Instructions for Line of Position Navigation Sight Reduction using Stark Lunar Tables

Robin Stuart
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The instructions below refer to line of position (LOP) sight reduction using *Stark Tables for clearing lunar distances*, B. Stark, Starpath Publications, Seattle, WA (2010).

The terms "Greater of" and "Lesser of" in the following description refer to the magnitudes or absolute values of the quantities concerned.

Altitude

Bring down the longitude of the assumed position (AP) and Greenwich hour angle (GHA) of the celestial object placing the lesser under the greater and add them. Convert the result to a value between 0° and 360° by adding or subtracting multiples of 360°. This is the local hour angle (LHA).

If the LHA is greater than 180° then subtract 180° and copy its K function from the tables. If the LHA is less than 180° use it in K table directly. Bring down the greater of the AP latitude and declination of the celestial object and write the lesser of the two below it. Ignoring their signs, enter the tables and copy their log Dec.functions below the K previously recorded. Add all three values together.

Find the difference between latitude of the AP and declination taking due care of the relative signs but ignoring any negative sign in the result. Record its K value from the tables to the left of the sum of three values calculated above. Copy the lesser of these under the greater and subtract it. Use the remainder to enter the Gaussians table. Copy the Gaussian under the other (lesser) value and subtract it from the logarithm above. This is the K of the object's Zenithal Distance (ZD). Subtract the ZD from 90° ($89^\circ 60'$) to give the altitude, H.

To avoid confusion strike through the number used to look up the Gaussians as soon as it is no longer needed.

Azimuth

Subtract the object's declination from 90° (89° 60′) to obtain its co-declination. Bring down the greater the altitude and AP latitude and write the lesser of the two below. Ignoring signs find their log Dec. find their log Dec. functions and add them together.

Find the difference between altitude and AP latitude taking due care of the relative signs but discard any negative sign in the result. Label this (H~L).

Bring down the greater the co-declination and (H~L) and write the lesser of the two below. Form the sum and difference of the two numbers. Ignoring any negative signs in the results look up the K functions of the two resulting angles and add the functions together. Take half of the sum. Bring down the sum of log Dec's previously calculated and subtract it. Discard any negative sign. This is K of the azimuthal angle, A. If the LHA is greater than 180° then A is the required true azimuth, otherwise subtract it from 360°.

Line of Position Sight Reduction using Stark Tables

Inputs								
Assumed Position								
Latitude	N	35 °	30.0 '					
Longitude	W	9 °	30.0 '					
Declination	N	38 °	40.2 '					
GHA		62 °	16.0 '					
Altitude								
Greater of Lon. & GHA		62 °	16.0 '					
Lesser of Lon. & GHA		-9 °	30.0 '					
add → LHA		52 °	46.0 '					
		°	'					
		52 °	46.0 '			K	0.70450	add
Greater of Lat.& Dec.		38 °	40.2 '			log Dec.	0.10749	
Lesser of Lat.& Dec.		35 °	30.0 '			log Dec.	0.08931	
subtract		3 °	10.2 '	K	3.11622		0.90130	
		89 °	60.0 '		0.90130	or	0.00264	
Zenithal Distance, ZD		41 °	37.9 '	K	~~2.21492~~		0.89866	
Altitude, H		**48 °**	**22.1 '**					
Azimuth								
		89 °	60.0 '					
Dec.		38 °	40.2 '					
subtract → coDec.		51 °	19.8 '					
Greater of H & Lat.		48 °	22.1 '			log Dec.	0.17761	add
Lesser of H & Lat.		35 °	30.0 '			log Dec.	0.08931	
subtract → (H~L)		12 °	52.1 '				0.26693	
Greater of coDec & (H~L)		51 °	19.8 '					
Lesser of coDec & (H~L)		12 °	52.1 '					
coDec - (H~L)		38 °	27.6 '			K	0.96464	add
coDec + (H~L)		64 °	11.9 '			K	0.54918	
							1.51381	
						half	0.75691	
							0.26693	
A		69 °	21 '			K	0.48998	subtract
Azimuth, Z		**290 °**	**39 '**	If LHA > 180° then Z = A				
				Otherwise Z = 360° - A				

Formulas

For Line of Position (LOP) sight reduction the relevant formulas are

$$\operatorname{hav}\hat{h} = \operatorname{hav}(L - \delta) + \cos L \cos\delta \operatorname{hav}(\mathrm{LHA}) \qquad (1)$$

and

$$\operatorname{hav} A = \frac{\sqrt{\operatorname{hav}\left(\hat{\delta} - (h - L)\right)\operatorname{hav}\left(\hat{\delta} + (h - L)\right)}}{\cos h \cos L} \qquad (2)$$

where

A is the azimuthal angle

δ is the celestial object's declination and $\hat{\delta} = 90^\circ - \delta$ is its codeclination

h is the celestial object's altitude and and $\hat{h} = 90^\circ - h$ is its coaltitude

L is the observer's assumed latitude and LHA is the local hour angle

The haversine function is defined by

$$\operatorname{hav}\theta = \frac{1}{2}(1 - \cos\theta) = \sin^2\frac{\theta}{2}$$

For an angle θ the K and log Dec. functions in the Stark Tables are

$$\mathrm{K} = \log_{10}\frac{1}{\operatorname{hav}\theta}; \qquad \log \mathrm{Dec.} = \log_{10}\frac{1}{\cos\theta}.$$

The Gaussian logarithm table lists values of

$$\mathrm{Glog}_{10}\, x = \log_{10}\left(1 + 10^{-x}\right)$$

which has the property that

$$\log_{10}\frac{1}{y} - \mathrm{Glog}_{10}\left(\log_{10}\frac{1}{x} - \log_{10}\frac{1}{y}\right) = \log_{10}\frac{1}{x + y}$$

and is used in the evaluation of formula (1).

Both formulas (1) and (2) follow from the application of the haversine distance formula to the standard navigational triangle. Formula (2) is obtained by means of the additional relation

$$\operatorname{hav} A - \operatorname{hav} B = \sqrt{\operatorname{hav}(A + B)\operatorname{hav}(A - B)}$$