The Darned Old Cocked Hat- still captivating! Part 1: Distribution of fixes at the True Location

Remember this issue? In Dec. 2010, FER, T. Sult, G. Huxtable, G. Kolbe, G. Brandenburg, P. Fogg and particularly J. Karl had a sometimes heated discussion about it. The question was this: Three LOP's do not, in general, intersect in one point but rather create a - hopefully small – triangle. Having this triangle, where to put the fix? It is this triangle they called the The Darned Old Cocked Hat (DOCH) because of the stubbornness of the problem. Although serious number crunchers are in this group this topic might quite well interest still others; so let me share some recent thoughts. Critique is invited!

Some assume the True Location (TL) lies somewhere within the DOCH, maybe at its "center of gravity". They might also assume the probability of position of a fix may be described by a normal distribution ("bell shaped in 3D") with the peak at TL. Some argued that the chance for the DOCH to even surround the TL should be just abt 12.5%. I believe no agreements were found.

I am interested in pursuing the issue. [NavList 21842]. To start with, I have found good reasons for these statements: a) the location of the TL is not a normal distribution, and b) the probability of a fix to hit the TL is exceedingly small.

Let me present arguments for these statements with a simulation and with graphs.

First, however, some insights: The distance of a fix from the TL has two independent coordinates, lat and Long. For a precise fix, both must be determined individually with precision. Generally, we will make errors in each coordinate, both errors being different and independent from each other. But it is virtually impossible to make small errors for lat and Long at ***the same time*** which would be necessary for an exact fix! This is the core of my argument.

To convince yourself just make a simple spreadsheet simulation *[1]* : Let column A be a list of latitudes and column C a list of Longitudes of, say, 500 fixes around TL. Assume each coordinate as individually distributed in its own bell-shaped fashion ("normally") with mean= 0 and, arbitrarily, std.dev.= 5 sm. Now proceed like this:

```
col A: =randnorm(0,5)        autofill 500 rows
col C: =randnorm(0,5)        autofill 500 rows

col F:  1:15                autofill  15 rows
col G: =sqrt(A1*A1+C1*C1)   autofill 500 rows
```

As a result G becomes the desired list of absolute distances from TL i.e. the errors of the fixes. For a display of the results in form of a histogram,

(a) highlight col's F,G (b) hit "Insert Chart"
(c) select "Statistics" (d) select "Histogram"

You should now see a bar chart like the one in Fig 1. The peak of this histogram occurs at B = 5sm from TL. B turns out to be equal to the std.dev's of the individual coordinates. Nice surprise!

[1] e.g. GNUMERIC SPREDSHEET which is freeware. Commands may differ in other programs.

Fig 1 is annotated with additional info plus a continuous curve, the " Rayleigh Distribution with parameter B ". It approximates the histogram and v.v. As claimed, it is not " bell-shaped ".

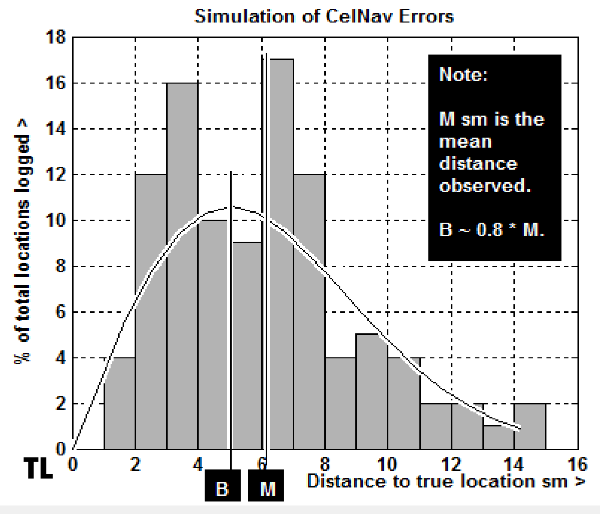
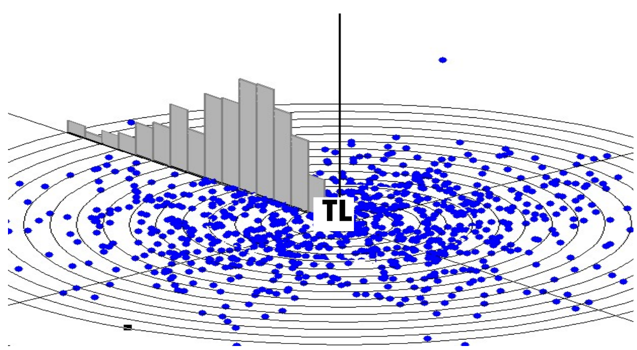


Fig 1 CelNavError Dist.fig

B: Theoretically most probable distance of fixes to the true location TL, no matter the direction. Equals here 5 sm, our reference.

Note: The most probable fixes occur on a circle of radius B = 5 sm around center at TL. As claimed, no fix can occur at TL itself.

Fig2. 3D display of the situation:



Circles indicate distances form TL at the center. The dots are fixes; the histogram indicates the probability of a fix to fall into the space between adjacent circles.

Close to TL, the area between two circles is small and so there is little opportunity for a fix to fall there. At very large distances the area between two circles increases dramatically but extreme errors of the fixes are rare. Around the distance B sm between those extremes, however, the areas between adjacent circles are of significant size ***and*** fixes with medium size errors are quite probable. Both facts cooperate to maximize the probability of fixes to concentrate B sm away from TL. The histogram confirms these views.

Next: Simulating The Darn Old Hat.