Local Apparent Noon Methods - Part II :
An exact solution from Culmination to Meridian Passage

FOREWORD

Obtaining within a short time-span one's position from just ONE SINGLE Celestial Body has long been a Holy Grail for a number of Celestial Navigators.

Directly observing the unknown Meridian Passage Time and Height of a single Celestial Body is not achievable with only a Sextant and a Chronometer at sea. Hence **Local Apparent Noon** Methods - also known as **LAN** - have resorted to observing and recording a set of heights vs. times surrounding the selected Body local culmination.

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All LAN Methods follow **3 successive steps**:

Step 1 : Observing & recording a reasonable number - typically 7 to 15 - of such "*parabola type*" observations.

Step 2 : Deriving a best fit of these data to "reconstruct" the actual Culmination Time and Height. And finally:

Step 3 : From this Culmination Time and Height obtaining the Moving Observer's Meridian Transit Time and Height of the Celestial Body. Deriving the Observer's Latitude and Longitude has then become an immediate operation.

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An abundant literature has already been devoted to many LAN issues. See a list of a few chosen references further down this document. In particular ***Step 2*** has been extensively studied and Navigators now have an ample choice between many different methods to obtain *a best estimate* for Culmination Time and Height: either statistical computations, or (sometimes very cleverly) hand-drawn sketches, even resorting to a quite clever too "semi-transparent folded paper sheet" method, or other ... *Its scope being now more than adequately covered in depth it can then be considered that no real breakthrough may or will significantly improve **Step 2** from now on.*

Two remarks here:

a - With the exception of the *very clever and simple **Wilson 2** Method (Ref. 5)* all "simple" LAN methods involving "manual operations" - e.g. hand-drawn sketches or folding paper sheets - have to perform ***Step 2*** then ***Step 3*** separately and in order. The same holds true for all the statistical methods doing simple "parabola type" regression fits on just the data themselves without any recourse to the Body celestial coordinates. Accordingly once their own ***Steps 2*** are completed all these methods require ***some mathematical tool*** to undertake ***Step 3***.

b - More refined statistical methods taking in account the Body Celestial coordinates are believed to *internally merge **Step 2*** and ***Step 3***, in order to directly deliver the Observer's Position at Meridian Passage Time.

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Step 3** still needs attention* because it could be improved. In particular - *and again assuming Culmination Time and Height already secured by **Step 2 - from all the publications studied by the Author and to his best knowledge ***Step 3** has **not been** **specifically documented** **yet** as having been exactly solved mathematically*.

This document demonstrates how ***Step 3*** can be solved *exactly* by the ***new Iteration Method*** described here-after.

This Method ***iterates*** between 2 Calculus Formulae: one which is well known and its exact 1st order derivative. It assumes *all their higher order derivatives to be negligible*. *Under such assumptions - already applicable to all previous LAN methods - **this new method delivers an exact mathematical result and thus entirely solves for Step 3***.

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Au Roc Saint Luc en Pissotte, le 19 Décembre 2023, Antoine M. Couëtte

PART II

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This new Iterative Solution and its Principle

(1) - Starting Elements - As per **§3.2.1** compute the appropriate "starting elements" to derive h_{oo} , ϕ_o , μG_o , and μT_o. Assume T_{oo} = 0° and from ϕ_o and repeatedly use **Formula (2)** to compute successively refined values of "T_i" until no significant change in their values. Use "T_i" last value renamed as "T_{10}" to compute a *Preliminary Observer's Position* (ϕ_o, G_o) from which h_{oo} is refined into the required ***benchmark value "h_o"***. With all these [updated] Elements, enter the *Full Loop*.

(2) - Full Loop : **(2.1)** - Assume T_i=T_{10} then again as per **§4** repeatedly use **Formula (2)** to compute successively refined values of "T_i" until no significant change in their values.

(2.2) - Feed **Formula (1)** with the last obtained "T_i" and "ϕ" values to derive a value for "h_i" and compare it to the ***benchmark value "h_o"***. *Update "ϕ_i" through a correction equal to exactly* (h_i - h_o), *a quite simple **REGULA FALSI** working extremely well here*, then update "μG_i" and "μT_i" accordingly.

(2.3) - With these updated elements "ϕ_i", "μG_i" and "μT_i" perform **§4** again and repeat as necessary until no significant difference between "h_i" and "h_o".

(3) - Loop Exit - When last "h_i" equals "h_o", the iteration is complete, exit loop. **The Observer's position is :**

***At UT** Culmination* **: Latitude = Last updated value of ϕ_i *and* Longitude = Body GHA - (Last updated value of T_i)**

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FULL DESCRIPTION OF THE ITERATIVE METHOD

All **constant quantities** displayed as **bold characters** (e.g. **TT-UT, HoE, Temp., QNH, UT culm, H culm)**. Abbreviations here-after explained as necessary: **h_0: geocentric height (benchmark value)**, **D°: Body Decl., µD = dD/dUT °/h**, **RA°: Body Right Ascension, µRA = dRA/dUT °/h**, **GHA°: Body Greenwich Hour Angle, µGHA = dGHA/dUT °/h**, **µγ = dGHAγ/dUT °/h**, T°: Body Local Hour Angle, µT = dT/dUT °/h, (ϕ, G): Observer's Latitude & Longitude, (West Longitudes positive in the course of computations) **µϕ = dϕ/dUT °/h**, µG = dG/dUT °/h, **Smg: Speed made good in knots, Cmg °: Course made good.**
IMPORTANT NOTE : *We assume here for all variable quantities - i.e. T, µT, (ϕ, G) and µG - to be sufficiently linear functions of UT over a short time interval (+/- 30 min around UT culm).*

1 - Height [well known] Formula

(1) : sin h = sin ϕ **sin D** + cos ϕ **cos D** cos T *with T being measured from the Observer*

2 - Differential equation derived from Formula (1)

cos h dh = cos ϕ dϕ **sin D** + sin ϕ **cos D** dD - sin ϕ dϕ **cos D** cos T - cos ϕ **sin D** dD cos T - cos ϕ **cos D** sin T dT
Cos h≠0, and: cos h dh = dD (sin ϕ **cos D** - cos ϕ **sin D** cos T) + dϕ (cos ϕ **sin D** - sin ϕ **cos D** cos T) - sin T dT cos ϕ **cosD**

(2.0) : cos h (dh/dUT) = **µD**/µT (tg ϕ - **tg D** cos T) + **µϕ**/µT (**tg D** - tg ϕ cos T) - sinT. *Solving for dh/dUT = 0 yields :*

(2) : sin T **= µD**/µT (tg ϕ - **tg D** cos T) **+ µϕ**/µT (**tg D** - tg ϕ cos T)

With T on both sides, **solving (2) requires iteration**, greatly simplified here since T remains small, hence cos T ≃ 1.

3 - Derivation of the Starting Elements for the Iterations i 0 1 2 3 4 5 7 8 9

3.1 - General Formulae

3.1.1 - **µϕ °/h** = **(Smg/60) cos Cmg** ,
3.1.2 - **µγ °/h = 15°/h * (366.2422 / 365.2422) = 15.04106864 °/h**

3.2 - Determination of h_0

Since the ***geocentric height "h_o"*** is our ***benchmark value*** it needs a most careful determination.

