Sun – Moon – Jupiter observations, 9 Feb 2011

Averages of altitudes and distances:

WT	$00^h02^m56^s$	Sun LL	11°19,4’
	$01^h42^m36^s$	Moon LL	57°19,2’
	$01^h49^m16^s$	Jupiter	31°42,4’
	$02^h07^m17^s$	Jupiter – Moon near	29°38,4’

The moon’s altitude changes -18,4’/04^m32^s; during the time interval until the lunar observation of 24^m41^s this gives -18,4’ x $1481^s/272^s$, assuming no calculator is available:

1481	log	3,171
272	log	2,435
		0,736
18,4’	log	1,265
	log	2,001

This log equals 100’ = 1°40’, resulting in a moon altitude at the time of the lunar of 55°39’.

The same calculation for Jupiter, -31,1’/03^m57^s, during 18^m01^s gives -31,1’ x $1081^s/237^s$

1081	log	3,034
237	log	2,375
		0,659
31,1’	log	1,493
	log	2,152

equaling 142’ = 2°22’, resulting in Jupiter’s altitude at the time of lunar 29°20’.

Using Stark’s tables with above extrapolated altitudes gives a cleared lunar distance of 30°19,7’ corresponding to

GMT	$18^h08^m09^s$
WT	$02^h07^m17^s$
WE_{GMT}	$16^h00^m52^s$

Details of the lunar clearing are given on page 3.

The sextant altitude of the Sun corrected according to the tables in NA gives an observed altitude of 11°27,0’. With an assumed latitude of 47° N (Nantes area), this altitude is used to find the local hour angle and the azimuth. Declination as per NA at GMT $16^h03^m48^s$ of 14°38,3’ S.

Latitude	47°	log sec	0,16622		log sec 0,16622
Polar distance	104°38,3’	log cosec	0,01433	hav 0,62636	
Altitude	11°27,0’				log sec 0,00873
sum	162°65,3’				
half	81°32,7’	log cos	9,16741		
Altitude	11°27,0’				
remainder	70°05,7’	log sin	9,97325		
		log hav	9,32121		
Latitude-Alt.	35°33’			hav 0,09320	
				hav 0,53316	log hav 9,72686
					log hav 9,90181

The log hav of 9,32121 gives LHA 54°28,9’. This local hour angle corresponds to local apparent time

LAT	$3^h37^m56^s$ pm
	12^h
EoT	14^m12^s
LMT	$15^h52^m08^s$
GMT	$16^h03^m48^s$
time diff	11^m40^s which equals a longitude of 2°55’ W.

The log hav of 9,90181 gives the sun’s azimuth N 127° W, or 233°. Now we have a sun LOP passing through 47° N 2°55’ W at right angle to the azimuth, or 143°, on (or near) which the observer was located. Crossing with a moon LOP will give the place.

Correcting the sextant altitude of the moon observation as per tables in NA gives an observed altitude of 57°58,9'.

WT	$01^h42^m36^s$				
WE_{GMT}	$16^h00^m52^s$				
GMT	$17^h43^m28^s$				
	5°24,0' (13,6')				
	10°22,3'				
	9,9'				
GHA	15°56,2'				
Longitude	2°55'				
LHA	13°01,2'	log hav	8,10905		
Latitude	47°	log cos	9,83378		
	16°33,3' (9,1')				
	6,6'				
Declination	16°39,9'	log cos	9,98136		
		log hav	7,92419	hav	0,00840
Latitude-Dec.	30°20,1'			hav	0,06846
				hav	0,07686

This last haversine gives the zenith distance, but with a properly arranged table the altitude can be read directly as

Altitude calc.	57°48,6'
Altitude obs.	57°58,9'
intercept	10,3' towards

The azimuth is easily found with the use of ABC-tables:

A	4,64	
B	-1,33	
C	3,31	giving the azimuth as 204°

Now it remains to plot the two LOPs, or to calculate the fix using a traverse table:

We have a right-angled triangle with one corner at the assumed latitude 47° N and the calculated longitude 2°55' W. From this point we have the intercept of 10,3' in direction 204°, then a right angle, then a side with unknown length in direction 204°-90°=114°, crossing the sun LOP (hypotenuse) that starts in the given position and runs in direction 143°. The crossing angle is found to be 143°-114°=29° and thus the length of the hypotenuse can be calculated as 10,3'/sin 29°, or more conveniently as 10,3' x cosec 29° if logarithms are used.

