TIME CORRECTION TO MERIDIAN PASSAGE OBSERVATIONS.

This useful thread, together with Frank's recent "*Latitude and longitude around noon*" problem, have stimulated me to increase my understanding of this effect. I set out in the attached PDF various snippets that I have gleaned. They will bore the cognoscenti but may be of interest to those who share my curiosity. Please note that the tabular information therein is solely to illustrate the magnitude of the effects under study and should not be used as a source of data.

SECTION 1.

Firstly, I investigated how the time correction for Sun meridian passage sights varies with season and latitude for a stationary observer. The correction (in seconds) should be applied to the time of observed maximum altitude to obtain the time of meridian transit. Dates are the 20th of each month.

TABLE 1

Month (20th)	**Jan**	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
Sun Dec Rate (mins/hour)	+0.5	+0.8	+1.0	+0.9	+0.5	0	-0.5	-0.9	-1.0	-0.8	-0.5	0
Latitude												
60N	-16	-24	-26	-21	-13	0	10	21	26	23	16	0
45N	-10	-15	-15	-11	-6	0	5	11	15	15	10	0
30N	-7	-9	-9	-5	-2	0	2	5	9	9	7	0
15N	-5	-6	-4	-1	1	0	-1	1	4	6	5	0
0N	-3	-2	0	3	3	0	-3	-3	0	2	3	0
15S	-1	1	4	7	6	0	-5	-7	-4	-1	-1	0
30S	2	5	9	11	9	0	-7	-11	-9	-5	-2	0
45S	5	10	15	17	13	0	-10	-17	-15	-10	-5	0
60S	10	19	26	27	19	0	-16	-27	-26	-19	-10	0

SECTION 2.

Next I wanted to assess the effect of observer motion alone. To do this I had to choose a body of constant declination. The effect still varies with the observer's latitude, his velocity and the difference between his latitude and the body's declination. For table 2, I used Betelgeuse (declination N07° 25'), hence the symmetry about that latitude. I also used a constant speed of 10 knots: other (reasonable) speeds gave pro rata corrections.

TABLE 2

COG LAT	**000**	**045**	**090**	**135**	**180**	**225**	**270**	**315**
60N	+4m 03s	+2m 47s	0s	-2m 47s	-4m 03s	-2m 47s	0s	+2m 47s
45N	+2m 12s	+1m 31s	0s	-1m 31s	-2m 12s	-1m 31s	0s	+1m 31s
30N	+1m 08s	0m 47s	0s	-0m 47s	-1m 08s	-0m 47s	0s	0m 47s
15N	+0m 21s	0m 15s	0s	-0m 15s	-0m 21s	-0m 15s	0s	0m 15s
0N	-0m 20s	-0m 14s	0s	+0m 14s	+0m 20s	+0m 14s	0s	-0m 14s
15S	-1m 00s	-0m 42s	0s	+0m 42s	+1m 00s	+0m 42s	0s	-0m 42s
30S	-1m 47s	-1m 15s	0s	+1m 15s	+1m 47s	+1m 15s	0s	-1m 15s
45S	-2m 52s	-1m 59s	0s	-+m 59s	+2m 52s	-+m 59s	0s	-1m 59s
60S	-4m 43s	-3m 14s	0s	+3m 14s	+4m 43s	+3m 14s	0s	-3m 14s

I would make these comments about table 2.

1. The table would be completely different for bodies of other declinations.
2. The values illustrate that only the North/South component of velocity affects the correction when the body's declination is constant, which is always the case for stars but usually not so for the Sun, Moon or planets.
3. If the body's declination is not constant, then East/West motion becomes an important part of the time correction. The value of the correction is complex and depends on (at least) five variables – body declination rate, body declination, observer's latitude and Northerly and Westerly components of observer's velocity. Use of a formula is essential.

SECTION 3. The Moon.

The Moon's declination range in a single earth orbit varies over a cycle lasting 18.6 years. At a maximum, (major lunar standstill), it changes from +28.5 to -28.5 degrees in 14 days. At a minimum, 9.3 years later, (minor lunar standstill) it changes from +18.5 to -18.5 degrees in a similar half orbit. Depending on the stage of that cycle, her peak rate of change of declination varies between 18.2 and 11.8 arc minutes per hour compared with the Sun's maximum rate of about 1 arc minute per hour.