3.2.1 - First approximation and updates on h_o , ϕ, μG and μT

3.2.1.1 - From adequate ***Software*** or from the ***Nautical Almanac*** obtain the following Celestial data:
D, **GHA**, **Parallax/Semi-Diameter**, **µD**, **µGHA**.
3.2.1.2 - Start from the given sextant height and perform standard 2D Tabular Corrections to get a *provisional value for h_o* to be addressed as *h_{oo}, pending its further refinement into h_o in 3.2.1.9* .
3.2.1.3 - Assume $T_{oo} = 0°$
3.2.1.4 - Assume regula falsi ϕ_o = **D +/-** (90° - h_{oo}) **+** *if Body South* /**-** *if Body North* .
3.2.1.5 - **$\mu\phi$ = (Smg/60) cos Cmg**
3.2.1.6 - μG_o °/h = - **(Smg/60) sin Cmg** / cos ϕ_o
3.2.1.7 - μT_o °/h = $[d(GHA\gamma - RA - G)/dUT]_o$ °/h = (**15.04107** - **µRA** - μG_o) °/h = (**µGHA** - μG_o) °/h
3.2.1.8 - Compute T from **(2)** and with G_o = (**GHA** - T_{10}) get a ***1st Approximate Position at (ϕ_o , G_o)***.
3.2.1.9 - From a 3D software *compute a better value for **h_o***, e.g. through a *Marcq Saint Hilaire intercept/azimuth sight* from (ϕ_o, G_o) to the Celestial Body. Use its resulting geocentric height as the ***reference / benchmark h_o.***
3.2.1.10 - From ***h_o*** :
3.2.1.10.1 - As per 3.2.1.4 update ϕ_o into regula falsi ϕ_1, and :
3.2.1.10.2 - As per 3.2.1.5 update μG_o into μG_1, and finally :
3.2.1.10.3 - As per 3.2.1.6 update μT_o into μT_1.

4 - Iteration from the starting elements here-above

4.1 - Iteration on T

As per **(2)** compute:
$\sin T_{11} = \mu D/\mu T\,(\mathrm{tg}\,\phi_1 - \mathrm{tg}\,D \cos T_{10}) + \mu\phi/\mu T\,(\mathrm{tg}\,D - \mathrm{Tg}\,\phi_1 \cos T_{10})$, and iterate as necessary :
$\sin T_{12} = \mu D/\mu T\,(\mathrm{tg}\,\phi_1 - \mathrm{tg}\,D \cos T_{11}) + \mu\phi/\mu T\,(\mathrm{tg}\,D - \mathrm{Tg}\,\phi_1 \cos T_{11})$,
$\sin T_{13} = \mu D/\mu T\,(\mathrm{tg}\,\phi_1 - \mathrm{tg}\,D \cos T_{12}) + \mu\phi/\mu T\,(\mathrm{tg}\,D - \mathrm{Tg}\,\phi_1 \cos T_{12})$ ... until no significant change in T_i
Then make "T_{20} = last value of T_i" and use T_{20} as the current best "T" provisional value.

4.2. - Update on ϕ and μT

4.2.1 - Then from **(1)** compute:
$\sin h_1 = \sin\phi_1 \sin D + \cos\phi_1 \cos D \cos T_{20}$, from which we compute h_1, then :
regula falsi $\phi_2 = \phi_1$ +/- (h_1 - ***h_o***) *+ if Body North / - if Body South*

4.2.2 - From 3.2.1.6 update μG_1 into μG_2 and from 3.2.7 update μT_1 into μT_2

4.3 - Back to § 4.1 and restart the iteration
4.3.1 - From T_{20} , ϕ_2 and μT_2 get T_{30} then ϕ_3 and μT_3. *Then as necessary:*
4.3.2 - From T_{30}, ϕ_3 and μT_3, get T_{40} then ϕ_4 and μT_4. *Then again until no significant change in T_i and ϕ_i .*

Iteration completed when (h_i - h_o) = 0 .

Checking that "dh/dUT = 0°/h"

Formula (2.0) permits only a **trivial check** since through ***Formula (2)*** the iteration solves for [exactly] dh/dUT = 0 .
Independent checks: At the computed **UT Meridian Passage** check for Body Azimuth lying at [exactly] North or South.
Even better: reverse-engineer the heights values and verify that **h (UT Culmination)** reaches an extremum value then.

The Observer's position obtained from the iteration is :

At UT Culmination : **LATITUDE Culm = Last updated ϕ_i** *and* **LONGITUDE Culm = Body GHA - (Last updated T_i)**

At UT Meridian Passage : **ΔUT_{mp-c} = (UT Meridian Passage - UT Culmination) = Last ($-T_i/\mu T_i$)**

LATITUDE Mp = LATITUDE Culm - $\mu\phi\,\Delta UT_{mp-c}$ *and* **LONGITUDE Mp = LONGITUDE Culm - $\mu G\,\Delta UT_{mp-c}$**

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THE HISTORICAL "MATHEMATICAL TOOL" DERIVING UT Meridian Passage FROM UT Culmination

As mentioned here-above a number of LAN Methods are performing ***Step 2*** then ***Step 3*** separately.

Once the Culmination Height and Time are known they most often obtain the Meridian Passage Time from the following usual and fairly well-known **Formula (3)** :

$$(3.0) : \Delta UT_{mp\text{-}c} \textit{ (in seconds of time)} = (UT_{\text{Meridian Passage}} - UT_{\text{Culmination}})_s = 48/\pi * (\text{tg}\phi - \text{tg}D)\ (\mu\phi\text{-}\mu D)\ '/h$$

Formula (3.0) seems to have been first published in the 1930's - *see Ref (1) and (2*) - as well as regularly afterwards, - *e.g. in Ref (3) and Ref (4)* - not to mention its regular publication from the onset in a number of Navigation courses.

Another related formula - *almost always ignored* - shifts from "$h_{o\ \text{Culmination}}$" into "$h_{o\ \text{Meridian Passage}}$" as follows:

$$(4) : (h_{o\ \text{Meridian Passage}} - h_{o\ \text{Culmination}})\ ' = -1/2 * |(k\Delta UT_{mp\text{-}c})_h * (\mu\phi\text{-}\mu D)\ '/h|$$

Important remarks

1 - With $\mu T = dT/dUT = \mu GHA\text{-}\mu G$ and its leading coefficient equal to "**48/π**", **Formula (3.0)** ***assumes μT = 15°/h*** which matches the **Sun actual µGHA** very well **for a steady Observer**.

Even for the Sun (when seen from a moving Observer) and most importantly for Lady Moon (always), the coefficient "**48/π**" needs a multiplicative correction " $k = (15/\mu T)^2$ " in order to more adequately address the effects of μT onto the time difference between Meridian Passage and Culmination.

Accordingly **Formula (3.0)** *should* be replaced by the following formula:

$$(3.1) : k\Delta UT_{mp\text{-}c} \textit{ (in seconds of time)} = (UT_{\text{Meridian Passage}} - UT_{\text{Culmination}})_s = (15/\mu T)^2 * 48/\pi * (\text{tg}\phi - \text{tg}D)\ (\mu\phi - \mu D)\ '/h$$

2 - **Formula (3.1)** nonetheless ***remains incomplete*** since it carries only the 1st term of an infinite decreasing series.

1.2.1 - Its neglected 3rd order term - *an* ***odd term*** *ranking 2nd in the series* - becomes more and more significant with lower and lower culmination heights since the observations need longer and longer time-spans to cover a sufficient angular span. Like its higher order *odd terms* siblings, the 3rd order *odd term* corrects for the non-symmetrical heights increase/decrease around Cumination Time, an effect quite visible in the further examples involving low Culmination Heights.

1.2.2 - And its neglected 4th order term - *an* ***even term*** *ranking 3rd in the series* - also starts being significant for values reaching about 10 minutes of time, an effect which is also easily observed in the same examples.

3 - For these reasons mainly, **Formula (3) itself is restricted to sufficiently high Culmination Heights**. The relevant Literature generally indicates that no LAN Observations should be carried out unless their expected **Culmination Heights are to exceed a [very] minimum of 60°**. Even in such cases, it ***should best*** be replaced by **Formula (3.1)**.

4 - If and when used in conjunction with *unfortunately all too often ignored* **Formula (4)** , then **Formula (3.1)** itself can cope with much softer limitations as shown in the examples further given in **APPENDIX A** and in **APPPENDIX B**.

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We have seen earlier that the **LAN Iteration Method** solves exactly for ***Step 3***: first for Culmination then for Meridian Passage. Therefore at the very end of this document we are comparing this **LAN Iteration Method** - *then used as an exact Mathematical solution benchmark* - to one of the LAN Methods using **Formula (3.1)** and **Formula (4)** here-above, namely the **"Twin Altitude" LAN Method** described in **APPENDIX A** and in **APPENDIX B**.

