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The Accuracy of Astronomical Observations at Sea

IN 1952 the Institute set up a Working Party, under the chairmanship of Mr. D. H. Sadler, to investigate the accuracy attained in the practice of astronomical navigation at sea. The report that follows was presented in summary to the Technical Committee on 15 February, 1957, and formed the subject of a discussion at the Institute's meeting on 21 June.

1. SUMMARY OF CONCLUSIONS. The following conclusions relate to a total of 4245 observations received by the Working Party from 173 observers, and are representative of navigational practice as a whole only in so far as the observations, and the observers, are so representative. Only 3319 observations from 156 observers were retained as suitable for analysis, and in nearly half of these observations the ship's position is not known sufficiently accurately for them to be included in the main analysis.

The errors, from all causes, to be expected in an astronomical position line in good conditions for observation are:

Percentage error	Average observer	Best observer	Error exceeded in
50	0'·7	0'·5	10 out of 20 observations
90	2'·4	1'·4	2 ,, ,, ,, ,,
95	3'·1	2'·0	1 ,, ,, ,, ,,

These figures are based on 1539 observations from 129 observers for the 'average' observer, and on 383 observations from 8 observers for the 'best' observer.

There is strong evidence of a wide range in the standards of accuracy attained by different observers. But only 22 observers (a total of 2203 observations) sent in sufficient observations (25 or more) for a reliable estimate of their accuracy to be deduced, and little information is thus available about the great majority of the observers. The 'average observer' includes all observers—good, indifferent and poor—and this is the main reason why the 90 per cent and 95 per cent errors are relatively larger than the 50 per cent error would suggest. The 'best observer' is the average of the eight best observers from the 22 with more than 25 observations each. The poorest observers of these 22 have errors about three times as large as the best.

The observations are too mixed and too few in number for the effects of individual causes of error to be separated, except perhaps for one or two observers. The errors are, however, increased when the quality of the horizon is poor, by a factor of about two.

There is a strong indication of the occurrence of mistakes or blunders, either in the observations themselves or in the reductions.

2. INTRODUCTION. The formal terms of reference of the Working Party on 'The Accuracy of Astronomical Observations at Sea' are:

'To obtain—by research into the literature, by questionnaire from members and others, by experiment and special observations, by co-operation with other bodies interested in like matters, by theoretical analysis, and by such other means as the Working Party deems fit—*information relating to the accuracy of astronomical observations at sea*. To collate the information obtained, subject it to such analytic procedures as necessary and to report to the Technical Committee.'

This Report is restricted to the analysis of observations received in response to an appeal addressed widely to all classes of navigators. It thus covers only a small part of the stated means by which the Working Party was authorized to seek information; but it is by far the most important and the most difficult to obtain.

Details of the appeal, of the form used, the information requested, and of the circulation are given in Section 3.

The general aims of the Working Party, and the methods of obtaining and analysing data, were described by the Chairman in an article in the *Journal* (**5**, 296). To quote: 'The ultimate object of the Working Party is to obtain information as to the accuracy of position fixing at sea by means of astronomical observation in various conditions. To do so it may be necessary to examine in detail every one of the many different aspects of the observation, reduction, plotting, &c., that can arise; but it must not be overlooked that the navigator will in practice probably ignore all such questions as personal error (for which in theory corrections could be applied) so that it may be necessary to base the analysis on heterogeneous data chosen deliberately to include all the various sources of systematic and accidental error, and mistakes.'

Although provision was made on the forms for the recording of all data which might reasonably affect the accuracy of the observations, the Working Party always had in mind that the analysis of such mixed observations for the effects of specific factors must be subsidiary to the main aim of an overall figure of accuracy, averaged over all observers and conditions. In the light of the observations received, it might have been better to have concentrated solely on the main object, and not to have requested data which, in the event, could not be fully used.

The number of observations received is rather disappointing in view of the large number of forms circulated. Moreover, the distribution of the observations between observers is so uneven as to give undue influence to the few observers who have contributed most observations. However, it cannot be stressed too highly that these observations are the basic data from which results must be deduced objectively, independently of preconceived ideas and theories; the effects of particular factors must, if possible, be deduced from the observations themselves.

The analytic treatment of such heterogeneous (i.e., arising from mixed sources) observations is difficult, and the interpretation of the results must be made with great care; it may, for instance, be possible to explain some apparently anomalous result by examination of the individual observations, but this will weaken the significance of an apparently normal result. Moreover, the observations, even if reasonably representative, are probably selective: the keen observers will tend to send the most observations; only selected observations (selected perhaps randomly, but perhaps for some special reason) may have been sent; and it is to be noted that the error of the observation is known to the observer, although it is hoped that in all cases the observations were selected before reduction.

The Working Party is not entirely satisfied that the results deduced from these observations are representative of navigation as a whole; but they appear to be the only observations available which were not made for some special purpose by some special observers. The results, though uncertain, seem to be the best guide available to the accuracy actually attained in the every-day practice of astronomical navigation in the Merchant Navy.

It is a great pity that there are so few observations from the Royal Navy.

3. Design of form and distribution. The appeal for observations was made on a printed form, with the following introduction and notes.

AN APPEAL FOR OBSERVATIONS

A working party, of which Mr. D. H. Sadler is chairman, has been set up to obtain and discuss observations and information relating to the accuracy of astronomical observations at sea. The terms of reference of this working party, together with a list of its members, are given in the *Journal* for July 1952 (Volume V, p. 296); this is followed by notes by Mr. Sadler describing the aims of the working party and the methods of obtaining and analysing data which it is hoped to adopt.

By far the most important part of this investigation is the collection of reliable observations normally made at sea, and it is for these that the Institute appeals to members, and others.

The most useful observations, and those which will be the easiest to analyse, are those of the errors of individual position lines when the ship's position is accurately known; this form is designed primarily for such observations. However, three other series of observations will yield valuable information and should be made if there is no means of fixing the ship's position independently.

(i) simultaneous observations of several stars to obtain a fix, and thus the errors of individual position lines;
(ii) measured altitudes of the Sun, Moon, planets and stars at frequent intervals (say 20^m to 30^m during the day and 2^m to 3^m during twilight), the ship's speed and course being recorded;
(iii) observations by two or more observers at about the same time.

No special forms are being issued for observations (i), (ii) and (iii), but either the attached form may be adapted or observers can design their own. In (i) a fix should always be deduced and the errors of the position lines given.

While intercept methods are stressed, provision is also made for sights reduced by ex-meridian and longitude methods; the error of the position line must, however, always be given as the shortest distance from the fix.

Overleaf will be found particulars of the observations required and a form providing for the recording of the relevant details. The latter have been carefully considered by the working party and each piece of information will play an important part in the subsequent analysis and discussion of the observations.

The Institute realizes that these observations, their reduction and the completion of the forms will involve a considerable amount of work for volunteers; it is confident, however, that navigators will do their best to assist this purely objective investigation, which is likely to have fruitful results.

Copies of the final report of the working party will be sent to all who contribute observations. Reprints of Mr. Sadler's note on the aims of the working party, and additional supplies of these forms, will be sent on request.

NOTES ON OBSERVATIONS

The principal aim of this investigation is to discover the accuracy of astronomical observations at sea *as they are normally made at present*; the determination of the *best accuracy possible* with present instruments and tables is a secondary aim. It is thus important that, while taking reasonable care to avoid gross errors, observations should be taken as usual and without special care. Altitudes and times should be recorded in the normal way.

Please record *all* observations of the series whether they are grossly in error or not; the proportion of large errors (which always occur and cannot be entirely avoided) is of considerable interest in itself.

In addition to name and address, observers were asked to give particulars of ship, sextant, methods of observation and time-taking and methods of reduction; and opinions, based on their own experience, of the accuracy of sextant altitudes, position lines and fixes, and of the adequacy of instruments, almanacs and tables. These opinions, which do not enter into the analysis of the observations, are themselves analysed in Section 7.

