23.4 \mathrm{~s}$ N $46^{\circ} 43^{\prime} .1 \mathrm{~W} 002^{\circ} 25^{\prime} .4$ (see Note concluding STEP 1 b )

STEP 3c : From the position obtained hereabove, (UT34 $=18 \mathrm{~h} 07 \mathrm{~m} 23.4 \mathrm{~s} \mathrm{~N} 46^{\circ} 43^{\prime} .1 \mathrm{~W} 002^{\circ} 25^{\prime} .3$ ) and with D14=29 $38^{\prime} .36$ (Near Limb) let us enter FER's OLLC, we get :

## Error in Lunar: $\mathbf{0 '}^{\prime}$

Approximate Error in Longitude: $0^{\circ} 01.4^{\prime}$ (which - more accurately - means that the "Error in Lunar" is actually $0^{\prime} 028$ )
Let us transform this $0^{\prime} .028$ Error in Lunar value hereabove into a correction for time, with a Centers Distance hourly variation known to be equal to $28 . ' 72$ (a result given in the last part of the PRELIMINARY STEP hereabove).
We then find a correction of $+0 \mathrm{hOOm03.6s}$ to UT
No need any longer to continue our iteration any further. Nonetheless, lest us simply shift our last position to the West by an amount equivalent to 3.6 seconds of time, i.e. shift our last Longitude by 0 ' 9 to the West.

## Therefore our final result is :

UT of Sextant Average Distance $=18 \mathrm{~h} 07 \mathrm{~m} 27.0 \mathrm{~s}$, and Observer's Position at this time was N $46^{\circ} 43^{\prime} 1 \mathrm{~W} 002^{\circ} \mathbf{2 6}^{\prime} 2$

Where did I actually stand ? My actual position, as derived from Google Earth, was : N $46^{\circ} 42^{\prime} 9 \mathrm{~W} 002^{\circ} 23^{\prime} 7$ (I was standing
next to the threshold of RWY 04)

The position derived with my software is : UT of Sextant Average Distance $=18 \mathrm{~h} 07 \mathrm{~m} 33.8 \mathrm{~s}$, and Observer's Position at his time is $\mathrm{N} 46^{\circ} 43^{\prime} 2 \mathrm{~W} 002^{\circ} 28^{\prime} 1$. When I enter these results into FER's OLLC I am observing a difference of 2 ". 2 on the Lunar Distances between Frank's results and mine, a value consistent with our observed 1'. 9 difference in Longitude.