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OVERALL CONCLUSION TO PART I AND PART II

1 - All LAN Methods follow the same path : (a) recording “parabolic type” Time and Height data observed around Culmination Time, (b) “extracting” the *Culmination Time and Height* from them (c) and finally transforming the *Culmination Time and Height* into *Meridian Passage Time and Height* in order to easily derive the ensuing *Observer’s position at Meridian Passage*.

1.1 - In all LAN methods the Celestial Body and the moving Observer coordinates are assumed to not depart significantly from strict linear functions of Time.

1.2 - Many LAN Methods have been published to-day and mostly focus on the second part of this operation here-above (b) - and elsewhere referred to as “***Step 2***” - which can be performed in different ways.

2 - Elsewhere referred to as “***Step 3***” the shift from Meridian Passage ***Time*** to Culmination ***Time*** here-above (c) has long been carried out through ***Formula (3.0)***.

$$\textbf{(3.0)} : \Delta UT_{mp\text{-}c} \textit{ (in seconds of time)} = (UT_{\text{Meridian Passage}} - UT_{\text{culmination}})_s = 48/\pi * (tg\phi - tgD)\ (\mu\phi - \mu D)_{'/h}$$

2.1 - ***Formula (3.0)*** should be best replaced by ***Formula (3.1)***. *This is rarely documented in the relevant literature.*

$$\textbf{(3.1)} : k\Delta UT_{mp\text{-}c} \textit{ (in seconds of time)} = (UT_{\text{Meridian Passage}} - UT_{\text{Culmination}})_s = (15/\mu T)^2 * 48/\pi * (tg\phi - tgD)\ (\mu\phi - \mu D)_{'/h}$$

2.2 - Unfortunately also the counterpart formula for the ***Heights*** themselves - i.e. ***Formula (4)*** - *is almost never carried out or even published in spite of its sometimes quite significant contribution.*

$$\textbf{(4)} : (h_{o\ \text{Meridian Passage}} - h_{o\ \text{Culmination}})' = -1/2 * |(k\Delta UT_{mp\text{-}c})_h * (\mu\phi - \mu D)_{'/h}|$$

2.3 - These both omissions are kind of unfortunate since both ***Formula (3.1) and Formula (4) when used together*** carry much softer limitations of use than the early initial and sole ***Formula (3)*** restricted to only Culmination heights superior to 60°.

3 - The **LAN Iteration Method** described here *solves for **Step 3** only*. From both the given Culmination Time and Culmination Height it directly yields the “mathematically exact” Observer’s Position at the Culmination Time itself.

3.1 - It is interesting to see how wonderfully its internal simple ***regula falsi*** (or “***false position***”) works.

3.2 - As regards the expected final position - *and still assuming for this Method the usual linearity assumptions already applicable to all the others* - it converges quite well. **It stays free of all the explicit or implicit mathematical shortcomings / limitations inherent to all previously documented LAN Methods.** It therefore comes as a welcome addition to the LAN Methods Family.

4 - Since it has recourse to **Formula (3.0)**, the **Twin Altitude LAN Method** has been retained for the examples given in APPENDIX A and APPENDIX B. It actually uses here both **Formula (3.1)** and **Formula (4)** together. Accordingly it behaves very well within the usual ***60° operational limitation*** generally accepted for **Formula (3.0)** when used alone. Even outside this 60° limitation, the **Twin Altitude LAN Method** also behaves reasonably well, and in all cases much better than if it were relying onto **Formula (3)** only.

5 - Most LAN methods - if not all of them - remain sufficiently robust against noisy data.

Given the noise-free data in all the examples given here, the Culmination Heights are always obtained with no error.

In a noisy data environment and on the average the E-W distances errors will be up to 3 to 5 times greater (or even more) than the N-S distance errors. *In other words:*

In all LAN Methods, the Position uncertainties always stay (far) more important for Longitudes than for Latitudes.

6 - We should not forget about the following rule of thumb gained from experience:

Always arrange for a minimum difference of 30', and <u>never less than 20'</u> ,

between the extreme values of the heights surrounding Culmination.

Lack of so doing may start significantly weakening the reliability of the Longitudes obtained by <u>any</u> LAN Method.

7 - *Not an immediately related topic now* … but:

Often overlooked but unfortunately true: the MOON HEIGHTS *pencil corrections* keep requiring meticulous care. The French ***Éphémérides Nautiques*** and the ***Nautical Almanac*** (2D) Tables deliver similar ones, almost always within 0.1' from one another. Nonetheless the resulting geocentric heights *"h_{oo}"* may differ by up to 0.3' from the "true" values *"h_o"* computed through 3D Methods. It is an inherent limitation of such Tables.

8 - ***Back to LAN's*** *… if asked for a personal choice between the many LAN Methods available:*

8.1 - For a "hand drawn sketch" LAN Method, I would choose the wonderful ***Wilson 2 LAN Method*** - Ref (5) - which does not even need to compute **(UT Meridian Passage - UT Culmination)** as it immediately jumps to the Meridian Passage.

8.2 - For securing the best results from any given set of data I have eventually come to the conclusion that *statistical methods almost always remain the most reliable and accurate ones*.

So … in that case, I am going for the **6th Order LAN Method** :

- Its useful time-span around Culmination covers about twice the time-span of the classical 2nd order methods. Even if it is not *mathematically exact* since it lets all higher order terms aside, it behaves quite well.

- It copes adequately with quite noisy data and under most conditions it also generally computes with remarkable accuracy the quantity **(UT Meridian Passage - UT Culmination)**.

- And it has always faithfully stayed afloat in all and any configurations experienced or simulated until now.

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ACKNOWLEDGMENTS AND REFERENCES

(1) - ***HAUTEURS CIRCUMZÉNITHALES CORRESPONDANTES***, by **Imprimerie de l'École Navale** *(1934)*

(2) - ***ADMIRALTY NAVIGATION MANUAL*** *(near page 152)*, by **HM Stationary Office**, Vol III (*1938*)

(3) - ***CALCULATOR NAVIGATION*** by **Mortimer Rogoff**, by **NORTON Publishers**, ISBN 0-393-03192-6 *(May 1979)*

(4) - ***POSITION FROM OBSERVATION OF A SINGLE BODY***, by **James N. Wilson** in **Journal of the Institute of Navigation**, V.32 N.1 *(Spring 1985)*

(5) - ***THE NOON FIX***, by **James N. Wilson**, by **AUTHOR HOUSE Bloomington IN**, ISBN 978-1-4389-5866 (sc) *(25 March 2009)*

(6) - ***NAVLIST FORUM ARTICLE*** *by* **Frank E. Reed** *http://fer3.com/arc/m2.aspx/Latitude-Longitude-Noon-Sun-FrankReed-jun-2005-w24178 at https://navlist.net/* ***.*** ***<u>Note</u>****: This Article describes F.E. Reed's full "****Folded Paper Method****" which advocates pre-treating data as described here in* ***Section 4.0****. A simpler version without pre-treatment is described and demonstrated here with excellent end-results believed to be identical.*

(7) - ***210915 - LATITUDE LONGITUDE NEAR NOON METHODS COMPARISON,*** by **Antoine M. Couëtte** as ***Attachment*** *https://navlist.net/imgx/210908-Latitude-Longitude-Near-Noon-methods-comparison-.pdf* to a ***NAVLIST FORUM ARTICLE*** by **Antoine M. Couëtte** *https://navlist.net/Compare-Methods-LatLon-Near-Noon-Couëtte-sep-2021-g51145* , both currently at *https://navlist.net/* **.**

(8) - ***231203 - STEADY OBSERVER POSITION FROM H CULM AND UT CULM,*** by **Antoine M. Couëtte** as ***Attachment*** *https://navlist.net/imgx/Steady-Observer-position-from-H-culm-and-UT-culm-.pdf* to a ***NAVLIST FORUM ARTICLE*** by **Antoine M. Couëtte** *https://navlist.net/Non-moving-Observer-position-from-H-culm-UT-culm-Couëtte-dec-2023-g55034* , both currently at *https://navlist.net/* .