The next maximum (of +18.2 minutes per hour) will occur on 15th October 2024. In table 3, the observer is assumed to be stationary exactly on the Greenwich Meridian at the time of true transit, 22:25:19 UT. In addition to the corrections, columns 4 and 5 show the latitude and longitude that would result from a traditional 'equal altitudes' meridian passage calculation ignoring this effect.

TABLE 3

2024/10/15 Moon				
Latitude	**UT of Max Obs.Alt.**	**Correction**	**Uncorrected Latitude**	**Uncorrected Longitude**
60N	22:33:55	-8m 36s	N60° 01.3'	W002° 04.8'
45N	22:30:17	-4m 58s	N45° 00.8'	W001° 11.9'
30N	22:28:11	-2m 52s	N30° 00.5'	W000° 41.5'
15N	22:26:39	-1m 20s	N15° 00.2'	W000° 19.3'
0N	22:25:19	0s	S00° 00.1'	E000° 00.0'
15S	22:23:59	+1m 20s	S15° 00.2'	E000° 19.3'
30S	22:22:27	+2m 52s	S30° 00.5'	E000° 41.5'
45S	22:20:21	+4m 58s	S45° 00.7'	E001° 12.0'
60S	22:16:43	+8m 36s	S60° 01.3'	E002° 04.6'

A few comments about table 3 results.

1. At the time of maximum declination rate, the declination itself is zero. Hence, in this example, at transit, the Moon will be overhead at the equator – but it could be overhead at any latitude below 28.5° and the above data would then be nearly symmetrical about that latitude.
2. For the Moon, in addition to the time correction and corresponding longitude correction, there is also a small latitude correction. This occurs with all bodies, but is only significant in the case of the Moon on account of her greater declination rate.
3. The corrections shown, at a major lunar standstill, are about 18 times larger than those for the Sun in table 1.
4. This example is after dusk and (at best) only a moonlit horizon will be visible. Landlubbers with artificial horizons will be unconcerned.
5. A meridian passage sight can also be taken using planets and stars, though duration of twilight restricts the opportunities at sea. The magnitude of the time correction is of the same order as for the Sun, but see section 4 below.
6. Having obtained the time correction, always use the Sun longitude rate (15°/hour) to obtain the longitude correction. So, for a Moon correction of, say, -4m 0s, the corrected longitude is 1° 00' further East.

SECTION 4. FORMULAE.

I've avoided the math so far, but now it is time to look at the formulae!

4.1 Formula 1. On Apr 8th 2017, Frank Reed posted a snippet in this thread from page 154 of the 1914 *Admiralty Manual of Navigation.*

The 'general' formula for the time correction to the time of maximum altitude is

$$(\tan L \sim \tan D)\,(v_L + V_D)\,V_G^{-1}\,\left(1 + \frac{2v_G}{V_G}\right)\,\text{cosec}\ 1^s.$$

Initially I found the use of upper and lower case expressions v_G and V_G confusing, as was the cosec 1^s. The 1^s refers to 1 second of time which, in the case of the sun is 15" of arc whose cosec is 13751. The (upper case) V_G is the longitudinal rate movement of the body in minutes of arc per hour. In the case of the Sun, this is 900'/hour. Dividing 13751 by 900 gives 15.28 and approximating 2/900 to 0.002 gives the quoted Sun specific formula for the time correction (in seconds) as

$$15{\cdot}28 \tan L \,\tilde{+}\, \tan D)\,(v_L \,\tilde{+}\, V_D)\,(1 \pm {\cdot}002\ v_G).$$

Opening bracket after 15.28 omitted in source document.

The remaining (lower case) v_G is the same as "x" and the ($v_{L\ \sim/+}\ V_{D}$) term is the same as "y", both discussed in 4.2 below.

For later use, I mention that cosec 1^s divided by average body longitudinal rate gives the following constant values for other bodies.