For the details of the observations themselves, particulars were requested in respect of the following factors which were considered as possibly affecting the accuracy of observation: area of observation; sea and swell; conditions (rolling, glare, &c.); quality of horizon (good, satisfactory, poor); accuracy of ship's position and how obtained; single shot or mean of several; method of reduction; remarks. Otherwise provision was made for all the data that would enable the sight to be reduced and the error of the position line recalculated from the stated position of the ship. (In the light of experience, the Working Party realizes that it asked for more data than it could digest.) In Fig. 1 is reproduced a form on which have been copied, precisely as given, the details of two fairly typical observations as actually sent in.

The forms were distributed in April–June 1953

(*a*) with all copies of the *Journal*;
(*b*) to the Marine Superintendents of about 50 major shipping companies with a request for distribution (with their blessing!) to individual ships and officers;
(*c*) to all specialist navigators in the Royal Navy, and in the Royal Netherlands Navy;
(*d*) to selected Merchant Ships by Port Meteorological Officers, and to the weather ships;
(*e*) to Nautical Schools;
(*f*) to cable ships;
(*g*) to a few individuals.

Three thousand forms were printed, and most of them were distributed; in addition, 3000 'continuation sheets', containing only that part of the form used for the actual observations, were printed. In spite of this many observations were sent in on plain paper.

4. THE OBSERVATIONS. A total of 4245 observations was received from 173 observers. But, for various reasons, 867 of these observations could not be used; the principal omission was of the error of the position line (sometimes replaced by a comment such as 'good', or merely as 'towards' or 'away'). Longitude sights have been included where the data given allowed the position-line errors to be calculated. It has not been possible, however, to include the few series of sights at regular intervals of time or altitude, taken when the ship's position was not known accurately. These require separate analysis and, in any case, involve reduction; there are too few such observations to warrant systematic calculation by, say, punched-card machines.

Rejection of observations purely on account of the size of the errors to which they give rise is always a difficulty. There is a high probability that large errors are due to blunders, whose retention will give a false impression of the accuracy since they will increase the standard deviation (though not the actual percentage errors) disproportionately. The Working Party is, however, interested in the overall accuracy, including

DETAILS OF OBSERVATIONS

1. DATE	1. 25th JULY 1953	1. 11th AUGUST 1953
2. AREA (Red Sea, Atlantic, &c.)	2. N. ATLANTIC	2. INDIAN OCEAN OFF SOCOTRA
(a) Sea and swell	(a) MOD.	(a) MOD. SEA, HEAVY SWELL
(b) Conditions (Rolling, glare, &c.)	(b) SATISFACTORY	(b) ROLLING SLIGHTLY, SLIGHT GLARE
(c) Quality of horizon (Good, satisfactory, poor)	(c) SATISFACTORY	(c) GOOD
(d) Air temperature	(d) 57°	(d) 90°F.
(e) Pressure	(e) 1009 Mb.	(e) 29.75 ins.
SHIP'S POSITION	3. D.R. 50° 10.′0 N. 10° 51.′0 W.	3. 12.57 N. 53. 39 E.
(a) G.M.T. of fix	(a) 04 05 21	(a) 12. 02. 11
(b) How obtained? (Decca, bearings, &c.)	(b) TAUT WIRE AND CABLE RUN	(b) VISUAL BEARINGS
(c) Probable accuracy	(c) WITHIN ONE MILE	(c) WITHIN ½ MILE
(d) Course and speed	(d) 264° 7.0 KTS	(d) 079° at 15 KNOTS.
(e) Height of eye *	(e) 44′	(e) 38 FEET.
4. BODY OBSERVED (and limb)	4. MARKAB	4. SUN. LOWER LIMB
(a) Day and G.M.T. of observation	(a) 25 04-05-21	(a) Aug. 11th. 12. 02. 11
(b) Probable accuracy of time *	(b) ½ sec.	(b) WITHIN 1 sec.
(c) Sextant altitude (corrected for sextant and any known personal errors)	(c) 54° 23.′0	(c) 38° 36′ OBS. NIL CORR.
(d) Probable accuracy of readings *	(d) 0.′2	(d) To 0.′2
(e) Single shot or mean of how many?	(e) SINGLE	(e) SINGLE SHOT
(f) Remarks on observation (good, doubtful, &c.)	(f) GOOD	(f) GOOD
5. METHOD OF REDUCTION* (cos.-hav., longitude, Ex-mer., &c.)	5. COS HAV	5. LONGITUDE BY CHRON.
(a) Almanac (A.N.A., Air Almanac, &c.)	(a) A.N.A.	(a) A.N.A.
(b) Tables (H.D. 486 [H.O. 214], Burton, Hughes, &c.)	(b) NORIES	(b) BURTON
(c) Probable accuracy of reduction	(c) 0.′2	(c) To 0.′5
6. ERROR OF POSITION LINE (towards or away)	6. INTERCEPT 0.′2 To	6. 0.′4 AWAY
(a) How obtained? (by calculation or plotting)	CALCULATED	PLOTTING

Fig. 1. Illustrates the details of two observations, copied on the form exactly as they were sent in.

the blunders which are presumably representative. This is a reasonable approach, for it is quite likely that at least some navigators censored their observations before submitting them, and it is legitimate to take the view that those that were included here were representative sights. On balance it was decided to omit the 59 observations which gave rise to errors of more than $\pm 10'$; most of these were associated with poor conditions or inaccurately-known ship's position.

After rejection of the incomplete observations and those with position-line errors of more than 10′, 3319 observations from 156 observers remain for analysis. These form the material whose relationship to the totality of astronomical observations at sea determines the relevance of the results to navigational practice as a whole. Unfortunately, in 1461 of these observations the ship's position was stated not to be known to within 1′ and the Working Party has regretfully had to omit them in most of the analyses; the errors are, in general, significantly greater than in the remaining 1858 observations from 129 observers, but there are exceptions for individual observers (see Section 10).

The observations are very poorly distributed in respect to the observers. Of the total of 3319 observations, no fewer than 892 were contributed by a single observer, 1408 by five observers, and 2203 by 22 observers—all, in fact, who contributed 25 or more observations. Only 39 (making a total of 61) other observers contributed 10 or more observations, leaving 95 with less than 10 observations each.

When the observations made from accurate (better than 1′) positions are considered, 1022 out of a total of 1858 observations were contributed by 14 observers each with 25 or more observations; 45 observers contributed 10 or more observations leaving 84 observers with less than 10 observations. One observer contributed 268, and three observers 586 observations. Details are given in Table XI.

There is a wide range in the standard of presentation. The great majority of the observations are given in full detail, often in typescript, and show that exceptional care has been taken to provide all the information requested. In many cases additional information is given, sometimes in rather embarrassing detail for each observation—embarrassing because, although of interest and value, it cannot readily be used. However, the minority of the observations, representing a higher proportion of the observers, show evidence of lack of care and thought. The chief faults lie in giving vague descriptive estimates of accuracy instead of numerical ones; 'nil' is not suitable as a statement of the error of a position line, though its numerical meaning can often be deduced from the values of errors (which are not regarded as 'nil') given by the same observer. There is, however, a disproportionate number of 'nil' errors. Detailed examination of the position-line errors reveals also, as might perhaps be expected, a bias towards multiples of 1′.0, 0′.5 and 0′.2; see Fig. 2.

