London, Edinburg & Dublin = philosophical magazine.

THE

PHILOSOPHICAL MAGAZINE,

OR

ANNALS

OF

CHEMISTRY, MATHEMATICS, ASTRONOMY, NATURAL HISTORY, AND GENERAL SCIENCE.

BY

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AND

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"Nec aranearum sane textus ideo melior quia ex se fila gignunt, nec noster vilior quia ex alienis libamus ut apes." JUST. LIPS. *Monit. Polit.* lib. i. cap. 1.

VOL. IX.

NEW AND UNITED SERIES OF THE PHILOSOPHICAL MAGAZINE AND ANNALS OF PHILOSOPHY.

JANUARY—JUNE, 1831.

LONDON:

PRINTED BY RICHARD TAYLOR, RED LION COURT, FLEET STREET,
Printer to the University of London.

SOLD BY LONGMAN, REES, ORME, BROWN, AND GREEN; CADELL; BALDWIN AND CRADOCK; SHERWOOD, GILBERT, AND PIPER; SIMPKIN AND MARSHALL; UNDERWOOD; AND S. HIGHLEY, LONDON:— AND BY ADAM BLACK, EDINBURGH; SMITH AND SON, GLASGOW; AND HODGES AND M'ARTHUR, DUBLIN.

XXX. *On* Mr. Witchell's *Method of clearing a Lunar Distance. By* C. Rumker, *Esq.**

I HAVE remarked that a very imperfect approximate foreign method for clearing the lunar distance (under some circumstances liable to considerable errors) is now much in vogue amongst British mariners, although they have better methods of their own: amongst which Witchell's appears to me one of the best. I think approximate methods better calculated for mariners than direct ones, since small errors are more likely to vitiate the result and more easily escape discovery than in the former, where the computer after a little practice can nearly judge from the altitudes and distance what each correction will amount to: and Witchell's enables him morever to assign to himself by a rough sketch the reasons for his proceedings. But as analytical demonstrations are now more approved, I offer you the following one of his formula, preceded by a simpler practical rule than the one usually given.

Add together the logarithms of,

Cotangent of half the sum of both apparent altitudes.
Tangent of half their difference.
Cotangent of half the apparent distance.

The sum of these logarithms is the tangent of an arc A, which must be added to half the apparent distance, and also subtracted from it. Then add together the logarithms of,

Cotangent of the sum of A and half distance.
Cotangent of the *lesser* apparent altitude.
Proportional logarithm of the corresponding correction.

Cotangent of the difference of A and half distance.
Cotangent of the *greater* apparent altitude.
Proportional logarithm of the corresponding correction.

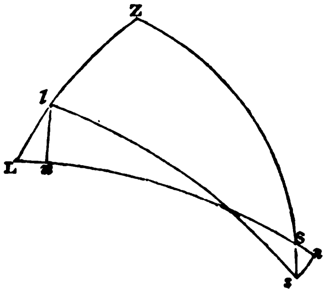
The sums are the proportional logarithms of two corrections in distance, whereof the difference must be subtracted† from the apparent distance as long as A is less than half the apparent distance; but if A is greater, their sum must be added to the apparent distance if the moon's altitude is greatest, but subtracted therefrom if that altitude is least. With this corrected distance find from Table XXXV. of Norie's *Req. Tables*, the corrections answering to the moon's correction in altitude and in distance: their difference added to the corrected distance if

* Communicated by the Author.

† I here suppose that the correction in distance depending on the moon's altitude is greater than that from the sun. In the very rare contrary case their difference must be added when A is less than half distance.

if this is less, but subtracted from it if it is greater, than 90°, gives the true distance.

Demonstration.



D = apparent distance L S.
h = apparent sun's altitude.
H = apparent moon's altitude.

Then is, with the omission of the third correction, which we shall explain hereafter, the true distance $ls = \mathrm{LS} - \mathrm{L}m + \mathrm{S}m$ $= \mathrm{LS} - \mathrm{L}l \cos \mathrm{L} + \mathrm{S}s \cos \mathrm{S}$.

But $\cos \mathrm{L} = \dfrac{\sin h - \sin \mathrm{H} \cos \mathrm{D}}{\cos \mathrm{H} \sin \mathrm{D}} = \dfrac{\sin h}{\cos \mathrm{H}\, 2 \sin \frac{1}{2}\mathrm{D} \cos \frac{1}{2}\mathrm{D}}$

$$- \frac{\sin \mathrm{H}}{\cos \mathrm{H} \operatorname{tang} \mathrm{D}} = \frac{\sin h \sec^2 \frac{1}{2}\mathrm{D} - \sin \mathrm{H}\,(1 - \operatorname{tang}^2 \frac{1}{2}\mathrm{D})}{2 \cos \mathrm{H} \operatorname{tang} \frac{1}{2}\mathrm{D}} =$$

$$\frac{\sin h - \sin \mathrm{H} + (\sin h + \sin \mathrm{H}) \operatorname{tang}^2 \frac{1}{2}\mathrm{D}}{2 \cos \mathrm{H} \,.\, \operatorname{tang} \frac{1}{2}\mathrm{D}} =$$

