



$$\begin{aligned} \operatorname{tg} \alpha - \alpha &= \frac{\alpha^3}{3} \left[+\frac{2}{15} \alpha^5 + \dots \right] \\ \cos \alpha - 1 &= -\frac{\alpha^2}{2} \left[+\frac{\alpha^4}{24} - \dots \right] \end{aligned}$$

$$\cos \alpha = \frac{R}{R+h} = \frac{1}{1+h/R} \approx (1-h/R)$$

$$\cos \alpha - 1 = -\frac{\alpha^2}{2} = -\frac{h}{R} \quad \alpha^2 = 2h/R$$

$$(1) \quad \boxed{\alpha = \sqrt{2h/R}} \quad \text{with } \alpha \text{ in radians}$$

with α in arc minutes: $\alpha = 60 \times \frac{180}{\pi} \sqrt{\frac{2}{R}} \sqrt{h}$

$$R = 6378137 \text{ m} = 20925646.33 \text{ FT}$$

We are to keep FULL FLEDGED geometrical values.

Hence, with h in meters
with h in Feet

$$\begin{aligned} \alpha' &= 1.92 \sqrt{h_m} \quad (\text{vs. } 1.77 \sqrt{h_m} \text{ Ephémérides Nautiques}) \\ \alpha' &= 1.06 \sqrt{h_{ft}} \quad (\text{vs. } 0.97 \sqrt{h_{ft}} \text{ Nautical Almanac}) \end{aligned}$$

$$OA = R \operatorname{tg} \alpha, \quad AS = R \alpha, \quad \Delta = OA - AS = R(\operatorname{tg} \alpha - \alpha) = R \frac{\alpha^3}{3}$$

Known quantities: Δ & R

$$\alpha^3 = \frac{3\Delta}{R} \quad \alpha^6 = \left(\frac{3\Delta}{R}\right)^2 \quad (\text{from (1)}) : 8(h/R)^3 = 9(\Delta/R)^2$$

$$(h/R)^3 = \frac{9}{8} \left(\frac{\Delta}{R}\right)^2 \quad \boxed{h/R = \frac{1}{2} \left[9 \left(\frac{\Delta}{R}\right)^2 \right]^{1/3}}$$

with $\Delta = 1.5 \text{ FT}$ $R = 20925646.33 \text{ FT}$

Get: $h = 375.55 \text{ FT} = 125.18 \text{ Y}$

Last check: α must be small

have $\alpha' = 1.06 \sqrt{375.55} = 20.5' = 0.34^\circ$

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