Observers were asked to give both the estimated accuracy of the ship's position and the method by which it was found. There is no

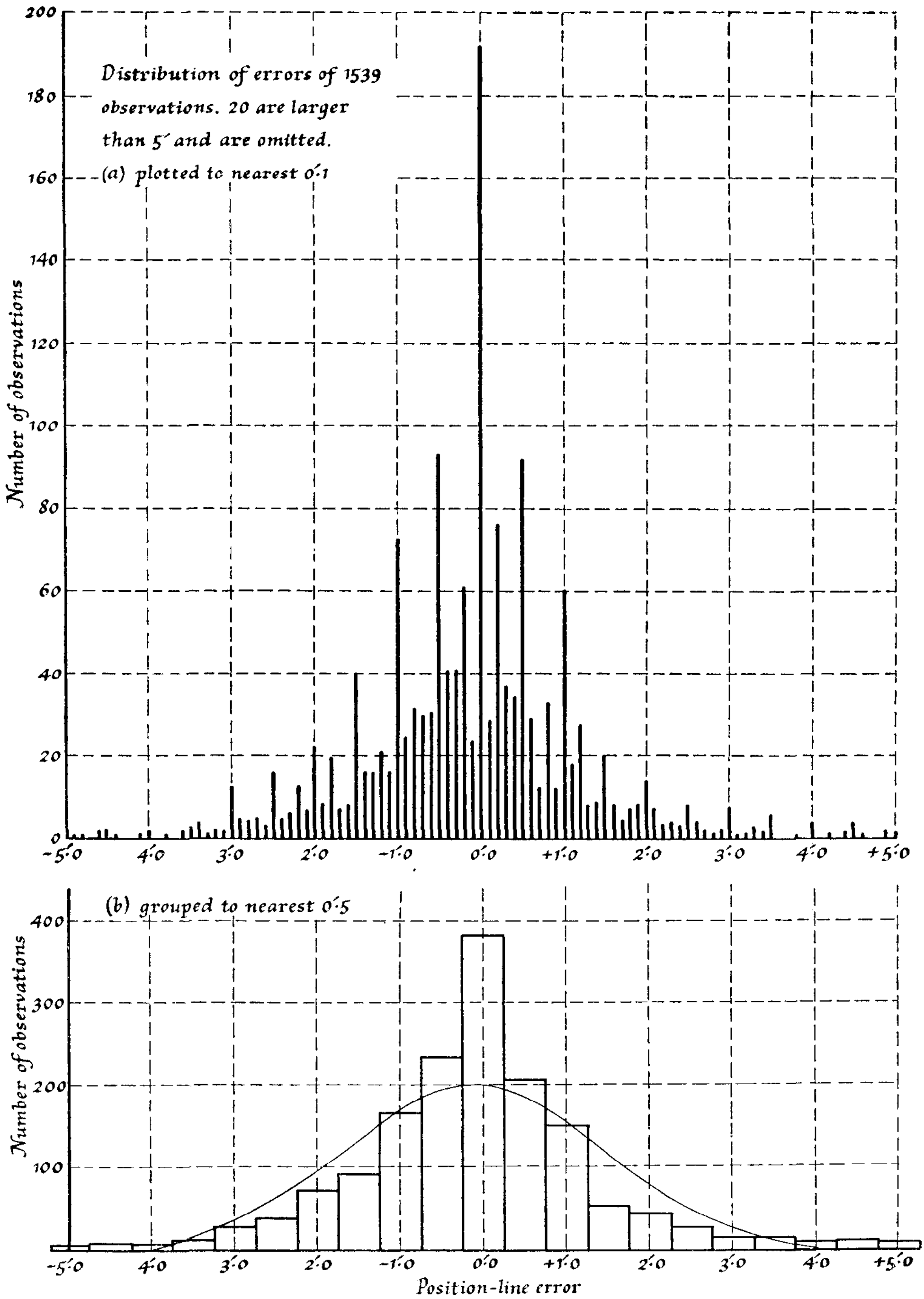

Fig. 2. Shows the frequency distribution of the position-line errors of the 1539 observations made by 129 observers from positions known to within 1′.0 and with good horizons. 20 errors greater than 5′.0 have not been included. (a) *Gives the numbers of each value of the error to the nearest 0′.1, and illustrates vividly the preponderance of multiples of 1′.0 and 0′.5, and, to a lesser extent, of 0′.2;* (b) *depicts the corresponding histogram when the observations are grouped into groups of length 0′.5 centred on values of 0′.0, 0′.5 &c. The continuous curve is the equivalent Gaussian distribution with a mean value of −0′.13 and a standard deviation of 1′.53. The excess of both small and large errors should be noted.*

practical alternative to accepting the observer's estimate for the purpose of grouping the observations, but different observers have widely differing views as to the accuracy of various methods. Some are, to say the least, optimistic while others are ultra-careful; with the ship's position uncertain by 3–4 miles it is a remarkable coincidence to get a whole series of position lines with errors of not more than 1′! 'Accurate' positions, stated to be known to within 1′, have been obtained mainly by visual bearings but some were deduced from star fixes and some by 'radar'.

Full details of the observations were recorded on to punched cards, each factor (such as area or quality of horizon) being coded numerically. The factors were divided more finely than could in fact be used, so that the divisions had later to be combined. The data were transcribed by girls who could not, even if it had been practicable, have taken account of individual comments outside the established codes. Thereafter, all the selection and analysis was done completely impersonally on punched-card machines.

A selection of the comments of observers and the remarks on the observations is given later in Section 8.

5. MAIN ANALYSIS. Bearing in mind the main object of an estimate of the accuracy of astronomical navigation averaged, as it were, over all navigators, the Working Party first analysed all the observations, without regard to the observers. As will be seen in Sections 10 and 12, the heterogeneity of the observations, together with the uneven distribution between observers, weakens the conclusions to be drawn of the effect of the various factors; but the overall picture is still the best that the observations can provide.

The methods of analysis used, and the statistical terminology of the following Sections, follow very closely the Institute's monograph on the use of statistics, *Observational Errors*, printed in the *Journal*, Vol. IX, p. 105, April 1956.

The punched-card machines can sort the cards, each card representing one observation, mechanically into sets corresponding to any (or all) desired combination of factors. Each such set can then be analysed to give 'statistics' describing the distribution of errors, which can then be compared between sets to indicate the influence that any particular factor may have. It must be emphasized that these 'statistics' are merely a convenient method of summarizing the observations in each set, in order to allow of a simple, though approximate, description of the frequency distribution. The statistics usually calculated are the mean and the standard deviation σ about the mean; the mean should here be zero and is, in fact, satisfactorily small in almost all cases. There is, on the whole, a slight bias towards negative (i.e. 'away') position-line errors. If the frequency distribution is Gaussian (or normal), as might be expected if the observations were homogeneous and unaffected by blunders, the 50 per cent error (known sometimes as the probable error and such that half of the errors are smaller and half larger) should be $0{\cdot}674 \times$ the standard deviation. The 50 per cent error can be, however, determined directly from the observations themselves by finding that quantity that is exceeded in half the cases. A comparison of the two determinations gives an indication of whether the distribution is Gaussian and whether the standard deviation is a useful statistic. Another comparison can be made between the average error η (the mean of all observations without regard to sign) and $0{\cdot}8 \times$ the standard deviation.

The distributions turn out to be so far from Gaussian that the conventional standard deviation is not a useful statistic for describing them, although it is still useful for comparing them. For description it is found better to deduce the 50, 90 and 95 per cent errors directly from the actual distribution. This is done by counting (on the sorter) the numbers of observations and noting the points corresponding to 50, 90 and 95 per cent of the total number of observations. At the same time the numbers falling into various groups can be counted, allowing the distributions to be presented in the form of histograms.

Table I gives the above statistics for all observations, and for all observations separated into categories corresponding to single factors. The actual distributions, which are not given, are very similar to that illustrated in Fig. 2. Bearing in mind the mixed nature of the observations, and the small number of observations in some categories, it is clear that very little can be deduced about the effect of most factors; and, for the same reasons, that the observations can be combined for these factors without great loss of accuracy. Consider each factor separately.

(*a*) *Area of observation*. There is no great variation in accuracy from one area to another, bearing in mind the small number of observations in some areas.

(*b*) *State of the sea*. There is not sufficient evidence that the state of the sea is an important factor as far as these observations are concerned.