Au Roc Saint Luc en Pissotte, le 19 Décembre 2023 - Antoine M. "Kermit" Couëtte - antoine.m.couette@club-internet.fr

APPENDIX A - NON MOVING OBSERVER

In **APPENDIX A** and **APPENDIX B** of this document we are comparing the **LAN Iteration Method** with a LAN Method using both **Formula (3.1)** here-above and the **Formula (4)** here-after, namely the **"Twin Altitude" Method**.

This "**Twin Altitude Method**" also referred to as the "**Double/Equal/Iso Altitude Method**" involves taking and recording the times of the **very same pre-set altitudes before and after Culmination**.

- The Culmination Time is taken as the averaged value of all such Observations. And:
- The Culmination Height is taken separately without recording its time.

The Meridian Passage Time is obtained from the Culmination Time through **Formula (3.1)**.

The Meridian Passage Height is computed from the Culmination Height through the following formula:

$$(4)\quad h_{ooMp} = h_{ooCulm} - 0.5|(\mu\phi - \mu D)k\Delta UT_{mp\text{-}c}|$$

And for historical reference:

The following **Twin Altitude LAN** rules were published for the **French Navy** in 1934 - *Ref (1)* :

(1) - Attempt using only h Culm's higher than 80°, but do not use h Culm's lower than 60°.

(2) - Carefully prepare the Observations, e.g. for 3 sets of heights observe as per the following schedule :

h Culm range	89° - 88°	88° - 86°	86° - 84°	84° - 82°	82° - 80°
Start observations ahead of best LAN estimate by:	6 minutes	7 min.	9 min.	11 min.	13 min.
Increase (decrease) successive Sextant heights by:	7'	5'	4'	3'	3'
Stop observations ahead of best LAN estimate by:	2 minutes	3 min.	5 min.	7 min.	9 min.

The following examples processing *noise-free data obtained through reverse-engineering* all involve one same Moon LAN Culmination Time and the same Upper Limb: one observed at 31° and one observed at 81°, one from a North Culmination position and one from a South Culmination position, and finally one from a steady Observer and one from a high speed moving Observer, which altogether makes 8 different configurations.

With the ***LAN Iteration Method*** used as a benchmark, each of these 8 examples is solved by both:

- The ***Twin Altitude LAN Method*** using as ***benchmark height*** the ***Nautical Almanac*** corrected UL Heights.

- A ***6th order LAN Method*** using as ***benchmark height*** the more accurate 3D corrected same UL Heights differing by about 0.25' from their ***NA*** corrected counterparts.

The performance of each LAN Method is checked for both:

- *"STEP 2"* PERFORMANCE :

How do $\mathbf{UT_{Culm}}$ and $\mathbf{h_{Culm}}$ obtained by each method compare to the "reference" initial values.

- *"STEP 3"* and then OVERALL END POSITION PERFORMANCE :

How do the **End Positions** derived by each method compare to the **"true" End Position** obtained by the **LAN Iteration Method**.

Last note: Many **non-significant decimals** published throughout for computation performance checks only.

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Example A1 : **NON-MOVING Observer, Southerly Culmination with UL h = 31°**

From *https://navlist.net/Weird-moonrise-direction-Ritchie-nov-2023-g55024* :

A1 - 23 Nov 2023 , TT-UT = + 69.2s, HoE = 15', Temperature = 50° F, QFE = 29.83", UT culm = 06:19:44, Southerly Moon Culminating at UL h = 31°09.2' seen from a non-moving Observer.

NOTE : The following steps refer to the identically numbered earlier precepts

A1 - 3.2.1.1 - From own Software accurate to +/- 4", get for the Moon:

GHA=149°16.54', µRA=0.5205°/h, µGHA=µT= 14.52061°/h, Dec=+1°39.17', µD=0.2790°/h, Parallax=59.20'

NOTE : Since the Observer is not moving, µϕ = 0°/h and µG = 0°/h

A1 - 3.2.1.2 - From the ***Éphémérides Nautiques (2D) Tables***, let's perform a preliminary estimate of $\mathbf{h_o}$:
Dip = -3.7' , LL Correction (-Refraction, +Parallax +SD) = +65.2' , UL D Correction = -32.2'
Total Correction = +33.0' , hence **h_o provisional value : h_{oo} = 31°38.5'**

A1 - 3.2.1.4 - From h_{oo} get regula falsi ϕ_o = D + (90° - h_{oo}) = 60°00.67' , i.e. a Northerly Latitude

A1 - 3.2.1.8 - From T_{oo} = 0°, ϕ_o = 60°00.67' and µT= 14.52061°/h, from **(2)** get T_{01}= 1.87589°, T_{02} = 1.87591°, T_{03} = 1.87591° , hence T_{10}= 1.87591°.
With G_o = (GHA - T_{10}) get : Observer's 1[st] Approximate Position at (ϕ_o = 60°00.67', G_o = 147°23.99')

A1 - 3.2.1.9 - Through a Marcq Saint Hilaire Line of Position yielding +0.8 NM /182.2° , the 3D correction onto Moon UL height gives **h_o = 31°38.18'**, (vs. earlier 31°38.5') to be kept from now on. From **h_o = 31°38.18'** :

A 3.2.1.10.1 - As per 3.2.1.4 update ϕ_o = 60°00.67' into regula falsi ϕ_1 = 60°00.99'

A1 - 4.1 - First Iteration on T

With T_{10} = 1.87591°, ϕ_1 = 60°00.99' and µT= 14.52061°/h from **(2)** get T_{11} = 1.87632° , T_{12} = 1.87632°, hence T_{20}= 1.87632°

A1 - 4.2 - First Update on ϕ

A1 - 4.2.1 - With T_{20}= 1.87632°, from **(1)** get : h_1 = 31°37.10' and (h_1 - $\mathbf{h_o}$) = 31°37.10' - **31°38.18'** = -1.08'
With (h_1 - $\mathbf{h_o}$) = -1.08', we need to decrease ϕ_1 in order to bring h_2 (next in **§ A 4.4**) closer to $\mathbf{h_o}$
Hence regula falsi ϕ_2 = ϕ_1 + (h_1 **-** $\mathbf{h_o}$) = 60°00.98' - (-1.08') = 59°59.91'

A1 - 4.3 - Second iteration on T

With T_{20}= 1.87631°, ϕ_1 = 59°59.90', from **(2)** get T_{21} = 1.87493° , T_{22} = 1.87493°, hence T_{30}= 1.87493°

A1 - 4.4 - Second Update on ϕ

With T_{30}= 1.87493°, from **(1)** get : h_2 = 31°38.18' . Since (h_2 - $\mathbf{h_o}$) = 0.00' , our iteration is then complete.