10,3'	log	1,013
29°	log cosec	0,314
	log	1,327

This logarithm corresponds to a distance of 21,2', sailed on course of 143°, from the given start position. From the traverse table we find a dLat of 17' S and a departure of 12,8' E. The departure is converted to a dLong of 18,7' E. We now get the position of the fix as

Latitude	46°43' N
Longitude	2°36' W

Clearing of lunar according to Stark's method and table. Those who have the form available will be able to follow and check the calculations.

Height of eye 17 feet, HP 54,6'

29°20,0'	55°39,0'		
4,0'	10,8'		
29°16,0'	55°49,8'	29,12'	534,7
	29°16,0'	0,91'	17,0
	26°33,8'	1,73'	0,0
	31,8'	31,76'	551,7
	27°05,6'		

15,1'		
29°38,4'		
29°53,5'		
26°33,8'		
3°19,7'		3,07397
56°27,3'		0,65033
		3,72430
		1,86215
		551(,7)
27°05,6'	1,26069	1,86767
	0,09594	1,26069
	1,16475	~~0,60698~~

The last K-value (which equals a negative log hav) of 1,16475 gives the cleared lunar of 30°19,7'.

Calculation of true distances:

18^h GMT

	45°47,4'	
	19°56,6'	
	25°50,8'	1,30087
16°42,4'		0,01873
00°21,0'		0,00001
16°21,4'	1,69387	1,31961
	1,31961	0,15303
30°15,8'	~~0,37426~~	1,16658

19^h GMT

	60°49,4'	
	34°29,3'	
	26°20,1'	1,28490
16°51,4'		0,01907
00°21,2'		0,00001
16°30,2'	1,68617	1,30398
	1,30398	0,15069
30°44,5'	~~0,38219~~	1,15329

3,9'	1,7891	
28,7'	0,9223	
	0,8668	resulting in an observed GMT $18^h08^m09^s$

This first result may benefit from a second round of calculations because we have used linearly extrapolated altitudes of Moon and Jupiter for the lunar clearing. Any error in the used rate of change are magnified in the extrapolation, we also know that the rate of change is not constant. On the other hand the lunar clearing is not very sensitive to minor errors in the altitudes. Let us see what happens.

Thus, calculate the altitudes of Moon and Jupiter at the time of the lunar, GMT $18^h08^m09^s$, and at the found position, 46°43' N 2°36' W. First the Moon:

	19°56,6' (13,7')				
	1°56,7'				
	1,9'				
GHA	21°55,2'				
Longitude	2°36'				
LHA	19°19,2'	log hav	8,44959		
Latitude	46°43'	log cos	9,83608		
	16°42,4' (9,0')				
	1,3'				
Declination	16°43,7'	log cos	9,98122		
		log hav	8,26689	hav	0,01851
Latitude-Dec.	29°59,3'			hav	0,06693
				hav	0,08544

This corresponds to an altitude of 56°00,5'. It has to be "uncorrected" to sextant altitude:

Calc.alt	56°00,5'
2nd corr in NA	-2,5'
1st corr in NA	-42,3'
App. Alt	55°15,7'
1st corr	+42,8'
2nd corr	+ 2,5'
Res.alt	56°01,0'

We see that the resulting altitude is 0,5' too high, thus we deliberately reduce the apparent altitude by the same amount and check again:

App. Alt	55°15,2'
1st corr	+42,8'
2nd corr	+ 2,5'
Res.alt	56°00,5'

Now the result agrees with the calculated altitude and by applying the dip correction of 4,0' we get the sextant altitude of Moon's LL as 55°19,2'.