(*c*) *Quality of horizon*. The quality of the horizon appears to have an appreciable effect on the accuracy of the observations, though the number taken with poor horizon is small.

(*d*) *Accuracy of position*. The errors in observations taken when the ship's position is uncertain by more than a mile include a contribution due to the unknown error of the ship's position, and this is significant when compared with the accuracy of the sights themselves. This is amply demonstrated by the figures in the table, which suggest that the errors of the observations and of the ship's position, when not known accurately, contribute about equally to the total error. If the total error is treated solely as error of observation (and reduction) it will thus be an overestimate. The main result must therefore be deduced solely from the 1858 observations for which the ship's position is

TABLE I. ANALYSIS OF 3319 OBSERVATIONS FROM 156 OBSERVERS ACCORDING TO SINGLE FACTORS

Factor	No. of observations	σ	50%	$0{\cdot}674\sigma$	η	$0{\cdot}8\sigma$
All observations	3319	2'·01	0'·9	1'·36	1'·31	1'·61
Area						
Arctic Ocean	70	2·24	0·8	1·51	1·43	1·79
N. Atlantic Ocean	987	1·84	0·9	1·24	1·21	1·47
S. Atlantic Ocean	134	1·34	0·7	0·90	0·91	1·07
Mediterranean Sea	452	2·20	0·9	1·48	1·45	1·76
Red Sea	206	1·98	1·0	1·34	1·33	1·58
Indian Ocean	1120	2·26	1·0	1·53	1·47	1·81
N. Pacific Ocean	173	1·60	0·7	1·08	1·07	1·28
S. Pacific Ocean	174	1·22	0·7	0·82	0·94	0·98
Antarctic Ocean	1					
Not given	2					
State of sea						
Good or satisfactory	2951	2·00	0·8	1·35	1·28	1·60
Moderate or poor	302	2·20	1·1	1·48	1·49	1·76
Not given	66					
Quality of horizon						
Good or satisfactory	2904	1·94	1·1	1·31	1·24	1·55
Hazy or poor	401	2·50	1·4	1·69	1·83	2·00
Not given	14					
Accuracy of position						
Known to 1'·0 or better	1858	1·74	0·8	1·17	1·14	1·39
Not known to 1'·0	1461	2·31	1·0	1·56	1·52	1·85
Body observed						
Sun	965	1·66	0·9	1·12	1·14	1·33
Moon	100	2·54	1·1	1·71	1·72	2·03
Stars and planets	2252	2·12	0·9	1·43	1·36	1·70
Not given	2					
No. of shots						
3 or more	159	1·93	0·9	1·30	1·30	1·54
2 ,,	929	2·11	1·0	1·42	1·44	1·69
1 ,,	2146	1·97	0·8	1·33	1·24	1·58
Not given	85					

σ = Standard deviation about the mean.
η = Average error, the mean of all observations without regard to sign.

stated to be known to 1'·0 or better. The error in the ship's position will then have a standard deviation of about 0'·4 or 0'·5, which will not significantly contribute to the total error, with standard deviation of about 1'·7.

(*e*) *Body observed*. Star and planet sights do not give greatly different results from the Sun and there are too few Moon observations for any conclusions to be drawn.

(*f*) *Number of shots*. As far as the present observations are concerned, the errors of single-shot observations appear to be just as accurate as those which are the mean of 2, 3, or more shots. All the observations can therefore be combined.

As mentioned above, little weight can be attached to the above analysis, which is solely a guide to whether the categories can be combined or not. Apparently anomalous results can easily arise because, for

TABLE II. ANALYSIS OF 1858 OBSERVATIONS MADE FROM 'ACCURATE' POSITIONS (KNOWN TO WITHIN 1′) ACCORDING TO COMBINATIONS OF FACTORS, AND DEDUCED BLUNDER RATIOS

Factors	No. of observations	Actual distribution 50%	90%	95%	Blunder ratio (per cent)	Corrected distribution 50%	90%	95%
All, position accurate	1858	0′.8	2′.8	3′.9	4	0′.8	2′.5	3′.0
Sun	709	0·9	2·5	3·0	3	0·9	2·1	2·8
Stars and planets	1084	0·8	2·9	4·2	5	0·8	2·6	3·3
Good horizon	1539	0·7	2·4	3·1	3	0·7	2·2	2·8
Poor horizon	254*	1·5	4·6	6·5	14	1·4	3·0	4·0
Sun, good horizon	622	0·8	2·4	2·9	2	0·8	2·3	2·8
Sun, poor horizon	87*	1·2	2·9	4·5	7	1·2	2·8	3·3
Stars, good horizon	917	0·7	2·6	3·7	4	0·7	2·4	3·1
Stars, poor horizon	167*	1·6	5·2	7·3	18	1·5	3·5	4·5
All observations (for comparison)	3319	0·9	3·2	4·6	7	0·9	2·9	3·8

* The number of observations is too few for the deduced figures to be very significant; but the blunder ratios are definitely higher than in other cases.

instance (as in fact is the case), practically all the observations based on the mean of two or more shots were made by only two observers. This point is analysed further in Sections 10 and 12. It transpires from this that these results are indeed misleading, but an independent study of the question of the connection between accuracy and number of shots, given in Section 6, enables fresh light to be shed on this problem.

The only factors which can be regarded as of significance, apart from accuracy of position, are quality of horizon and less definitely, body observed. In Table II are given the results for the combinations of the above factors which appear to be useful. There are first given the 50, 90, and 95 per cent points for the actual distribution, without correction of any kind; these are given to the nearest 0'.1, which is all the accuracy that the figures warrant. It will be noticed that the ratios of the 50:90:95 per cent errors differ considerably from those for a Gaussian distribution, namely, 0·674:1·645:1·960 or 1:2·4:2·9; the 50 per cent error is much smaller, and there are more large errors, than would normally be expected.

The departure of a typical distribution from the equivalent Gaussian form is well illustrated in Fig. 2. There is great excess of small values, due partially to the large number of 'nil' values. As a consequence, the frequency of the medium-sized errors is less than the Gaussian curve would indicate; but even so the curves cross for errors of about 4'.0 and, for larger errors, the actual frequency is greater.

This excess of large errors may arise from the heterogeneous nature of the observations and from the incidence of blunders or mistakes. When the observations of two or more observers of different standards of accuracy are combined there will be a greater spread in the frequency distribution than the Gaussian law warrants. The frequency of blunders is often almost independent of their size, unless they are very large, and could be likened, perhaps, to the effect of a very inaccurate observer. All observations (about 2 per cent) leading to errors of more than 10 miles have previously been rejected; it would appear from these distributions that many, about 7 per cent in fact, smaller errors may arise from observations made by inaccurate observers or through blunders. There is no certain way of distinguishing between these causes, except that the proportion of blunders is likely to be larger for the larger errors. It is, however, possible to estimate the distribution that would have arisen if both the blunders and the inaccurate observers had been omitted, and to deduce a 'blunder ratio' giving the total proportion of such omitted observations. A crude theory is all that is justified by the observations, and the results are correspondingly very approximate. The blunder ratio has, in fact, been determined by assuming that some of the large observations leading to too great a departure from the normal curve at the tails of the distribution are due to blunders of one kind or another, including errors due to inexperienced observers, extreme conditions, &c. Blunders do not necessarily give rise to large errors and the theory allows for the correct proportion of small errors being affected by blunders. The blunder ratios and the 50, 90, and 95 per cent errors of the corrected distributions are given in the second part of Table II.

The ratios of the percentage errors are still not Gaussian, but this is as likely to be due to the differences between observers as to a too low estimate of the blunder ratio. An undue proportion of large errors seems to be a significant and characteristic feature of these observations, shown equally by individual observers (see Section 12). It must again be emphasized that the blunder ratio has a specialized meaning and that the values given are only crude estimates.