$$\frac{\dfrac{\sin h - \sin \mathrm{H}}{\sin h + \sin \mathrm{H}} + \operatorname{tang}^2 \frac{1}{2}\mathrm{D}}{\dfrac{\operatorname{cotang} \mathrm{H} \,.\, 2 \sin \mathrm{H} \operatorname{tang} \frac{1}{2}\mathrm{D}}{\sin h + \sin \mathrm{H}}} = \frac{\operatorname{tang} \mathrm{H}\left(\dfrac{\sin h - \sin \mathrm{H}}{\sin h + \sin \mathrm{H}} \operatorname{cotang} \frac{1}{2}\mathrm{D} + \operatorname{tang} \frac{1}{2}\mathrm{D}\right)}{\dfrac{\sin h + \sin \mathrm{H} - (\sin h - \sin \mathrm{H})}{\sin h + \sin \mathrm{H}}}$$

$$= \frac{\operatorname{tang} \mathrm{H}\left(\operatorname{tang} \frac{1}{2}\mathrm{D} + \dfrac{\sin h - \sin \mathrm{H}}{\sin h + \sin \mathrm{H}} \,.\, \operatorname{cotang} \frac{1}{2}\mathrm{D}\right)}{1 - \dfrac{\sin h - \sin \mathrm{H}}{\sin h + \sin \mathrm{H}} \operatorname{cotang} \frac{1}{2}\mathrm{D} \,.\, \operatorname{tang} \frac{1}{2}\mathrm{D}}$$

$$= \frac{\operatorname{tang} \mathrm{H}\left(\operatorname{tang} \frac{1}{2}\mathrm{D} + \dfrac{\operatorname{tang} \frac{1}{2}(h - \mathrm{H})}{\operatorname{tang} \frac{1}{2}(h + \mathrm{H})} \,.\, \operatorname{cotang} \frac{1}{2}\mathrm{D}\right)}{1 - \operatorname{tang} \frac{1}{2}\mathrm{D} \dfrac{\operatorname{tang} h (\frac{1}{2} - \mathrm{H}) \operatorname{cotang} \frac{1}{2}\mathrm{D}}{\operatorname{tang} \frac{1}{2}(h + \mathrm{H})}}.$$

and making $\operatorname{tang} \mathrm{A} = \operatorname{cotang} \frac{1}{2}\mathrm{D} \dfrac{\operatorname{tang} \frac{1}{2}(h - \mathrm{H})}{\operatorname{tang} \frac{1}{2}(h + \mathrm{H})}$

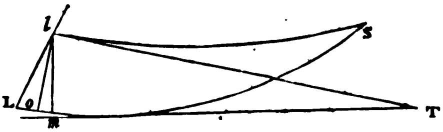
we have $\cos \mathrm{L} = \operatorname{tang} \mathrm{H} \,.\, \operatorname{tang}(\frac{1}{2}\mathrm{D} \pm \mathrm{A})$ accordingly as $h \gtrless \mathrm{H}$.

and also $\cos \mathrm{S} = \operatorname{tang} h \operatorname{tang}(\frac{1}{2}\mathrm{D} \mp \mathrm{A})$. Q. E. D.

In case that $\mathrm{A} > \frac{1}{2}\mathrm{D}$, the sign of the cosine of either L or S, and consequently that of the corresponding correction, will be changed. It may easily be proved that A is the part of the apparent distance intercepted between its middle and a perpendicular from the zenith upon it. It remains now to explain the third correction, which is nearly applicable to all approximate methods:

We have hitherto supposed $s\,l = s\,m$, which is incorrect.

Describe from s as pole the circle $l\,o$, then is $o\,m$ the third cor-



rection. Draw at m a tangent and make $T\,l$ = tang D, then is

$$\sin T = \sin lm \, . \, \text{cotang}\, D.$$

but

$$mo \, . \sin 1'' = \text{tang}\, D\,(1 - \cos T) = \text{tang}\, D \, . \, 2 \sin^2 \tfrac{1}{2} T = \tfrac{1}{2} \text{tang}\, D \; 4 \sin^2 \tfrac{1}{2} T = \tfrac{1}{2} \text{tang}\, D\,(2 \sin \tfrac{1}{2} T)^2 = \tfrac{1}{2} \text{tang}\, D \sin^2 T$$

$$= \tfrac{1}{2} \text{tang}\, D \,\text{cotang}^2 D \sin^2 lm = \frac{\sin^2 Ll - \sin^2 Lm}{2 \,\text{tang}\, D}.$$

Hamburgh, Jan. 16, 1831. C. RUMKER.

XXXI. *Facts bearing on the Theory of the Formation of Springs, and their Intensity at various Periods of the Year. By* W. J. HENWOOD.

THAT those springs which exist during the winter and disappear as summer approaches, owe their origin to rain, has not I believe been disputed. But whether we may ascribe to the same cause those on which changes of the seasons appear to exert but little influence, has been frequently discussed. In the mining districts of Cornwall, registers of the performance of the steam-engines employed for pumping water, is periodically published by Messrs. John and Thomas Lean, of Camborne. These documents supply information from which it is not difficult to calculate the quantity of water drawn by each engine in a month, and consequently the intensity of springs at the spot. The particulars contained in the following columns are of some consequence in this investigation.

In some of the extensive mines several steam-engines are required; and as they are usually erected at a considerable distance from one another, each drains the whole of a certain district. Hence I think we may safely assume the water drawn by one engine as representing the intensity of the springs at that spot. The numbers in Table I. denote cubic feet of water drawn by one engine; in Table II. the averages of the respective mines are for one engine on each; but in Table III. the numbers are intended for cubic feet of water drawn by all the engines on each of the respective mines.

Mine