See full results on next page

A1 - 5 - FINAL POSITION FROM ITERATION METHOD, AND COMPARISON CHECKS WITH DATA FOR LAN METHODS

EXAMPLE A1 - 23 Nov 2023, UT culm = 06:19:44 Southerly Moon UL = 31°09.2' Cmg/Smg = 153°/ 0 kt

FINAL POSITION OBTAINED BY THE ITERATION METHOD

Observer's position at UT $_{Culmination}$: 06h19m44.0s / N 59°59.91' / W 147°24.04'

vs. Author's Position at UT $_{culmination}$: 06h19m44s / N 60°00.0' / W147°24.0'

ΔUT_{mp-c} = UT $_{Meridian\ passage}$ - UT $_{Culmination}$ = -7m44.8s *(vs. Author's result at -7m45s UT)*

Observer's position at UT $_{Meridian\ passage}$: 06h11m59.2s / N 59°59.91' / W 147°24.04'

vs. Author's Position at UT $_{Meridian\ passage}$: 06h11m59s / N 60°00.0' / W147°24.0'

Quick Check : At 06h11m59.2s MOON height UL = 31°08.107' , Az = 180.00010° : quick check OK

A1 - Data for Twin-altitude LAN Method

Difference between extreme heights: 17.9'

	UT	UL Heights		UT
	6	31°09.2'		06:19:30.0
	UT	UL Heights		UT
5	06:00:07.0	31°02.2'	7	06:39:20.5
4	05:57:09.0	30°59.9'	8	06:42:23.0
3	05:54:11.5	30°57.4'	9	06:45:18.0
2	05:51:14.0	30°54.5'	10	06:48:18.0
1	05:48:17.5	30°51.3'	11	06:51:10.5

A1 - Position by the Twin-altitude LAN Method with a corrective factor applied to both ΔUT_{mp-c} and $-\mu\phi\ \Delta UT_{mp-c}$: "k" = $(15/\mu T)^2$ = 1.0671

D≃+1°39' μD=+16.7'/h ϕ ≃ 60°00' μϕ = 0'/h

μT=14.521°/h **ΔUT_{mp-c} = -7m15s**

$-0.5|(\mu\phi-\mu D)\Delta UT_{mp-c}|$ = -1.0'

UTculm = 06:19:45 ***k*ΔUT_{mp-c} = -7m44s**

UT_{mp} = 06:12:01 $-0.5|(\mu\phi-\mu D)k\Delta UT_{mp-c}|$ = -1.1'

hUL_{culm}=31°09.2' h_{OOculm}=31°38.5' h_{OOmp}=31°37.4'

Position at UT_{mp} : N 59°59.6' / W 147°24.5'

"*STEP 2*" PERFORMANCE :

Error in UT Culm : 1 s Error in H culm : 0.0'

"*STEP 3*" and OVERALL POSITION PERFORMANCE

Error in ***k*ΔUT_{mp-c} : 0.8 s**

When using "k" : Position Error = 0.4 NM

When not using "k" : Position Error = 3.7 NM

A1 - Data for other LAN Methods

Difference between extreme heights: 22.4'

	UT	UL Heights		UT	UL Heights
1	05:45:00.0	30°47.4'	10	06:30:00.0	31°07.3'
2	05:50:00.0	30°53.2'	11	06:35:00.0	31°05.0'
3	05:55:00.0	30°58.1'	12	06:40:00.0	31°01.8'
4	06:00:00.0	31°02.1'	13	06:45:00.0	30°57.7'
5	06:05:00.0	31°05.3'	14	06:50:00.0	30°52.6'
6	06:10:00.0	31°07.5'	15	06:55:00.0	30°46.8'
7	06:15:00.0	31°08.8'	16		
8	06:20:00.0	31°09.2'	17		
9	06:25:00.0	31°08.7	18		

A1 - Position by a 6th order LAN method

UTculm = 6h19m44.1s ΔUT_{mp-c} = -7m45.0s

Position : UT_{mp} = 6h11m59.1s / N 59°59.8' / W147°24.0'

SDEV=0.0 NM , Valid interval width +/- 1h24m @ UTculm ,

i.e. Valid Observation time span: (04:55-07:44) εT'/εh'≤5.0

Longitude Error Uncertainty (1σ) = 0.1'

"*STEP 2*" PERFORMANCE :

Error in UT Culm : 0.1 s Error in H culm : 0.0'

"*STEP 3*" and OVERALL POSITION PERFORMANCE

Error in **ΔUT_{mp-c} : -0.2 s**

Actual Position Error : 0.1 NM

****

Example A2 : **NON-MOVING Observer, Northerly Culmination with UL h = 31°**

A2 - 23 Nov 2023 , TT-UT = + 69.2s, HoE = 15', Temperature = 50° F, QFE = 29.83", UT culm = 06:19:44, Northerly Moon Culminating at UL h = 31°09.2' seen from a non-moving Observer.

NOTE : The following steps refer to the identically numbered earlier precepts

A2 - 3.2.1.1 - From own Software accurate to +/- 4", get for the Moon:

GHA=149°16.54', µRA=0.5205°/h, µGHA=µT= 14.52061°/h, Dec=+1°39.17', µD=0.2790°/h, Parallax=59.20'

NOTE : Since the Observer is not moving, µϕ = 0°/h and µG = 0°/h

A2 - 3.2.1.2 - From the ***Éphémérides Nautiques (2D) Tables***, let's perform a preliminary estimate of $\mathbf{h_o}$:
Dip = -3.7' , LL Correction (-Refraction, +Parallax +SD) = +65.2' , UL D Correction = -32.2'
Total Correction = +33.0' , hence **h_o provisional value : h_{oo} = 31°38.5'**

A2 - 3.2.1.4 - From h_{oo} get regula falsi ϕ_o = D + (90° - h_{oo}) = -56°42.33' , i.e. a Southerly Latitude

A2 - 3.2.1.8 - From T_{oo} = 0°, ϕ_o = 56°42.33' and µT = 14.52061°/h from **(2)** get T_{01} = -1.70801°, T_{02} = -1.70800°, T_{03} = -1.70800°, hence T_{10}= -1.70800°.
With G_o = (GHA - T_{10}) get : Observer's 1st Approximate Position at (ϕ_o = -56°42.33', G_o = 150°59.02')

A2 - 3.2.1.9 - Through a Marcq Saint Hilaire Line of Position yielding +0.7 NM /002.0° , the 3D correction onto Moon UL height gives **h_o = 31°38.18'**, (vs. earlier 31°38.5') to be kept from now on. From **h_o = 31°38.18'** :

A2 - 3.2.1.10.1 - As per 3.2.1.4 update ϕ_o = -56°42.33' into regula falsi ϕ_1 = -56°42.65'

A2 - 4.1 - First Iteration on T

With T_{10}= -1.70800°, ϕ_1 = -56°42.65' and µT=14.52061°/h, from **(2)** get T_{11}= -1.70834°, T_{12}= -1.70834° hence T_{20}= -1.70834°

A2 - 4.2 - First Update on ϕ

A2 - 4.2.1 - With T_{20}= -1.70834°, from **(1)** get : h_1 = 31°37.20' and (h_1 - **h_o**) = 31°37.20' - **31°38.19'** = -0.98'
With (h_1 - **h_o**) = -0.98', we need to increase ϕ_1 in order to bring h_2 (next in **§ A 4.4**) closer to **h_o**
Hence regula falsi ϕ_2 = ϕ_1 - (h_1 **- h_o**) = -56°42.65' - (-0.98') = 56°41.67'

A2 - 4.3 - Second iteration on T

With T_{20}= -1.70833°, ϕ_1 = -56°41.67', from **(2)** get T_{21} = -1.70729° , T_{22} = -1.70729°, hence T_{30}= -1.70729°

A2 - 4.4 - Second Update on ϕ

With T_{30}= -1.70728°, from **(1)** get : h_2 = 31°38.18' . Since (h_2 - **h_o**) = 0.00' , our iteration is then complete.