The conclusions that can be drawn from Tables I and II are thus:

(*a*) (Fairly definite.) Of the observations taken by an average observer, in good conditions, half will give rise to position-line errors exceeding $0'.7$, one-tenth to errors exceeding $2'.4$ and one-twentieth to errors exceeding $3'.1$. The distribution of such errors is shown in Fig. 2. With a poor horizon, these errors will be considerably increased and may be doubled.

(*b*) (Rather less definite.) There is an indication that about three per cent of the observations made in good conditions are affected by blunders. The proportion becomes larger as conditions deteriorate and, in extreme conditions, about one in every seven observations may be affected by blunders or extremely large observational errors. About nine per cent of all the observations are so affected, allowing for those previously discarded.

6. SUCCESSIVE SIGHTS. In the previous Section, analysis of all the observations into categories according to the number of shots indicated

TABLE III. ANALYSIS OF GROUPS OF SUCCESSIVE SIGHTS

Analysed as	Variance Expected	Variance Calculated	No. of observations
Single observations	a^2+b^2	2·13	247
Groups (average no., n)	$a^2+\frac{1}{n}b^2$	1·48	81
Differences	$2b^2$	2·12	166

that single-shot observations were as accurate as those based on the mean of two or more shots. This suggested that it would be interesting to analyse the 81 groups of observations, averaging three observations in each group, taken of the same body at short (few minutes) intervals of time and reduced separately. Accordingly, the 247 observations were analysed: first, as single observations; secondly, as groups taking the average of the observations in each group; and finally, by taking the differences between successive observations. The calculated values of the variance are given in Table III.

It is reasonable to assume that the errors of these observations consist of two parts, one which is constant for a whole group of sights (the main contributions probably being the error in the ship's position) and one which is completely random (observational error). Writing a as the standard deviation of the constant error and b as that of the random error, the expected values of the variance are as given in Table III, where n is the average number of observations in a group.

Taking n as 3, $2b^2$ is deduced from the first two analyses as $3(2\cdot13-1\cdot48)=1\cdot95$, in very good agreement with the value obtained from the differences. Taking $b^2=1\cdot0$, $a^2=1\cdot14$ giving $a=1'.1$ and $b=1'.0$.

The standard deviation of a single observation is 1'.46 as compared with 2'.01 for all observations and 1'.74 for observations from accurate positions; these particular observations were made, in fact, by the more careful observers. It is clear from these figures that taking the average of a number of sights does not necessarily reduce the error by the square root of the number; the constant error is larger than that due to the errors in the ship's position. The effect is likely to be no less in taking the average of several shots.

From this analysis is deduced a figure, 1'.0, for the standard deviation of the random or observational error. It is based on an inadequate number of observations and must be regarded as an indication only.

7. THE OBSERVERS. Forms were received from 173 observers, but on a few forms sights were recorded from more than one actual observer; thus one set of forms, credited to one observer, contains sights taken by

TABLE IV. EXPERIENCE AND SIZE OF SHIP OF THE 173 OBSERVERS

Experience	Size of ship: Less than 2000 tons	2000–10,000 tons	More than 10,000 tons	Not given	Totals
More than 5 years	6	49	31	5	91
2–5 years	—	13	6	1	20
Less than 2 years	1	2	3	2	8
Naval officer	8	7	2	—	17
Not given	1	20	7	9	37
Totals	16	91	49	17	173

58 cadets. The observers are distributed as regards experience and size of ship, as given in Table IV.

The following Tables V–VII present a straightforward analysis of the information supplied by the observers as to their methods of observation and reduction.

TABLE V. ACCURACY OF SEXTANT READING AND MAGNIFICATION USED

Accuracy of sextant reading	No. of observers
0'.1	11
0·2	76
0·3	1
0·4	—
0·5	44
not given	41

Magnification	No. of observers: Sun	Stars
1·5	1	1
2·0	5	4
2·5	46	42
3·0	30	28
3·5	5	6
4·0	13	15
4·5	—	1
5·0	5	6
6·0	4	4
7·0	—	1
not given	64	75

TABLE VI. METHOD OF TIMING OBSERVATIONS

Method	No. of observers
Deck watch	9
Counting seconds	99
Chronometer	13
Timed by assistant	16
Stop watch	17
Not given	19

TABLE VII. ALMANACS, TABLES AND METHODS USED IN THE REDUCTION

Almanacs:	*Abridged Nautical Almanac*	112
	Brown's Nautical Almanac	3
	Not given	58

Tables: Burton, 24; Burton (4-figures), 2; Nories', 100; H.D. 486 (or H.O. 214), 21; A.P. 3270 (H.O. 249), 2; others 5; not given 19.

Methods: Methods largely determined by the tables used; most of those who did not give a table stated they use the cosine-haversine method of calculation.

Observers were asked to give their opinions, based on their own experience, of the accuracy of the four specific determinations in good conditions; it was made clear that the figure given should be an estimate of the 50 per cent error. The estimates are analysed in Table VIII.

TABLE VIII. OBSERVERS' ESTIMATES OF 50 PER CENT ERROR OF

(i) a single sextant measure of altitude, disregarding dip and refraction;
(ii) a single position line, including the error arising through altitude correction and in the reduction;
(iii) a fix deduced from three or more position lines;
(iv) a Sun-run-Sun or a Sun-run-mer. alt. position.

50% error		0'.00	0'.05	0'.10	0'.15	0'.20	0'.25	0'.30	0'.35	0'.40
No. of	(i)	1	—	2	—	4	5	4	—	1
observers	(ii)	—	—	—	—	1	—	1	—	1
	(iii)	1	1	—	—	—	3	1	1	2
	(iv)	—	—	—	—	17	1	—	—	—

50% error		0'.50	0'.60	0'.70	0'.75	0'.80	0'.90	1'.00	1'.20	1'.25
No. of	(i)	76	—	—	2	—	—	19	—	—
observers	(ii)	15	1	6	3	1	—	56	—	3
	(iii)	29	—	1	5	—	—	41	—	—
	(iv)	5	—	—	1	—	—	14	2	1

50% error		1'.50	1'.75	2'.00	2'.30	2'.50	3'.00	4'.00	4'.50	5'.00	Not given
No. of	(i)	1	—	2	—	1	—	—	—	—	55
observers	(ii)	9	2	17	—	1	—	1	—	2	53
	(iii)	13	1	13	—	2	1	—	—	1	57
	(iv)	17	2	34	1	5	—	5	1	3	64

It will be noticed that the median estimate for (ii), which is the determination considered in this Report, is almost exactly 1'.0. Apart from the obvious misunderstandings, no fewer than 17 observers have indicated a 50 per cent error of 0'.2 for a Sun-run-Sun or Sun-run-mer. alt. position; this, surely, must verge on the optimistic side!

Observers were also asked for their opinions on the adequacy of the accuracy of (i) instruments and methods of observation, and (ii) almanacs and methods of reduction; figures are given in Table IX.

TABLE IX. ACCURACY OF INSTRUMENTS AND ALMANACS

Adequacy	No. of observers (i) *instruments*	(*ii*) almanacs
Inadequate	3	5
Adequate	137	117
More than adequate	3	13
Not given	30	28

8. OBSERVERS' REMARKS. Most observers either wrote accompanying letters or attached remarks to their forms or to individual observations. It is clearly not possible to subject these to formal analysis, but a fairly comprehensive selection of the substance of these remarks is given below; each remark is preceded by the observer's number.

10. Single observations of the Sun are in error by up to 5' by comparison with taut wire checks.

11. A single shot, taken with care, is better than several taken hurriedly.

12. Largest errors are due to poor horizon, glare or haze.

13. A cocked hat of 2' is good in practice; 1' may be got in very good conditions.

21. Sights can often be taken during hours of darkness.

22. Would like quicker means of reduction; Sun-run-mer. alt. is overrated. Star sights are better.

25. G.H.A. to ¼' would be adequate.

27. Altitude correction tables for the Sun need revision, together with effects of irradiation.

29. No difference in accuracy between H.O.214 and cos-haversine.

30. Artificial horizons should be perfected and more widely used. Tables should be simplified and condensed.

32. Accuracy of sights is affected by the strength of the wind.

37. A.N.A. worthy of special mention for ease, speed and accuracy.

38. Average accuracy with A.P. 3270 found to be 2'.5, with a maximum of 6'.

40. Mercury should be included in the A.N.A.

43. Cites example of abnormal refraction.

44. Has noticed systematic errors between morning and evening sights in the Arabian Sea.

62. In Caribbean Sea, *Polaris* gave errors of 3' even under excellent conditions. Would like log cos (dec.) included in A.N.A.

