Position and time of a vessel by celestial observations

Andrés Ruiz
Navigational Algorithms
http://sites.google.com/site/navigationalalgorithms/

The observations

t – time (WT, UTC, UT1, TT)

H – Altitude of a celestial body

LD – Lunar distance

The equations

For an altitude

H = H(B, L, Dec, GHA) = H(t, B, L)

$$H = \arcsin\left(\sin Dec \sin B + \cos Dec \cos B \cos(GHA + L)\right)$$

$$dH = \frac{\partial H}{\partial B} dB + \frac{\partial H}{\partial L} dL + \frac{\partial H}{\partial t} dt$$

$$dH = Ho - H$$

The analytical expressions for altitude derivatives are:

$$\frac{\partial H}{\partial B} = \frac{1}{\cos H}\left(\sin Dec \cos B - \cos Dec \sin B \cos(GHA + L)\right)$$

$$\frac{\partial H}{\partial L} = \frac{-1}{\cos H}\left(\cos Dec \cos B \sin(GHA + L)\right)$$

$$\frac{\partial H}{\partial t} = \frac{H_{i+1} - H_i}{t_{i+1} - t_i} = \frac{H(t_{i+1}, B_i, L_i) - H(t_i, B_i, L_i)}{t_{i+1} - t_i}$$

For a lunar

LD = LD(t, B, L)

$$dLD = \frac{\partial LD}{\partial B} dB + \frac{\partial LD}{\partial L} dL + \frac{\partial LD}{\partial t} dt$$

$$dLD = LDo - LDc$$

$$\frac{\partial LD}{\partial B} = \frac{LD_{i+1} - LD_i}{B_{i+1} - B_i} = \frac{LD(t_{i+1}, B_i, L_i) - LD(t_i, B_i, L_i)}{B_{i+1} - B_i}$$

$$\frac{\partial LD}{\partial L} = \frac{LD_{i+1} - LD_i}{L_{i+1} - L_i} = \frac{LD(t_{i+1}, B_i, L_i) - LD(t_i, B_i, L_i)}{L_{i+1} - L_i}$$

$$\frac{\partial H}{\partial t} = \frac{LD_{i+1} - LD_i}{t_{i+1} - t_i} = \frac{LD(t_{i+1}, B_i, L_i) - LD(t_i, B_i, L_i)}{t_{i+1} - t_i}$$

The solution

In matrix form for three altitudes and a lunar:

$$\begin{bmatrix} dH_1 \\ dH_2 \\ dH_3 \\ dLD \end{bmatrix} = \begin{bmatrix} \frac{\partial H_1}{\partial B} & \frac{\partial H_1}{\partial L} & \frac{\partial H_1}{\partial t} \\ \frac{\partial H_2}{\partial B} & \frac{\partial H_2}{\partial L} & \frac{\partial H_2}{\partial t} \\ \frac{\partial H_3}{\partial B} & \frac{\partial H_3}{\partial L} & \frac{\partial H_3}{\partial t} \\ \frac{\partial LD}{\partial B} & \frac{\partial LD}{\partial L} & \frac{\partial LD}{\partial t} \end{bmatrix} \begin{bmatrix} dB \\ dL \\ dt \end{bmatrix}$$

An initial value for the unknowns is needed: B0, L0, t0

The improved solution is:

B = B0 + dB

L = L0 + dL

t = t0 + dt

Iterate until error < epsilon

The motion of the observer

B – latitude

L – longitude

R – course

V – speed

B = B(Bo, Lo, R, V, t-to)

L = L(Bo, Lo, R, V, t-to)

t → Observation

to → Celestial position(Bo, Lo) and Motion(R, V)

d = V(t-to) = VΔt

B = Bo + d cos R = Bo + VΔt cos R

$$L = Lo + d\,\frac{\sin R}{\cos Bm} = Lo + \mathrm{V}\Delta t\,\frac{\sin R}{\cos Bm}$$

Bm = (Bo+B)/2 = Bo + ½VΔt cos R

Changes in R,V for multiple leg tracks: the simplest way to take it into account, is to introduce the adequate equation of restriction for the motion to each observation equation:

B = Bo + ΣΔB

L = Lo + ΣΔL

Generalization

If the unknowns are: time, latitude and longitude, the general differential equation of the measured variable F is:

$dF = \partial F/\partial t\ dt + \partial F/\partial B\ dB + \partial F/\partial L\ dL$

or in finite differences:

$\Delta F = \partial F/\partial t\ \Delta t + \partial F/\partial B\ \Delta B + \partial F/\partial L\ \Delta L$

navigational data from different sources can be combined, F =

1. Altitude
2. Lunar distance
3. Star-star distance
4. Azimuth (measured on land with an astronomical instrument)
5. Bearing
6. Distance
7. Horizontal angle
8. Radio-bearing
9. ...

dF = Fobserved – Fcalculated

and the partial derivatives are obtained with finite differences:

$\partial F/\partial x = (F_{i+1} - F_i)/(x_{i+1} - x_i)$

$$\begin{bmatrix} 1 \\ 2 \\ dH \\ \\ n \end{bmatrix} = \begin{bmatrix} \frac{\partial F}{\partial B} & \frac{\partial F}{\partial L} & \frac{\partial F}{\partial t} \\ \frac{\partial F}{\partial B} & \frac{\partial F}{\partial L} & \frac{\partial F}{\partial t} \\ \frac{\partial H}{\partial B} & \frac{\partial H}{\partial L} & \frac{\partial H}{\partial t} \\ \\ n & & \end{bmatrix} \begin{bmatrix} dB \\ dL \\ dt \end{bmatrix}$$

If number of equations n > 3, the method of the least squares is used to solve the system.