See full results on next page

A2 - 5 - FINAL POSITION FROM ITERATION METHOD, AND COMPARISON CHECKS WITH DATA FOR LAN METHODS

EXAMPLE A2 - 23 Nov 2023, UT culm = 06:19:44 Northerly Moon UL = 31°09.2' Cmg/Smg = 153°/ 0 kts

FINAL POSITION OBTAINED BY THE ITERATION METHOD

Observer's position at given $UT_{Culmination}$ 06h19m44.0s : S 56°41.67' / W 150°58.98'

$\Delta UT_{mp-c} = UT_{Meridian\ passage} - UT_{culmination} = +7m03.3s$

Observer's position at $UT_{Meridian\ passage}$: 06h26m47.3s / S 56°41.67' / W 150°58.98'

Quick Check : At 06h26m47.3s MOON height UL = 31°08.205' , Az = 0.00011° : quick check OK

A2 - Data for Twin-altitude LAN Method Difference between extreme heights: 30.7'

	6	31°09.2'		06:20:00.0
	UT	UL Heights		UT
5	05:50:53.5	30°52.7'	7	06:48:33.0
4	05:48:17.0	30°49.6'	8	06:51:06.0
3	05:45:43.0	30°46.3'	9	06:53:40.0
2	05:43:17.5	30°42.9'	10	06:56:07.5
1	05:40:22.0	30°38.5'	11	06:59:02.0

A2 - Position by the Twin-altitude LAN Method with a corrective factor applied to both ΔUT_{mp-c} and $-\mu\phi\Delta UT_{mp-c}$: "k" = $(15/\mu T)^2$ = 1.0671

D≃+1°39' μD=+16.7'/h ϕ ≃ -56°30' μϕ = 0'/h

μT=14.521°/h ΔUT_{mp-c}=+6m33s

$-0.5|(\mu\phi-\mu D)\Delta UT_{mp-c}|$ = -0.9'

UT_{culm} = 06:19:42 $k\Delta UT_{mp-c}$ = +6m59s

UT_{mp} = 06:26:41 $-0.5|(\mu\phi-\mu D)k\Delta UT_{mp-c}|$ = -1.0'

hUL_{culm}=31°09.2' $h_{oo culm}$=31°38.5' $h_{oo mp}$=31°37.5'

Position at UT_{mp} : S 56°41.4' / W 150°57.5'

"*STEP 2*" PERFORMANCE :

Error in UT $_{Culm}$: -2 s Error in H $_{culm}$: 0.0'

"*STEP 3*" and OVERALL POSITION PERFORMANCE

Error in $k\Delta UT_{mp-c}$: -4.3 s

When using "k" : Position Error = 0.9 NM

When not using "k" : Position Error = 4.3 NM

A2 - Data for other LAN Methods Difference between extreme heights : 30.7'

	UT	UL Heights		UT	UL Heights
1	05:40:22.0	30°38.5'	10	06:56:07.5	30°42.9'
2	05:43:17.5	30°42.9'	11	06:59:02.0	30°38.5'
3	05:45:43.0	30°46.3'	12	*This example carries the same data as the Twin-Altitude Lan Method running on the left side of this page*	
4	05:48:17.0	30°49.6'	13		
5	05:50:53.5	30°52.7'	14		
6	06:20:00.0	31°09.2'	15		
7	06:48:33.0	30°52.7'	16		
8	06:51:06.0	30°49.6'	17		
9	06:53:40.0	30°46.3'	18		

A2 - Position by a 6th order LAN Method

UT_{culm} = 6h19m43.6s ΔUT_{mp-c} = +7m03.4s

Position : UT_{mp} = 6h26m47.0s / N 56°41.6' / W150°58.9'

SDEV=0.0 NM , Valid interval width +/- 1h21m @ UT_{culm} ,

i.e. Valid Observation time span: (04:52-07:40) εT'/εh'≤4.7

Longitude Error Uncertainty (1σ) = 0.1'

"*STEP 2*" PERFORMANCE :

Error in UT $_{Culm}$: 0.4 s Error in H $_{culm}$: 0.0'

"*STEP 3*" and OVERALL POSITION PERFORMANCE

Error in ΔUT_{mp-c} : -0.1 s

Actual Position Error : 0.1 NM

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Example A3 : **NON-MOVING Observer, Southerly Culmination with UL h = 81°**

A3 - 23 Nov 2023 , TT-UT = + 69.2s, HoE = 15', Temperature = 50° F, QFE = 29.83", UT culm = 06:19:44, Southerly Moon Culminating at UL h = 81°09.2' seen from a non-moving Observer.

NOTE : The following steps refer to the identically numbered earlier precepts

A3 - 3.2.1.1 - From own Software accurate to +/- 4", get for the Moon:

GHA=149°16.54', µRA=0.5205°/h, µGHA=µT= 14.52061°/h, Dec=+1°39.17', µD=0.2790°/h, Parallax=59.20'

NOTE : Since the Observer is not moving, µϕ = 0°/h and µG = 0°/h

A3 - 3.2.1.2 - From the ***Éphémérides Nautiques (2D) Tables***, let's perform a preliminary estimate of $\mathbf{h_o}$:
Dip = -3.7' , LL Correction (-Refraction, +Parallax +SD) = +25.3' , UL D Correction = -32.3'
Total Correction = -10.7' hence **h_o provisional value : h_{oo} = 80°58.5'**

A3 - 3.2.1.4 - From h_{oo} get regula falsi ϕ_o = D + (90° - h_{oo}) = 10°40.67' , i.e. a Northerly Latitude

A3 - 3.2.1.8 - From T_{oo} = 0° and ϕ_o = 10°40.67' from **(2)** get T_{01}= 0.17578°, T_{02}= 0.17578°, hence T_{10}= 0.17578°.
With G_o = (GHA - T_{10}) get : Observer's 1st Approximate Position at (ϕ_o = 10°40.67', G_o = 149°05.99')

A3 - 3.2.1.9 - Through a Marcq Saint Hilaire Line of Position yielding -0.2 NM /181.1° , the 3D correction onto Moon UL height gives **h_o = 80°58.23'**, (vs. earlier 81°58.5') to be kept from now on. From **h_o = 80°58.23'** :

A3 - 3.2.1.10.1 - As per 3.2.1.4 update ϕ_o = 10°40.67' into regula falsi ϕ_1 = 10°40.94'

A3 - 4.1 - First Iteration on T

With T_{10} = 0.17578°, ϕ_1 = 10°40.94' , from **(2)** get T_{11} = 0.17587° , T_{12} = 0.17587°, hence T_{20} = 0.17587°

A3 - 4.2 - First Update on ϕ

A3 - 4.2.1 - With T_{20}= 0.17587°, from **(1)** get : h_1 = 80°58.13' and (h_1 - $\mathbf{h_o}$) = 80°58.13' - **80°58.23'** = -0.10'
With (h_1 - $\mathbf{h_o}$) = -1.08', we need to increase ϕ_1 in order to bring h_2 (next in **§ A 4.4**) closer to $\mathbf{h_o}$
Hence regula falsi ϕ_2 = ϕ_1 - (h_1 - $\mathbf{h_o}$) = 10°40.94' - (-0.10') = 10°40.84'

A3 - 4.3 - Second iteration on T

With T_{20} = 0.17587°, ϕ_1 = 10°40.84', from **(2)** get T_{21} = 0.17583° , T_{22} = 0.17583° hence T_{30}= 0.17583°

A3 - 4.4 - Second Update on ϕ

With T_{30}= 0.17583°, from **(1)** get : h_2 = 80°58.23' . Since (h_2 - $\mathbf{h_o}$) = 0.00' , our iteration is then complete.

See full results on next page

EXAMPLE A3 - 23 Nov 2023, UT culm = 06:19:44 Southerly Moon UL = 81°09.2' Cmg/Smg = 153°/ 0 kt

FINAL POSITION OBTAINED BY THE ITERATION METHOD

Observer's position at UT Culmination: 06h19m44.0s / N 10°40.84' / W 149°05.99'

ΔUT_{mp-c} = UT Meridian passage - UT Culmination = -0m43.6 s

Observer's position at UT Meridian passage: 06h19m00.4s / N 10°40.84' / W 149°05.99'

Quick Check : At 06h19m00.4s MOON height UL = 81°09.09' , Az = 179.99890° : quick check OK

A3 - Data for Twin-altitude LAN Method

Difference between extreme heights: 35.2'

	4	81°09.2		06:20:00.0
	UT	UL Heights		UT
3	06:09:57.0	80°50.8	5	06:29:32.0
2	06:08:03.5	80°43.2	6	06:31:24.0
1	06:06:05.5	80°34.0	7	06:33:22.0

A3 - Data for other LAN Methods

Difference between extreme heights : 51.9'

