Minimum Geocentric Angular Separation between of 2 moving bodies

Consider 2 celestial bodies ***A*** and ***B*** with geocentric coordinates (α_a,δ_a), (α_b, δ_b) and angular velocities $(\mu\alpha_a , \mu\delta_a)$, $(\mu\alpha b , \mu\delta b)$ with all of them being functions of the Time variable " T ".

At time T_0 a time assumed to be in the vicinity of their minimum geocentric angular separation these coordinates and velocities are computed as :

$$(\alpha_{a0}, \delta_{a0}, \mu\alpha_{a0}, \mu\delta_{a0}) \text{ and } (\alpha_{b0}, \delta_{b0}, \mu\alpha_{b0}, \mu\delta_{b0})$$

With $\Delta T_1 = T - T_0$ and to the 1st order :

$\alpha_a = (\alpha_{a0} + \Delta T_1\ \mu\alpha_{a0})$, $\delta_a = (\delta_{a0} + \Delta T_1\ \mu\delta_{a0})$ and : $\alpha_b = (\alpha_{b0} + \Delta T_1\ \mu\alpha_{b0})$, $\delta_b = (\delta_{b0} + \Delta T_1\ \mu\delta_{b0})$

Define the following :

$$\Delta\alpha = (\alpha_b - \alpha_a),\ \Delta\delta = (\delta_b - \delta_a),\ \Delta\alpha_0 = (\alpha_{b0} - \alpha_{a0}),\ \Delta\delta_0 = (\delta_{b0} - \delta_{a0}) \text{ , and :}$$

$$\Delta\mu_0 = (\mu\alpha b_0 - \mu\alpha_{a0}),\ \Delta\nu_0 = (\mu\delta b_0 - \mu\delta_{a0}),\ \delta m_0 = ½\,(\delta_{b0} + \delta_{a0})\ .$$

This results into the following 1st order expressions : $\Delta\alpha = (\Delta\alpha_0 + \Delta T_1\ \Delta\mu_0)$ and $\Delta\delta = (\Delta\delta_0 + \Delta T_1\ \Delta\nu_0)$

Then, and again to the 1st order the angular distance D between ***B*** and ***A*** is :

$$D = (\Delta\alpha^2\cos^2\delta m_0 + \Delta\delta^2)^{½},$$

$$\text{hence } D^2 = (\Delta\alpha^2\cos^2\delta m_0 + \Delta\delta^2) \text{ or : } D^2= ([\Delta\alpha_0 + \Delta T\ \Delta\mu_0]\cos\delta m_0)^2 + (\Delta\delta_0 + \Delta T\ \Delta\nu_0)^2$$

$$d/dT\ (D^2)= 2\ \Delta\mu_0\cos\delta m_0\ ([\Delta\alpha_0 + \Delta T_1\ \Delta\mu_0]\cos\delta m_0) + 2\ \Delta\nu_0\ (\Delta\delta_0 + \Delta T_1\ \Delta\nu_0)$$

To get an extremum (here a minimum) for D, we need to solve for ΔT_1 in the following equation:

$$\boldsymbol{\Delta\mu_0\cos\delta m_0\ ([\Delta\alpha_0 + \Delta T_1\ \Delta\mu_0]\cos\delta m_0) + \Delta\nu_0\ (\Delta\delta_0 + \Delta T_1\ \Delta\nu_0) = 0} \text{ , which gives :}$$

$$\Delta T_1\ ([\Delta\mu_0\cos\delta m_0]^2 + [\Delta\nu_0{}^2]\ ) = -\ (\Delta\alpha_0\ \Delta\mu_0\cos^2\delta m_0 + \Delta\delta_0\ \Delta\nu_0) \text{ , to be solved as :}$$

$$\boldsymbol{\Delta T_1 = \frac{-\ (\alpha b_0 - \alpha_{a0})(\mu\alpha b_0 - \mu\alpha_{a0})\cos^2 ½\ (\delta b_0 + \delta_{a0})\ -\ (\delta b_0 - \delta_a)(\mu\delta b_0 - \mu\delta_{a0})}{(\mu\alpha b_0 - \mu\alpha_{a0})^2\cos^2 ½\ (\delta b_0 + \delta_{a0})\ +\ (\mu\delta b_0 - \mu\delta_{a0})^2}}$$

All values in blue print being known, we can easily compute ΔT_1.

We can then update all coordinates for $T_1 = T_0 + \Delta T_1$ and iterate the entire computation, until the successive ΔT_i values become inferior to some specified value.