TABLE 4.1

Body	1^s as arc	Cosec of arc	Longitudinal rate	**Formula constant**
Sun	15"	13751	900'/hour	**15.28**
Moon	14.495"	14230	869.7'/hour	**16.36**
Star	15.04"	13714	902.4'/hour	**15.20**
Planet	15.02"	13732	901.2'/hour	**15.24**

4.2. Formula 2.

The previous day, Frank posted the 1938 *Admiralty Manual of Navigation Vol 3*. On page 154, this formula is quoted for the time correction. This is like Formula 1, but preferable as the 2/900 approximation has been withdrawn. It is specific to the Sun.

$$H = 15{\cdot}28\,y\left[1 \pm \frac{2x}{900}\right](\tan l \pm \tan d)$$

"***x***" is the observer's rate of change of longitude in minutes of arc per hour. This can be obtained by the expression x = -1 * SOG * sin (COG) / cos(abs(Lat)). The (-1) is to make Westerly motion positive (see below), whilst the "abs" is to prevent Southerly latitudes making it negative again! (COG and SOG are Course and Speed over ground – track and groundspeed to non-sailors.)

"***y***" is the observer's rate of change of latitude, minus the body's rate of change of declination. Both terms expressed in minutes of arc per hour (which equals speed in knots).

The +/- expressions (and the ~/+ in 4.1 above) need further thought. The formulae are sensitive to the "name" of the latitude and declination and to the direction of the observer's motion along both axes. The sources above quote rules which determine whether to use the upper or lower signs. "Wilson" (see 4.3 below) suggests the following convention which I find easier to understand.

1. Northerly latitudes and declinations are +ve.
2. Westerly motion of the observer is +ve.
3. Northerly motion of the observer is +ve.
4. If a declination is changing so that the it is becoming more Northerly, then the change is +ve. (IE From S21° 30.0' to S21° 25.0' is a positive change of 5'.)

Then the (Sun only) formula becomes: -
Correction = **15.28 * y * (1+ 2x/900) * (tan Lat – tan Dec),** where y and x are as described above. By applying the above convention, the rules of arithmetic resolve the matter, remembering that subtracting a minus quantity is addition.

4.3. Formula 3.

A different formula is quoted in "Position from Observation of a single body" by James A Wilson. It was published in the Journal of the Institude of Navigation. Vol 32. No 1. Spring 1986.

$$\boxed{\Delta t = \frac{10{,}800}{\pi(\Delta \mathrm{LHA})^2}(\mathrm{Sn} - d)(\tan \mathrm{Lat} - \tan \mathrm{Dec})}$$

Here, Sn is the Northerly component of observer's speed in knots and d is the body's Declination Rate in <u>minutes</u> per hour. The ΔLHA is the body's longitude rate minus the observer's longitude rate, both in <u>degrees</u> per hour.
The (Sn – d) equates to "y" and, for the Sun (15° per hour) and an observer with no East/West motion, the first term reduces to the now familiar 15.28 and the formula becomes
15.28 * y * (tan Lat – tan Dec).

For other bodies and/or an observer with East/West motion, the first term (15.28) must be changed to **10800 / (PI * (Body Longitude Rate – x/60)^2).**

For other bodies and an observer with no East/West motion, the first term (15.28) becomes **10800 / (PI * (Body Longitude Rate)^2).** This gives the following formula constant values which are identical to those in Table 4.1 derived from formula 1.

TABLE 4.2

Body	LHA rate	(LHA rate)^2	**Formula constant**	
Sun	15	225.00	**15.28**	Mean rates used for Moon and planets
Moon	14.495	210.11	**16.36**	
Star	15.04	226.20	**15.20**	
Planet	15.02	225.60	**15.24**	

SECTION 5. Comparison of formulae.

In formula 1, the effect of the observer's longitudinal motion is expressed by (1+ 0.002 x).
In formula 2, it is expressed by the more accurate (1 + 2x/900) term, whilst the rest is similar. So, formula 1 will be discussed no further.
In formula 3, the observer's longitudinal motion is expressed as part of the ΔLHA term. Otherwise it is like formula 2, albeit expressed differently. Formulae 2 and 3 give very similar results for a Sun observation.

