h vs. T departure from a straight line

Assume φ & D constant

φ: Latitude

D : Declination

T: Local Hour Angle

$$(0)\quad \sin h = \sin\varphi \sin D + \cos\varphi \cos D \cos T$$

$$(1)\quad \frac{dh}{dT} = -\frac{\cos\varphi \cos D}{\cos h} \sin T$$

$$(2)\quad \frac{d^2h}{dT^2} = tg\, h \left(\frac{dh}{dT}\right)^2 - \frac{\cos\varphi \cos D}{\cos h} \cos T$$

$$(3)\quad \frac{d^3h}{dT^3} = 3\, tg\, h \frac{d^2h}{dT^2}\frac{dh}{dT} + \left(\frac{dh}{dT}\right)^3 + \frac{\cos\varphi \cos D}{\cos h} \sin T$$

note: $\frac{d^3h}{dT^3} = 0$ for $T = 0$

$$(4)\quad \frac{d^4h}{dT^4} = 4\, tg\, h \frac{d^3h}{dT^3}\frac{dh}{dT} + 6 \frac{d^2h}{dT^2}\left(\frac{dh}{dT}\right)^2 - 3\, tg\, h \left(\frac{d^2h}{dT^2}\right)^2 + tg\, h \left(\frac{dh}{dT}\right)^4 + \frac{\cos\varphi \cos D}{\cos h} \cos T$$

$$\Delta h_{rad} = \frac{dh}{dT}\Delta T_{rad} + \frac{1}{2}\frac{d^2h}{dT^2}\Delta T_{rad}^2 + \frac{1}{2\times 3}\frac{d^3h}{dT^3}\Delta T_{rad}^3 + \frac{1}{2\times 3\times 4}\frac{d^4h}{dT^4}\Delta T_{rad}^4 + \ldots$$

$$\Delta h_{\circ} = \frac{dh}{dT}\Delta T_{\circ} + \frac{1}{2}\left(\frac{d^2h}{dT^2}\right)\frac{\pi}{180}\Delta T_{\circ}^2 + \frac{1}{6}\left(\frac{d^3h}{dT^3}\right)\left(\frac{\pi}{180}\right)^2\Delta T_{\circ}^3 + \frac{1}{24}\left(\frac{d^4h}{dT^4}\right)\left(\frac{\pi}{180}\right)^3\Delta T_{\circ}^4 + \ldots$$

Excerpt from Nov 1991 Full study

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June 30th, 2023