JotM's method

Time of transit (LAN) and ex-meridian fix from 3 or more random sights near transit

As most non-Dutch have trouble pronouncing Jaap van der Heide, let's refer to this method as "JotM's method" (JotM being an acronym of the literal translation of my name into English).

Abstract

In order to determine time of transit of a heavenly body one needs at least 3 randomly taken sights taken within the time span of half an hour before to half an hour after transit, not necessarily spread to before and after time of transit, but a bit spread among themselves. From at least 2 of the thus available pairs of sights – for each pair consider the sights taken at t_I and t_{II} with respect to a chosen origin; the time of the last sight taken seems like an appropriate origin – one determines points supporting a straight line on a graph which are at the coordinates $\left(\left(\frac{t_I+t_{II}}{2}\right), \left(H_{t_{II}} - H_{t_I}\right)\right)$. Through these "supporting points" one draws or determines (algebraically) the said straight line and at the value of t (horizontal axis) where the line crosses "0" (zero) on the vertical axis (or algebraically, where the resulting value of the function is zero), is time of transit. From the slope of the determined line the rate of change of the altitude of the heavenly body over time can be determined and with that the altitude of the heavenly body at transit, based on either one of the sights taken, from which a fix of the observers position can be calculated.

Method

According to the explanation on the tables "altitude factors" and "change of altitude in given time from meridian transit" in the American Practical Navigator ["Bowditch"], within certain boundaries the change of altitude of a heavenly body near transit can be described as

$$C = \frac{a}{60} \cdot t^2 \qquad (1)$$

in which *C* is the change of altitude in minutes of arc, *a* is the "altitude factor" in seconds of arc and *t* the meridian angle in minutes of time.

The "altitude factor" *a* can be determined by means of

$$a = 1.9635'' \cos L \cos d \csc(L{\sim}d) \qquad (2)$$

in which *a* is the change of altitude in one minute from meridian transit (expressed in seconds of arc), *L* is the latitude of the observer and *d* is de declination of the celestial body.
I will not be using "a" (equation (2)), except for either a rough check on the results or improved accuracy using the graphical method for determination of time of transit I will come to later.

The boundaries to the validity of this formulae – according to Bowditch – are:

- Altitude between 6° and 86°
- Latitude <60°

- Declination <63°
- "t" smaller than the limits of table "change of altitude in given time from meridian transit" (-> do not wander off more than about half an hour)

Beyond these boundaries the formulae yield reduced accuracy.

Now with the above the altitude of a heavenly body around upper transit becomes

$$H(t) = H_{transit} - \frac{a}{60} \cdot t^2 \tag{3}$$

in which *t* is the meridian angle with respect to transit in minutes of time (at transit, *t* = 0), $H_{transit}$ is the (unknown) altitude of the heavenly body at transit in minutes of arc and *H(t)* the altitude of the heavenly body at *t* minutes of time with respect to transit.

Formula (3) is actually a specific specimen of a formula that can more generally be written as

$$y = \alpha x^2 + \beta \tag{4}$$

The first differential of (4) being

$$y' = 2\alpha x \tag{5}$$

From (4) and (5) it can be found that for any pair x_1 and x_2:

$$dx = x_2 - x_1 \tag{6}$$

$$dy = (\alpha {x_2}^2 + \beta) - (\alpha {x_1}^2 + \beta) = \alpha({x_2}^2 - {x_1}^2) \tag{7}$$

$$\frac{dy}{dx} = \alpha \frac{({x_2}^2 - {x_1}^2)}{(x_2 - x_1)} = 2\alpha \frac{(x_2 + x_1)(x_2 - x_1)}{2(x_2 - x_1)} = 2\alpha \left(\frac{x_2 + x_1}{2}\right) \tag{8}$$

and

$$y'(x_1) = 2\alpha x_1 \tag{9a}$$

$$y'(x_2) = 2\alpha x_2 \tag{9b}$$

$$\bar{x} = \frac{x_1 + x_2}{2} \tag{10}$$

$$\overline{y'}(x_1, x_2) = \frac{2\alpha x_1 + 2\alpha x_2}{2} = 2\alpha \left(\frac{x_1 + x_2}{2}\right) \tag{11}$$

From which follows

$$\overline{y'} = y'(\bar{x}) = \frac{dy}{dx} \tag{12}$$

When $\alpha = -\frac{a}{60}$ is substituted into (3) the altitude formula takes on the form

$$H(t) = \alpha t^2 + H_{transit} \tag{13}$$

and

$$H'(t) = 2\alpha \cdot t \tag{14}$$

As already known, but also to be seen from (), *H'(t)* – which is a linear function – is 0 (zero) at transit. Pairs of sights can now be used to construct a linear function – or straight line – and from that the value of *t* (time of transit) for which *H'(t)* is 0. To find $t_{transit}$ one needs at least two supporting points, describing the line sought. From (12) can be seen that a single point on this line can be found from any combination of two times at which a sight is taken. The number *k* of combinations available from *n* sights taken being

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \tag{15}$$

From (15) can be found 2 sights render 1 pair, 3 sights render 3 pairs, 4 sights render 6 pairs etc. As every pair renders one point on the line sought and at least 2 point are needed to define a line it is clear at least 3 sights are needed. Although more supporting points can thus be obtained by using all possible combination, the resulting support points are not independent and not evenly spread through the range in time that is evaluated. It therefore seems it is better only to make combinations of adjacent observations. This means that from *n* observations would result in *n-1* combinations and thus *n-1* support points.

PROCEDURE

[to be added later]