For Moon observations, small differences in results between formulae 2 and 3 become apparent, so an exacting test was devised using the Moon at major standstill on Oct 15th, 2024. The observer is exactly at S60° 00.0' E000° 00.0' at the instant of transit and is moving due West at 60 knots.

Preliminary 1. Find UT of transit using Astron.
(Display changed to D.d format and 6 decimal places)

UT	Moon Zn (degrees)	
22:25:18	0.004906	**Transit 22:25:19**
22:25:19	0.000259	
22:25:20	359.995612	

Transit time agrees exactly with USNO data.

Preliminary 2. Find UT of maximum altitude using Astron.
(Longitude output changed to 4 decimal places and Hc displays changed to D.d format.)

UT	Observer Latitude	Observer Longitude	Hc.	
22:13:44		E000° 23.1666'	30.02833984	**Maximum altitude at 22:13:45**
22:13:45	**S60° 00.0'**	**E000° 23.1336'**	30.02833999	
22:13:46		E000° 23.1000'	30.02833977	

Thus, maximum altitude occurs 11m 34s before transit and **the correction is +11m 34s**.
USNO data gives Hc 30° 01.7' (=30.028333) for all three seconds above – it only permits entry of location to 0.1'.

Prelimary 3.
Moon declination at 22:13:45 is S00° 03.453' **(-0.05755** degrees)
Moon declination 1 hour later: N00° 14.742'
Thus Moon Dec Rate is **+18.195'/hour**

Preliminary 4.
Moon GHA at 22:13:45: 357.2070
Moon GHA 1 hour later: 11.6938
Thus Moon Longitudinal Rate is **14.4868** degrees/hour or **869.21** minutes/hour.
Hence, for formula 2, the Moon constant (average 16.35) becomes **16.3806. (**Normally 16.35 would be used, but this is done here for a fair comparison.)

(Theoretically, preliminaries 3 and 4 should bracket the observation time, say at –30 and +30 minutes. However, the 'hour later' method (or any whole hour method) is easier and sufficiently accurate because, at times of high rate change, the rate of change is nearly linear, whilst at times of low rate change it has no significant effect on the result.)

First, calculate using Formula 2.
"y" is the observer's rate of change of latitude minus the body's rate of change of declination, so y= 0 minus +18.195 = **-18.195**.
"x" is the observer's rate of change of longitude.
x = -1 * SOG * sin (COG) / cos(abs(Lat)).
x = -1 * 60 * sin(270) / cos(abs(-60)) = -1 *60 * -1 / 0.5 = **+120** minutes per hour.
Correction = **Constant for Moon * y * (1+ 2x/Moon Rate) * (tan Lat – tan Dec),**
Correction = 16.3806 * -18.195 * (1+ 2x/869.21) * (tan (-60) – tan (-0.05755))
Correction = 16.3806 * -18.195 * (1+ 240/869.21) * (-1.73205 – (-0.00100))
Correction = 16.3806 * -18.195 * 1.27611 * -1.73105
Correction = +658.4 secs = **+10m 58s.** (36 secs less than calculated, or 5½%)

Now calculate using Formula 3.
Correction =
(10800 / (PI * (Body Longitude Rate – x/60)^2)) * y * (tan Lat – tan Dec).
From above, "x" is **+120,** "y" is **-18.195** and the tangent expression is **-1.73105**
Correction = (10800 / (PI * (14.4868 – 120/60)^2)) * -18.195 * -1.73105.
Correction = (10800 / (PI * (12.4868)^2)) * -18.195 * -1.73105.
Correction = (10800 / (PI * 155.9201)) * -18.195 * -1.73105.
Correction = (10800 /489.83) * -18.195 * -1.73105.
Correction = 22.0481 * -18.195 * -1.73105.
Correction = +694.4 secs = **+11m 34s** (equal to calculated value)

Conclusion.
All three formulae give adequately accurate results for Sun, star or planet observations at normal yacht speeds and moderate latitudes. The above harsh conditions were chosen to see which was the more accurate formula – for an observation of the Moon, formula 3 should be used. The derivations of both formulae use (differing) approximations on the basis that there would be no significant accuracy loss due to the small angles involved, assumptions fully justified in the case of the Sun, the body most commonly used for Upper Meridian Passage observations.