	UT	UL Heights		UT	UL Heights
1	6:03:03.0	80°17.3'	10	6:21:39.0	81°08.5'
2	6:05:06.5	80°28.9'	11	6:23:42.0	81°06.1'
3	6:07:07.5	80°39.0'	12	6:25:51.0	81°01.9'
4	6:09:08.5	80°47.7'	13	6:27:53.0	80°56.4'
5	6:11:11.0	80°55.1'	14	6:29:59.0	80°49.0'
6	6:13:17.0	81°01.1'	15		
7	6:15:25.0	81°05.6'	16		
8	6:17:32.0	81°08.2'	17		
9	6:19:34.5	81°09.2'	18		

A3 - Position by the Twin-altitude LAN Method with a corrective factor applied to both ΔUT_{mp-c} and $-\mu\phi\Delta UT_{mp-c}$: "k" = $(15/\mu T)^2$ = 1.0671

D≃+1°39' µD=+16.7'/h ϕ ≃ +10°30' µϕ = 0/h

µT=14.521°/h **ΔUT_{mp-c} = -40s**

$-0.5|(\mu\phi-\mu D)\Delta UT_{mp-c}|$ = -0.1'

UT_{culm} = 06:19:44 ***$k\Delta UT_{mp-c}$ = -43s***

UT_{mp} = 06:19:01 **$-0.5|(\mu\phi-\mu D)k\Delta UT_{mp-c}|$ = -0.1'**

hUL_{culm}=81°09.2' $h_{oo culm}$=80°58.5' $h_{oo mp}$=80°58.4'

Position at UT_{mp} : N 10°40.6' / W 149°06.1'

"*STEP 2*" PERFORMANCE :

Error in UT Culm : 0 s Error in H culm : 0.0'

"*STEP 3*" and OVERALL POSITION PERFORMANCE

Error in $k\Delta UT_{mp-c}$: -0.6s

When using "k" : Position Error = 0.3 NM

When not using "k" : Position Error = 0.9 NM

A3 - Position by a 6th order LAN Method

UT_{Culm} = 6h19m44.2s ΔUT_{mp-c} = -43.5s

Position : UT_{mp} = 6h19m01.1s / N 10°40.8' / W149°06.2'

SDEV=0.1 NM , Valid interval width +/- 0h14m @ UT_{culm} , i.e. Valid Observation time span: (06:05-06:34) εT'/εh'≤2.8

Longitude Error Uncertainty (1σ) = 0.2'

"*STEP 2*" PERFORMANCE :

Error in UT Culm : -0.2s Error in H culm : 0.0'

"*STEP 3*" and OVERALL POSITION PERFORMANCE

Error in ΔUT_{mp-c} : -0.1 s

Actual Position Error : 0.2 NM

****

Example A4 : **NON-MOVING Observer, Northerly Culmination with UL h = 81°**

A4 - 23 Nov 2023 , TT-UT = + 69.2s, HoE = 15', Temperature = 50° F, QFE = 29.83", UT culm = 06:19:44, Northerly Moon Culminating at UL h = 81°09.2' seen from a non-moving Observer.

NOTE : The following steps refer to the identically numbered earlier precepts

A4 - 3.2.1.1 - From own Software accurate to +/- 4", get for the Moon:

GHA=149°16.54', µRA=0.5205°/h, µGHA=µT= 14.52061°/h, Dec=+1°39.17', µD=0.2790°/h, Parallax=59.20'

NOTE : Since the Observer is not moving, µϕ = 0°/h and µG = 0°/h

A4 - 3.2.1.2 - From the ***Éphémérides Nautiques (2D) Tables***, let's perform a preliminary estimate of $\mathbf{h_o}$:
Dip = -3.7' , LL Correction (-Refraction, +Parallax +SD) = +25.3' , UL D Correction = -32.3'
Total Correction = -10.7' hence **h_o provisional value : h_{oo} = 80°58.5'**

A4 - 3.2.1.4 - From h_{oo} get regula falsi ϕ_o = D + (90° - h_{oo}) = -7°22.33' , i.e. a Southerly Latitude

A4 - 3.2.1.8 - From T_{oo} = 0° and ϕ_o = -7°22.33' from **(2)** get T_{01}= -0.17417°, T_{02}= -0.17417°, hence T_{10}= -0.17417°.
With G_o = (GHA - T_{10}) get : Observer's 1st Approximate Position at (ϕ_o = -7°22.33', G_o = 149°26.99')

A4 - 3.2.1.9 - Through a Marcq Saint Hilaire Line of Position yielding -0.1 NM /001.1° , the 3D correction onto Moon UL height gives **h_o = 80°58.25'**, (vs. earlier **80°58.5'**) to be kept from now on. From **h_o = 80°58.25'** :

A4 - 3.2.1.10.1 - As per 3.2.1.4 update ϕ_o = -7°22.33' into regula falsi ϕ_1 = -7°22.58'

A4 - 4.1 - First Iteration on T

With T_{10} = -0.17417°, ϕ_1 = -7°22.58' , from **(2)** get T_{11} = -0.17425° , T_{12} = -0.17425°, hence T_{20}= -0.17425°

A4 - 4.2 - First Update on ϕ

A4 - 4.2.1 - With T_{20}= -0.17425°, from **(1)** get : h_1 = 80°58.15' and (h_1 - $\mathbf{h_o}$) = 80°58.15' - **80°58.25'** = -0.10'
With (h_1 - $\mathbf{h_o}$) = -0.10', we need to decrease ϕ_1 in order to bring h_2 (next in **§ A 4.4**) closer to $\mathbf{h_o}$
Hence regula falsi ϕ_2 = ϕ_1 + (h_1 **-** $\mathbf{h_o}$) = -7°22.58' - (-0.10') = 7°22.48'

A4 - 4.3 - Second iteration on T

With T_{20}= -0.17425°, ϕ_1 = 7°22.48' , from **(2)** get T_{21} = -0.17422° , T_{22} = -0.17422°, hence T_{30}= -0.17422°

A4 - 4.4 - Second Update on ϕ

With T_{30}= -0.17422°, from **(1)** get : h_2 = 80°58.25' . Since (h_2 - $\mathbf{h_o}$) = 0.00' , our iteration is then complete.

See full results on next page

A4 - 5 - FINAL POSITION FROM ITERATION METHOD, AND COMPARISON CHECKS WITH DATA FOR LAN METHODS

EXAMPLE A4 - 23 Nov 2023, UT culm = 06:19:44 Northerly Moon UL = 81°09.2' Cmg/Smg = 153°/ 0 kt

FINAL POSITION OBTAINED BY THE ITERATION METHOD

Observer's position at UT $_{Culmination}$: 06h19m44.0s / S 07°22.48' / W 149°26.99'

ΔUT_{mp-c} = UT $_{Meridian\ passage}$ - UT $_{Culmination}$ = +0m43.2s

Observer's position at UT $_{Meridian\ passage}$: 06h20m27.2s / S 07°22.48' / W 149°26.99'

Quick Check : At 06h20m27.2s MOON height UL = 81°09.097' , Az = 0.00041° : quick check OK

A4 - Data for the Twin-altitude LAN Method

Difference between extreme heights: 33.5'

	4	81°09.2'		06:19:30.0
	UT	UL Heights		UT
3	06:10:30.0	80°52.6'	5	06:28:59.0
2	06.08:31.0	80°44.9'	6	06:30:57.0
1	06:06:30.5	80°35.7'	7	06:32:58.5

A4 - Position by the Twin-altitude LAN Method with a corrective factor applied to both ΔUT_{mp-c} and $-\mu\phi\Delta UT_{mp-c}$: "k" = $(15/\mu T)^2$ = 1.0671

D≃+1°39' μD=+16.7'/h ϕ ≃ -07°30' μϕ = 0/h

μT=14.521°/h ΔUT_{mp-c} = +41s

$-0.5|(\mu\phi-\mu D)\Delta UT_{mp-c}|$ = -0.1'