Same procedure for Jupiter:

	45°47,4' (2,0')				
	2°02,3'				
	0,3'				
GHA	47°50,0'				
Longitude	2°36'				
LHA	45°14,0'	log hav	9,16994		
Latitude	46°43'	log cos	9,83608		
	0°21,0' (0,2')				
	0,0'				
Declination	0°21,0'	log cos	9,99999		
		log hav	9,00601	hav	0,10139
Latitude-Dec.	46°22,0'			hav	0,15498
				hav	0,25637

This corresponds to an altitude of 29°09,6'. It has to be "uncorrected" to sextant altitude:

Calc.alt	29°09,6'	
corr in NA	+ 1,7'	
App. Alt	29°11,3'	We see at a glance that this apparent altitude gives correct result when corrected.
Dip	+ 4,0'	
Sextant alt	29°15,3'	

Comparing with the extrapolated values used earlier, there is a 20’difference in Moon altitude and 5’for Jupiter. Now a second round in Stark’s tables:

29°15,3’	55°19,2’		
4,0’	10,8’		
29°11,3’	55°30,0’	29,36’	532,5
	29°11,3’	0,91’	17,0
	26°18,7’	1,73’	0,0
	32,0’	32,00’	549,5
	26°50,7’		

15,1’		
29°38,4’		
29°53,5’		
26°18,7’		
3°34,8’		3,01068
56°12,2’		0,65389
		3,66457
		1,83228(,5)
		549(,5)
26°50,7’	1,26854	1,83778
	0,10368	1,26854
	1,16486	~~0,56924~~

The last K-value of 1,16486 gives the cleared lunar of 30°19,5’. A 0,2’ difference from the previous result.

3,7’	1,8120	
28,7’	0,9223	
	0,8897	resulting in an observed GMT $18^h07^m44^s$, and a new watch error:

GMT	$18^h07^m44^s$
WT	$02^h07^m17^s$
WE_{GMT}	$16^h00^m27^s$

The time sight of the Sun is now reworked with the latitude found earlier:

Latitude	46°43’	log sec	0,16392
Polar distance	104°38,3’	log cosec	0,01433
Altitude	11°27,0’		
sum	162°48,3’		
half	81°24,2’	log cos	9,17457
Altitude	11°27,0’		
remainder	69°57,2’	log sin	9,97286
		log hav	9,32568

The log hav of 9,32568 gives LHA 54°47,1’. This local hour angle corresponds to local apparent time

LAT	$3^h39^m08^s$ pm	
	12^h	
EoT	14^m12^s	
LMT	$15^h53^m20^s$	
GMT	$16^h03^m23^s$	(WT + WE_{GMT} = $00^h02^m56^s$ + $16^h00^m27^s$)
time diff	10^m03^s	which equals a longitude of 2°30,8’ W.

Now calculate a new LOP from the Moon altitude observation:

WT	$01^h42^m36^s$				
WE_{GMT}	$16^h00^m27^s$				
GMT	$17^h43^m03^s$				
	5°24,0’ (13,6’)				
	10°16,3’				
	9,9’				
GHA	15°50,2’				
Longitude	2°30,8’				
LHA	13°19,4’	log hav	8,12896		
Latitude	46°43’	log cos	9,83608		
	16°33,3’ (9,1’)				
	6,6’				
Declination	16°39,9’	log cos	9,98136		
		log hav	7,94640	hav	0,00884
Latitude-Dec.	30°03,1’			hav	0,06721
				hav	0,07605

Altitude calc.	57°59,0’
Altitude obs.	57°58,9’
intercept	0,1’ away

With this small intercept there is no reason to bother with the azimuth calculation and plotting a LOP; we can consider both the sun and the moon altitudes to be valid at the found position of

Latitude 46°43’ N
Longitude 2°31’ W

We have also found that the watch is **$16^h00^m27^s$ slow** on GMT.

Compared with the published known position of N4642.9W00223.7, the latitude is spot on and the longitude only 7’ or 28^s off.

The “improvement“ of 5’ longitude achieved compared to the initial solution, a little more than three miles in distance, must be regarded as insignificant for ocean navigation. It is based on a change of the cleared lunar of only 0,2’, thus in the same range as the measurement uncertainty itself. A longitude based on only one short series of lunar distances must be handled with a certain amount of scepticism, and the prudent navigator will use the information accordingly.

The benefit of this recalculation is rather that it confirms the first solution, i.e. no arithmetical blunders seem present. Although many of the logarithms have been reused, it gives a reassurance that the result is trustworthy.