Then, use such updated T_i value to compute the Bodies **A** and ***B*** final coordinates and their minimum geocentric angular separation.

This is a quite cumbersome manual task, but it can be easily performed once the algorithm here-above has been programmed onto a Computer/Calculator with automatic chain computation of the coordinates and velocities as functions of time.

Two examples are given here-after on the same cases treated by Paul Hirose at :
http://fer3.com/arc/m2.aspx/Upcoming-Planets-2023-close-encounters-Hirose-feb-2023-g53946

1 - 22 Jan 2023 , with Venus being Body "A" and Saturn being body "B":

(1)- At Time $TT_0 = 00h00m00.0s$

Venus : $\alpha_{a0} = 326.50163°$, $\delta_{a0} = -15.14220°$, $\mu\alpha_{a0} = 0.05074$ °/h , $\mu\delta_{a0} = 0.01732$

Saturn : $\alpha_{b0} = 327.40468°$, $\delta_{b0} = -14.46528°$, $\mu\alpha_{b0} = 0.00465°$, $\mu\delta_{b0} = 0.00159°$

then : $\Delta\alpha_0 = (\alpha_{b0} - \alpha_{a0}) = 0.90306°$ $\Delta\delta_0 = (\delta_{b0} - \delta_{a0}) = 0.67692°$. Get : **actual $D_0 = 3.977.1''$**

$\Delta\mu_0=(\mu\alpha b_0 - \mu\alpha_{a0})=-0.04609°/h$, $\Delta\nu_0=(\mu\delta_{b0}-\mu\delta_{a0}) = -0.01573$ °/h, $\delta m_0 = ½ (\delta_{b0} + \delta_{a0}) = -14.80374°$

Get : **$\Delta T_1 = 22h11m23.6s$** and **$T_1 = T_0 + \Delta T_1 = 22h11m23.6s$**

And from angular velocities from T_0 to T_1 get **approximate D_1= 1251.9"**

(2)- At Time $TT_1 = 22h11m23.6s$

Venus : $\alpha_{a1} = 327.62542°$, $\delta_{a1} = -15.14220$, $\mu\alpha_{a1} = 0.05054$ °/h , $\mu\delta_{a1} = 0.01758$

Saturn : $\alpha_{b1} = 327.50805°$, $\delta_{b1} = -14.42994°$, $\mu\alpha_{b1} = 0.00467°$, $\mu\delta_{b1} = 0.00160°$

then : $\Delta\alpha_1 = (\alpha_{b1} - \alpha_{a1}) = -0.11737°$ $\Delta\delta_1 = (\delta_{b1} - \delta_{a1}) = 0.32510°$. Get : **actual $D_1 = 1.239.7''$**

$\Delta\mu_0=(\mu\alpha b_1- \mu\alpha_{a1})=-0.04587°/h$, $\Delta\nu_1=(\mu\delta_{b1}-\mu\delta_{a1}) = -0.01598$ °/h, $\delta m_1 = ½ (\delta_{b1}+ \delta_{a1}) = -14.59249°$

Get : **$\Delta T_2 = 00h04m20.9s$** and **$T_2 = T_1 + \Delta T_2 = 22h15m44.5s$**

And from angular velocities from T_1 to T_2 get **approximate D_2= 1239.7"**

Successive ΔT_i / T values :

$\Delta T_3 = 00h00m00.5s$, $T_3 = 22h15m45.0s$, estimated and actual D_3= 1239.7"
$\Delta T_4 = 3$ E-7 h , $T_4 = 22h15m45.0s$, estimated and actual D_4= 1239.7" (unchanged results)

Final result to be compared with Paul Hirose's one (`minimum separation 1240.5" at 22:15:12`) where most probably 99% the difference between respective results is explained by the different Ephemeris being used, with Paul's result being the most accurate one.

2 - 1978 Sep 13, Mercury – Saturn (Jean Meeus' example) also covered by Paul Hirose :

Own solution here at 15:06:53.4 TT and 3'44.2" to be compared with Paul's results : "*My solution with the DE431 ephemeris quickly converges to 3'44.8" at 15:06:35 TT, compared to 3'44" at 15:06 UT by Meeus.* "

Overall conclusion: Kermit's "brute force" angular velocities method here is quickly convergent for existing solutions. Was it possibly/probably published earlier or independently by someone else ???

Method described by Paul Hirose mentioned here-above is very clever and I did enjoy discovering it.

Thanks and congratulations again to you, Paul

This method was first worked out by 1987
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