64. Suggests that errors are largely due to interpolation in tables.

66. Position lines sometimes give a false position, even when conditions are good.

71. Sights taken near land are likely to be considerably in error.

73. Navigators do not like to admit personal error; and introduce errors by taking means in unsuitable places.

78. Steady platform of large ship increases accuracy.

82. Usually takes two Sun sights about one minute apart; from 130 pairs the differences of the resulting position lines are distributed:

$0'.5$ or less, 95; $0'.6$ to $1'.0$, 23; $1'.1$ to $2'.0$, 8; $2'.1$ to $3'.0$, 4.

He points out that horizon errors, or abnormal refraction, would tend to affect both position lines equally.

84. Would like quicker reduction tables; the present average time is 15 minutes.

86. Prefers Sun–Moon, or Sun–Venus, combination for noon position.

88. Sun sights less reliable than star sights.

94. Extracts from letter:

'H.M.S. ——— was leader of the Squadron consisting of five ships to which Commonwealth and United States ships were frequently attached during the period that these observations were taken. All British ships carried qualified Navigating Officers.

The following are the average differences in position as compared between the observer's ship and other ships in company at noon and twilight. The sights were taken in a wide variety of conditions and were taken in the ordinary course of passages—many of them with no thought of subsequent analysis. They were all in the area of the China Sea, West Pacific and Yellow Sea.

	Average difference	Normal maximum	No. of observations
Noon positions	$1'.8$	$3'.0$	52
Twilight positions	$1'.2$	$2'.3$	88

It is thought that the main causes of error are personal error of observation, and index error, which is always liable to change and is often not found with sufficient accuracy. Quite apart from a constant personal error which can be corrected or allowed for, it is surprising how large is the random personal error as seen in a series of cuts of the Sun or stars taken on the meridian. This is frequently up to $1'$ for myself (average $0'.4$ Sun, $0'.7$ star) but includes even larger errors even when one has been convinced that a good cut was obtained. The method of observation is by equal intervals of altitude, using $2'$ or $1'$ depending on the rate of change, 5 cuts for the Sun and 3 for stars, and comparing the centre time with the average for an instant check.

A.P. 3270 and the Rapid Star Reduction Method have been compared on 24 occasions with H.D. 486 and on no occasion did the difference in position exceed one mile. In fact the reduced chance of errors in calculation and the incentive to plot six stars instead of three, which is offered by the speed of the method, probably gives one more confidence in the resulting fix.'

100. Uses Rude Star Identifier for azimuth.

103. Would like inexpensive, easy to handle, artificial horizon.

106. A position within $2'$ is satisfactory. The *C*-correction in the Sun-run-mer. alt. position is likely to lead to errors in high latitudes.

110. Interpolation of reduction factor for decimals of *A* is inadequate, for large hour angles, in Nories' ex-meridian tables.

111. The G.H.A. almanac is little, if any, improvement on the old almanac.

113. Unnecessary to read the sextant to better than $0'.5$.

115. Simultaneous sights of the same object often differ by up to $2'$.

126. Suggests that haversine tables should be given for every $0'.2$ to save interpolation, and that the 'short' method in Nories' Tables should be extended to give azimuth as well as zenith distances.

127. (Cable ship) Most frequent cause of error is personal error.

130. Reductions take too long and use too many figures, leading to arithmetical blunders.

131. *Air Almanac* adequate for use at sea.

136. Single shots only are necessary, except when the horizon is difficult; fix to be based on four sights 90° apart.

140. Considers only good fix is with three or more position lines, with at least two consistent observations of each object.

142. Position lines from 3 or 4 stars should intersect inside a circle of radius $0'.3$ in good conditions and up to $2'.0$ in poor conditions. An almanac to $0'.5$ would be sufficient.

146. Would like log cos (dec.) of selected stars included on the bookmark of the A.N.A.

147. The additional altitude corrections for Venus in the A.N.A. do not assist daylight observations.

151. Would like G.H.A. increments in the A.N.A. to be like those in the *Air Almanac.*

153. Has obtained good sights with a night horizon.

156. Suggests using polaroid shades for sextants when the horizon is poor.

167. Sun sights are less reliable than planets by day, and still less than stars by night.

173. Considers that more errors enter in timing observations than from any other source.

9. ACCURACY OF REDUCTION. The principal aim of the Working Party is to describe the accuracy of astronomical position lines as found in practice, so that errors in the reduction should be included. However, a sample of 97 observations was investigated in detail, and it seems worthwhile to include the results here. The selection was at first intended to consist of one observation taken at random from each observer, but this was not completed; and for various reasons more than one observation from one or two observers were included. The details of the observations, their errors and possible explanations for them are given in Table X.

Of the 97 observations only 38 agreed to within $0'.5$ or less with the accurate recalculation made from the data given on the forms; the recalculation was done using H.D. 486 and checked independently. This number increases to 42 if those cases are included where the intercept is numerically correct, but no sign (away or towards) is attached. A further 12 observations agree to within $1'.0$, but this is well outside the limit of agreement between two independent reductions working to a nominal accuracy of $0'.1$. Of the remaining 43 observations 33 are grossly

TABLE X. COMPARISON OF 97 POSITION-LINE ERRORS AS GIVEN AND AS RECALCULATED

Obs. No.	Pos.-line error Given	Found	Notes
1	1'·75A	2'·0A	✓
5	0'·4T	>10'A	L
9	5·5A	6·3A	?
10	0·2T	0·8A	?
14	0·5A	1·2T	S
16	0·5A	0·4A	✓
18	+1·7	1·9T	C✓
19	0·5A	0·2A	✓
19	0·5	0·1A	C✓
23	0·2A	0·8A	?
26	1·0A	0·6A	✓
26	0·5T	0·3T	✓
27	1·2T	1·8T	?
28	0·75T	0·5T	✓
29	0·0	0·2T	✓
30	6·5T	2·5T	L
31	0·5A	1·6A	S
32	1·0A	1·0T	S
34	0·1A	0·3T	✓
35		1·6A	X
36	2·0A	3·0T	L
38	6·6	6·8T	C✓
39	1·3T	1·0T	✓
40	1·0A	1·2A	✓
41	0·6A	6·7A	L?
42	2·0A	1·6A	✓
43	1·2T	0·3T	?
44	0·5T	1·7T	S
45	2·1A	1·6A	✓
46	0·5A	0·8A	✓
47	2·5T	1·2T	S
47	1·0A	5·0A	L
47	0·5T	2·4A	L
47	0·0	0·4T	✓
47	3·0A	5·6A	L

Obs. No.	Pos.-line error Given	Found	Notes
48	0'·4T	8'·8T	L✓
49	8·0T	8·2T	✓
50	1·0T	9·7T	L
50	0·4T	0·4A	?
51	1·7A	>10'A	L✓
53	1·4A	1·2A	✓
54	0·0	7·4A	L✓
57	0·5	0·4A	C✓
59	0·4T	0·4T	✓
60	0·1A	0·0	✓
63	0·2A	0·4A	✓
64	0·8A	0·6A	✓
65	0·5T	7·0T	L
65	0·5A	1·6A	S
67	0·7T	8·9T	L✓
68	4·8T	2·9T	S
69	0·3A	3·5A	L
71	0·2T	7·2A	L?
71	3·25A	3·8A	?
72	1·5T	1·2A	L
73	0·0	3·9T	L✓
74	0·25T	>10'	L
75	1·2T	4·0A	L
76	0·0	0·3A	✓
77			X
79	0·3A	0·5A	✓
80	0·1T	>10'	L
82	0·5A	0·4A	✓
83	0·5T	0·4T	✓
85	1·0T	>10'	L
87	1·0A	0·8A	✓
88	0·2A	0·2A	✓
89	0·5T	0·8T	✓
90	0·0	0·5A	✓
91	4·0T	>10'A	L

