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<http://www.usno.navy.mil/USNO/astronomical-applications/data-services/cel-nav-data>

For N 33° 22.0', W 97° 14.1' at 2011 Dec 14 0000Z

**Celestial Navigation Data for 2011 Dec 14 at 0:00:00 UT**

For Assumed Position: Latitude N 33 22.0  
Longitude W 97 14.1

Object	GHA	Almanac Data				Altitude Corrections			
		Dec	Hc	Zn	Refr	SD	PA	Sum	
JUPITER	53 21.4	N10 26.8	+43 46.1	109.3	-1.0	0.4	0.0	-0.6	
POLARIS	40 15.0	N89 19.2	+33 44.1	0.7	-1.5	0.0	0.0	-1.5	

From John Karl's "Celestial Navigation in the GPS Age", we have the two fundamental formulas:

$$\sin H_c = \sin L \sin d + \cos L \cos d \cos LHA$$

$$\cos A = (\sin d - \sin L \sin H_c) / (\cos L \cos H_c)$$

Now, let's do some armchair celestial navigation.

Assume that the Hc (altitude of the object above the horizon) given in the above Almanac Data is a measured altitude, corrected for dip, refraction, sextant error, parallax and semi-diameter. That is, we have Ho (H observed, the corrected measurement), which we might have obtained from a sextant sighting.

To find our location, the process is (1) select an Assumed Position (AP); (2) from the almanac, find the GHA and Dec for the observation date/time. (3) From this information, find a computed Hc and azimuth. (4) compare the results of (3) with the observed Ho. (5) deduce the difference in distance and direction from your AP.

Step 1. AP= N 33°, W 97°

Step 2. At 2011 Dec 14 0000Z :

Jupiter d (declination) N 10° 26.8' ,

$$LHA \text{ (local hour angle)} = GHA - \text{longitude} = 53^\circ 21.4' - 97^\circ + 360 = 316^\circ 21.4'$$

At 2011 Dec 14 0000Z (typically this would be at a different time)

Polaris d N 89° 19.2'

$$LHA = 303^\circ 15.0'$$

Step 3. Find the Hc and Zn for both observations, using the above equations. Note that L is the assumed latitude, N 33° .

Jupiter

$$Hc = \text{asin}(\sin(33)\sin(10.4467) + \cos(33)\cos(10.4467)\cos(316.3567)) = 44.0749^\circ$$

$$H_c = 44^\circ 04.5'$$

$$A = \arccos[(\sin(10.4467) - \sin(33)\sin(44.0749)) / (\cos(33)\cos(44.0749))]$$

$$A = 109.1^\circ, Z_n = A = 109.1^\circ \text{ (Northern hemisphere, LHA} > 180)$$

$$\text{Polaris } H_c = 33^\circ 22.3', Z_n = 0.7^\circ$$

Step 4. Find intercept and  $Z_n$  for the AP N  $33^\circ$ , W  $97^\circ$  at 2011 Dec 14 0000Z

Jupiter	Ho $43^\circ 46.1'$ Hc - $44^\circ 04.5'$ -----	Polaris	Ho $33^\circ 44.1'$ Hc - $33^\circ 22.3'$ -----
	- 18.4 (or Away)/109.1 $^\circ$		+ 21.8 (or Toward)/00.7 $^\circ$

Step 5. According to Step 4, we are some distance from our AP. To find the coordinates, we could plot the two LOPs, or solve directly for the intersection of the two lines.

One convenient way is to use Henning Umland's

<http://www.celnav.de/fixcalc.htm>

It finds N  $33^\circ 21.95'$ , W  $97^\circ 14.22'$ , which is a reasonable match to the original GPS coordinates. (It puts you somewhere in the ballpark, literally.)

If you want to directly solve for the coordinates:

$$\Delta L = \frac{I_2 \sin(Z_{n_1}) - I_1 \sin(Z_{n_2})}{\sin(Z_{n_1} - Z_{n_2})} \text{ in arc minutes}$$

$$\Delta \lambda = \frac{I_1 \cos(Z_{n_2}) - I_2 \cos(Z_{n_1})}{\sin(Z_{n_1} - Z_{n_2}) \cos(L)}$$

Here we find

$$\Delta L = 21.9'$$

$$\Delta \lambda = -14.2'$$

and the fix is (adding in the AP, and noting west is negative) :

$$\text{N } 33^\circ 21.9', \text{ W } 97^\circ 14.2'$$