Obs. No.	Pos.-line error Given	Found	Notes
92	0'·2A	6'·0T	L?
96			X
97	Nil	0·4T	✓
100	0·5T	8·6A	L?
102	0·4T	0·2T	✓
103	0·5A	7·9A	L?
103	12·8T	9·1T	L
104	0·15T	0·1T	✓
106	1·5A	0·9A	?
108	?1·5A	>10'A	L
109	0·5A	0·1A	✓
109	0·8A	4·1T	L
109	2·95A	6·3T	L?
109	0·57T	9·1T	L✓
110	0·1	>10'A	L
111	1·5A	1·0A	✓
113	0·0	0·6A	?
114	0·6A	0·5A	✓
115	2·5A	1·7A	?
117	0·5A	1·1A	?
118	1·5A	2·0A	✓
119	0·5A	1·8T	L
120	1·25A	1·3A	✓
121	1·4A	0·4A	?
121	1·9T	1·7T	✓
121	0·5A	2·8A	L
122	0·8A	0·9A	✓

Notes Code	No.	Notes Code	No.
✓	38	S	7
L✓	6	L	21
C✓	4	X	3
?	12		
L?	6		

The following code is used in the column 'notes':

- ✓ Difference 'given-found' 0'·5 or less.
- ? Difference 'given-found' between 0'·6 and 1'·0.
- S Small discrepancy 'given-found' not greater than 2'·0; no explanation.
- L Large discrepancy 'given-found' greater than 2'·0; no explanation.
- L ✓ Large discrepancy explained by careful investigation, attributed to errors in auxiliary data (usually the corrected altitude being given instead of the sextant altitude), are reduced to 0'·5 or less.
- L ? As above, but reduced to between 0'·6 and 1'·0.
- C ✓ No direction given; discrepancy reduced to 0'·5 or less if direction assumed.
- X No position-line error given and, in two cases, insufficient data to calculate it.

in error, inadequate information to permit a reduction or comparison accounts for 3 more, 6 are small errors less than 2′, which could conceivably arise through careless reduction, and the remaining one could either be a small error or a good reduction with a wrong sign attached to the intercept. Some 12 of those grossly in error can be explained, and reduced at least to errors of 1′.0 or less, on the assumption that the corrected altitude has been entered instead of sextant altitude, uncorrected for dip and refraction. Even so there remain 21 observations for which the position-line error given cannot be reconciled with the details of the observation. There is at least a fair chance that most of these 21 cases are due to incorrectly copied data; but some are definitely errors of reduction. To summarize, out of 97 observations, the position-line errors of no fewer than 31 are inexplicably inconsistent with the other data, some of the data are wrong in other observations, and only 50—or rather more than half—are consistent to within 1′.0 with the data given.

Although it would have been possible to re-reduce a larger sample, or even all of the observations, little would have been gained. Even when a discrepancy is discovered and explained, by much individual work on that particular observation, it is not certain whether the data or the position-line error is wrong. A few incorrect reductions might have been corrected, but the labour would have been great and the results would have been unrepresentative.

It may be thought that these figures indicate a 'blunder ratio' much larger than those given in Table II and obtained by analysis of the observations themselves. Apart from the doubt as to where the blunder lies—in the position-line error or in the data—the above analysis is based essentially on 'one observer, one observation', whereas the observations are controlled by a relatively few observers. The above table primarily relates to observers, while the blunder ratio of Table II relates to observations.

10. Distribution of observations between observers. The whole mass of observations has so far deliberately been treated without reference to individual observers. Apart from the aim of presenting an overall picture, there are in general far too few observations to warrant the determination of observers' accuracy and thus to allow for the widely differing numbers of observations. It is, however, of interest to examine the degree of dependence of the results on the numbers of observations contributed by individual observers; to some extent this will indicate whether the sample of observations is at all representative. In Table XI are given the observers contributing 25 or more observations, together with a determination of their standard deviation σ: the table relates first to all observations, and secondly to those ('accurate' positions) in which the ship's position is known to within 1′.0 and 0′.5. Similar figures are given for the group of the remaining observers each with fewer than 25 observations, for comparison with the figures for the group of those with more than 25 observations.

TABLE XI. OBSERVERS CONTRIBUTING 25 OR MORE OBSERVATIONS

	All observations		Position accurate to:			
			1'·0		0'·5	
Observer No.	No. of obs.	σ	No. of obs.	σ	No. of obs.	σ
		'		'		'
19	73	2·2	29	1·1		
23	77	2·3	41	2·7		
27	69	1·4	39	1·2		
50	892	2·2	268	2·7	113	2·8
54	28	1·6				
65	26	3·0*	26	3·0*		
94	25	0·8	25	0·8	25	0·8
104	45	0·9				
105	211	1·7	211	1·7	211	1·7
109	71	1·6	65	1·6	57	1·7
124	27	1·0				
126	112	0·8	107	0·8	69	0·8
129	94	0·9				
136	49	0·8	45	0·8		
140	47	0·9	35	0·9	35	0·9
142	70	0·8	70	0·8	50	0·7
150	99	1·4				
152	62	1·7				
155	30	2·5				
158	27	2·0	27	2·0	27	2·0
161	36	2·6				
171	34	1·1	34	1·1		
Groups of observers with more or fewer than 25 observations:						
25 or more	22 obrs. 2203	2'·10	14 obrs. 1022	1'·87	8 obrs. 587	1'·81
others	134 ,, 1116	1·93	115 ,, 836	1·56	113 ,, 731	1·51

(*9 observations by No. 65 were subsequently suspected of being intercepts from the D.R. position; they have not been corrected.)

The range of standard deviation for the observers (all observations) varies from 0·8 to 3·0 (though this is probably accidentally high). The most remarkable feature, however, is the small apparent effect on the standard deviation of an increase in the accuracy of the estimate of the ship's position. It will be noticed how the large number of observations by observer No. 50 dominates the first group determinations; there is a significant increase in his standard deviation, as the observations are progressively restricted to more accurate positions! In all these categories the remaining observers, treated as a whole, are more accurate than those who contributed most observations and, in particular, than observer No. 50.

Although the figures are not given here, values of σ were calculated for all 61 observers who contributed 10 or more observations; the values, which mean little because of the small number of observations, range from 0·4 to 5·1. Similar figures are not available for observations from 'accurate' positions, but 45 and 36 observers respectively contributed 10 or more observations for position accuracies of 1'.0 and 0'.5.

There would seem some reason to suppose that the results are unduly influenced by observer No. 50, but the effect is not large since he contributes only one-seventh of the total number of 'accurate position' observations. In any case it is clearly impossible to do anything other than weight the observers according to the number of observations, which leads precisely to giving each observation equal weight.

11. MOST ACCURATE OBSERVERS. An estimate can be made of the accuracy attained by the most accurate observers by combining the observations of, say, the best 8 observers in Table XI. These are obtained by omitting observers Nos. 23, 50, 65, 105, 109 and 158, and give rise to a spread of standard deviation of only 0'.7 to 1'.2, with a total of 383 observations; the standard deviation of the whole 383 observations works out at 0'.9. The actual frequency distribution of these 383 observations indicates 50, 90 and 95 per cent errors of 0'.5, 1'.4 and 2'.0 respectively.

Not a great deal of weight should be attached to these results, because of the small number of observations, but also because selection of observations is likely to be a more significant feature when only a few observers are concerned.

The observers Nos. *19, 27, 94, 126, 136, 140, 142* and *171* are experienced navigators (more than 5 years) and one is a naval officer; but they serve on different sizes of ships and use different types of sextant and methods.

12. OBSERVERS WITH MORE THAN 50 OBSERVATIONS. There are only five observers with more than 50 observations from an accurately known position:

Observer no.	50	105	109	126	142
No. of observations	268	211	65	107	70
Standard deviation	2'.7	1'.7	1'.6	0'.7	0'.8

Observer No. 105 is in reality the group observations of 58 cadets, but the others are individual observers.

Some analysis has been done for each of these observers to see whether supposedly homogeneous observations indicate the influence of any of the factors (quality of horizon, state of sea, &c.) for which details were recorded. The results are disappointing in that, in general, the observations (for each observer) almost all fall into one category. Thus observers No. 50 and 105 almost always take two shots, whereas the others rarely take more than one shot. And also the few observations of the Sun were made almost entirely by one observer (No. 105), who made few observations of the stars. The only factor which does offer some possibility of analysis is the quality of the horizon: details, for what they are worth, are given in Table XII.

Although observers Nos. 105 and 109 appear unaffected by a poor horizon, it certainly seems to diminish the accuracy of the other three observers.

Attention was first called to the non-Gaussian distribution by examination of all observations by observer No. 50, which were analysed first as a pilot investigation. The distribution of his 268 observations is, however, very nearly normal; nevertheless, there are some five or six observations which are larger than might be expected and which could be

TABLE XII. FIVE OBSERVERS WITH MORE THAN 50 OBSERVATIONS: QUALITY OF HORIZON

Observer No.		50	105	109	126	142	All observers
Good horizon	No. of obs.	221	177	41	78	62	579
	σ	2·3	1·7	1·6	0·6	0·5	1·8
Poor horizon	No. of obs.	47	34	24	29	8	142
	σ	3·8	1·7	1·6	1·0	1·3	2·5

explained on the assumption of a blunder ratio of about 5 per cent. This compares with a value of about 8 per cent for all observations. This suggests that a high proportion of large errors is not necessarily a consequence of the inhomogeneity of the observations, and the mixture of the observers, but occurs for individual observers, even when only 'accurate' observations are considered.

As a further step the effect of the quality of the horizon was eliminated by analysing only the observations taken with a good horizon. The effect persists for observer No. 50, but is not present for observers Nos. 126 and 142 who do not have any large errors among their relatively few observations; for observer No. 105 there is a smaller excess of large errors but a marked excess of small ones; observer No. 109 has too few observations for any deductions to be drawn, except that he has a large personal error of about $-0\overset{'}{.}8$.

13. COMPARISON WITH OTHER RESULTS. The Working Party is not here concerned with a systematic comparison of these results with those obtained in other investigations. But it may be of interest to give some recently published figures.

C. H. Smiley (*Navigation*, *Los Angeles*, **2**, 342, 1951). Probable error of a single measure of altitude with a marine sextant by a careful observer, after allowing for personal equation, is about $\pm 0\overset{'}{.}27$ ($\sigma = 0\overset{'}{.}40$). This does not include errors due to refraction, dip and indefiniteness of the horizon.

P. V. H. Weems (*Navigation*, *Los Angles*, **2**, 354, 1951). Gives data from which the probable error of a single position line (excluding systematic errors) is deduced as $\pm 0\overset{'}{.}6$ ($\sigma = 0\overset{'}{.}8$); observations made from a surveying ship.

G. M. Clemence (*Navigation*, *Los Angeles*, **3**, 36, 1951). Determines the errors of position lines as part of an investigation into refraction at low altitudes. The figures of (about) $1'.4$ and $5'.5$ for the 50 per cent and maximum errors refer to low-altitude observations of the Sun, but are shown to be largely independent of altitude.

B. Chr. Peterson (this *Journal*, **5**, 31, 1952). In the course of analysing nearly 2000 observations for refraction at low altitudes, the author deduces that the mean error of a single observation (of the Sun's altitude) varies between $0'.58$ and $1'.57$, depending on the quality of the horizon. This corresponds to a 50 per cent error of between $0'.4$ and $1'.0$.

Barcoo observations (this *Journal*, **7**, 317, 1954) lead to a determination of the accuracy of a single position line of a standard deviation of from $0'.7$ to $1'.2$ according to the observer, a mean value being $1'.1$; these observations were also made from a surveying ship.

It will be seen that the 'best observers' have errors comparable to those determined by surveyors; but the proportion of poor observers, with much larger errors, is certainly large.

14. MEMBERSHIP. The original members of the Working Party consisted of:

Lt.-Cdr. M. Blake, R.N. (representing H.M.S. *Dryad*).
Mr. S. M. Burton (author of Burton's Tables).
Rear-Admiral (now Vice-Admiral Sir) A. Day (then Hydrographer of the Navy).
Captain W. Hamilton (Nautical School).
Mr. J. B. Parker (Statistician).
Captain B. Chr. Petersson (instructor in a Swedish Nautical School).
Mr. D. H. Sadler (Superintendent, H.M. Nautical Almanac Office; Chairman).
Mr. W. A. Scott (Head of navigation section, H.M. Nautical Almanac Office).
Mr. A. J. R. Tyrrell (sea-going Merchant Navy Officer).

Lt.-Cdr. M. Blake resigned from the Working Party on being posted abroad, and his place was taken for two years by Lt.-Cdr. J. Blake, R.N. of H.M.S. *Dryad*; he in turn was posted abroad and has been replaced by Lt.-Cdr. R. B. Michell, R.N.

The Executive Secretary of the Institute, Mr. M. W. Richey was formally co-opted at an early meeting.

Captain Brett Hilder (sea-going practical navigator and formerly president of the Australian Institute of Navigation), and Commander J. M. Sharpey-Schafer, R.N. (a hydrographic survey officer) were later co-opted as a result of their expressed interest.

Mr. W. A. Scott acted as secretary to the Working Party throughout.

The whole of the transcription of the data from the forms to punched cards and the subsequent analysis was done in H.M. Nautical Almanac Office. The methods of analysis were devised in cooperation with Mr. J. B. Parker, and conducted to the general requirements of the Working Party.

The Report, which was amended and approved by the Working Party as a whole, was drawn up by the Chairman with assistance from Mr. Parker and Mr. Scott; it is therefore necessarily influenced by the style and, to a lesser extent, by the views of the Chairman, who accepts responsibility for any shortcomings in these respects. The Chairman would like to take this opportunity of thanking the members of the Working Party for their practical help and encouragement, particularly those whom distance has prevented from attending meetings but not from contributing ideas and guidance.

15. ACKNOWLEDGMENTS. First of all the Working Party wishes to express its gratitude to all the observers who have communicated their observations; without them there would have been no results and no report. Much was asked from practical navigators, who as a class are far more concerned with practice than with theory and with sights rather than with printed forms; the Working Party is most appreciative of the special efforts that must have been made. Particular thanks are due to those—and they are the majority—who took such pains to write, or type, their observations with meticulous care, and to add illuminating comments.

Mr. W. A. Scott, Head of the Navigation Section of H.M. Nautical Almanac Office, acted as secretary to the Working Party and has been in charge of all the collation, interpretation, computing and statistical analysis that has been involved in the preparation of this Report. The Working Party owes him a debt of gratitude, greater perhaps than most members realize. Much of the routine collation and computing was ably done by Miss A. B. Grogan, under Mr. Scott's direction, while Mr. D. A. Harragan organized and supervised all the punched-card work.

A tribute must also be paid to the Institute's staff who not only handled and acknowledged all the forms as they were received, but maintained records and card-indexes of observers.

ANNUAL GENERAL MEETING

THE *eleventh Annual General Meeting of the Institute will be held (at the Royal Geographical Society) at 3 p.m. on Wednesday 16 October. The theme of the Presidential Address will be the presentation of navigation intelligence.*