UT_{culm} = 06:19:44 $k\Delta UT_{mp-c}$ = +44s

UT_{mp} = 06:20:28 $-0.5|(\mu\phi-\mu D)k\Delta UT_{mp-c}|$ = -0.1'

hUL_{culm}=81°09.2' $h_{oo culm}$=80°58.5' $h_{oo mp}$=80°58.4'

Position at UT_{mp} : S 07°22.2' / W 149°27.2'

"*STEP 2*" PERFORMANCE :

Error in UT $_{Culm}$: 0 s Error in H $_{culm}$: 0.0'

"*STEP 3*" and OVERALL POSITION PERFORMANCE

Error in $k\Delta UT_{mp-c}$: -0.8 s

When using "k" : Position Error = 0.3 NM

When not using "k" : Position Error = 0.6 NM

A4 - Data for other LAN Methods

Difference between extreme heights : 35.6'

	UT	UL Heights		UT	UL Heights
1	06:06:03.0	80°33.5'	10	06:24.35.0	81°04.6'
2	06:08:06.0	80°43.1'	11	06:26:42.0	80°59.7'
3	06:10:10.5	80°51.5'	12	06:28:47.5	80°53.3'
4	06:12:14.0	80°58.2'	13	06:30:49.5	80°45.5'
5	06:14:17.0	81°03.4'	14	06:32:53.0	80°36.1'
6	06:16:20.5	81°06.9'	15		
7	06:18:25.0	81°08.9'	16		
8	06:20:27.0	81°09.1'	17		
9	06:22:29.5	81°07.7'	18		

A4 - Position by a 6th order LAN Method

UT_{culm} = 6h19m44.1s ΔUT_{mp-c} = +43.2s

Position : UT_{mp} = 6h20m27.3s / S07°22.5' / W149°27.0'

SDEV=0.0 NM , Valid interval width +/- 0h14m @ UT_{culm} ,

i.e. Valid Observation time span: (06:05-06:34) εT'/εh'≤2.8

Longitude Error Uncertainty (1σ) = 0.1'

"*STEP 2*" PERFORMANCE :

Error in UT $_{Culm}$: -0.1 s Error in H $_{culm}$: 0.0'

"*STEP 3*" and OVERALL POSITION PERFORMANCE

Error in $k\Delta UT_{mp-c}$: 0.0 s

Actual Position Error : 0.0 NM

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APPENDIX B - FAST MOVING OBSERVER

In **APPENDIX A** and **APPENDIX B** of this document we are comparing the **LAN Iteration Method** with a LAN Method using both **Formula (3.1)** here-above and the **Formula (4)** here-after, namely the **"Twin Altitude" Method**.

This "**Twin Altitude Method**" also referred to as the "**Double/Equal/Iso Altitude Method**" involves taking and recording the times of the **very same pre-set altitudes before and after Culmination**.

- The Culmination Time is taken as the averaged value of all such Observations. And:
- The Culmination Height is taken separately without recording its time.

The Meridian Passage Time is obtained from the Culmination Time through **Formula (3.1)**.

The Meridian Passage Height is computed from the Culmination Height through the following formula:

$$\textbf{(4)}\quad h_{ooMp} = h_{ooCulm} - 0.5|(\mu\phi - \mu D)k\Delta UT_{mp-c}|$$

And for historical reference:

The following **Twin Altitude LAN** rules were published for the **French Navy** in 1934 - *Ref (1)* :

(3) - Attempt using only h Culm's higher than 80°, but do not use h Culm's lower than 60°.

(4) - Carefully prepare the Observations, e.g. for 3 sets of heights observe as per the following schedule :

h Culm range	89° - 88°	88° - 86°	86° - 84°	84° - 82°	82° - 80°
Start observations ahead of best LAN estimate by:	6 minutes	7 min.	9 min.	11 min.	13 min.
Increase (decrease) successive Sextant heights by:	7'	5'	4'	3'	3'
Stop observations ahead of best LAN estimate by:	2 minutes	3 min.	5 min.	7 min.	9 min.

The following examples processing *noise-free data obtained through reverse-engineering* all involve one same Moon LAN Culmination Time and the same Upper Limb: one observed at 31° and one observed at 81°, one from a North Culmination position and one from a South Culmination position, and finally one from a steady Observer and one from a high speed moving Observer, which altogether makes 8 different configurations.

With the ***LAN Iteration Method*** used as a benchmark, each of these 8 examples is solved by both:

- The ***Twin Altitude LAN Method*** using as ***benchmark height*** the ***Nautical Almanac*** corrected UL Heights.

- A ***6th order LAN Method*** using as ***benchmark height*** the more accurate 3D corrected same UL Heights differing by about 0.25' from their ***NA*** corrected counterparts.

The performance of each LAN Method is checked for both:

- *"STEP 2"* PERFORMANCE :

How do **UT_{Culm}** and **h_{Culm}** obtained by each method compare to the "reference" initial values.

- *"STEP 3"* and then OVERALL END POSITION PERFORMANCE :

How do the **End Positions** derived by each method compare to the **"true" End Position** obtained by the **LAN Iteration Method**.

Last note: Many **non-significant decimals** published throughout for computation performance checks only.

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Example B1 : **FAST MOVING Observer, Southerly Culmination with UL h = 31°**

B1 - 23 Nov 2023, TT-UT = +69.2 s, HoE = 15', Temperature = 50° F, QFE = 29.83", UT culm = 06:19:44, Southerly Moon culminating at UL h = 31°09.2' seen from an Observer moving at Cmg/Smg = 153° /50 kts

NOTE : The following steps refer to the identically numbered earlier precepts

Here steps 3.2.1.1 and 3.2.2.2 are identical for both examples A and B.

B1 - 3.2.1.1 - From own Software accurate to +/- 4", get for the Moon:

GHA=149°16.54' , µRA=0.5205°/h , µGHA=14.52061°/h , Dec=+1°39.17' , µD=0.2790°/h , Parallax=59.20'

B1 - 3.2.1.2 - From the ***Éphémérides Nautiques (2D) Tables***, let's perform a preliminary estimate of $\mathbf{h_o}$:
Dip = -3.7' , LL Correction (-Refraction, +Parallax +SD) = +65.2' , UL D Correction = -32.2'
Total Correction = +33.0' , hence **h_o provisional value : h_{oo} = 31°38.5'**

B1 - 3.2.1.4 - From h_{oo} get regula falsi ϕ_o = D - (90° - h_{oo}) = 60°00.67' , i.e. a Northerly Latitude

B1 - 3.2.1.5. - **$\mu\phi$ = (Smg/60) sin Cmg = -0.74251 °/h**

B1 - 3.2.1.6 - μG_o °/h = - (Smg/60) sin Cmg / cos ϕ_o {=-0.37833 * 60 NM /h = -22.7 kts (East Speed) =} = -0.75691 °/h

B1 - 3.2.1.7 - μT_o °/h = (15.0410684 - µRA - μG_o) °/h = (µGHA - μG_o) °/h = 15.27752 °/h

B1 - 3.2.1.8 - From T_{oo} = 0° , ϕ_o = -56°42.33' and μT_o = 15.20980 °/h from **(2)** get T_{01} = 6.54179° ,
T_{02} = 6.51037°, T_{03} = 6.51067° , T_{04} = 6.51066°, T_{05} = 6.51066° hence T_{10}= 6.51066° . With G_o = (GHA - T_{10}) get:

Observer's 1st Approximate Position at : ϕ_o = 60°00.67', G_o = W 142°45.90'

B1 - 3.2.1.9 - Through a Marcq Saint Hilaire Line of Position yielding +12.7 NM / 187.60° , the 3D correction onto Moon UL height gives **h_o = 31°38.18'**, (vs. earlier 31°38.5') to be kept from now on.

B1 - 3.2.1.10 - From **h_o = 31°38.18'** :

B1 - 3.2.1.10.1 - As per 3.2.1.4 update ϕ_o = 60°00.67' into regula falsi ϕ_1 = 60°00.99' , and :

B1 - 3.2.1.10.2 - As per 3.2.1.5 update μG_o = - 0.75691 °/h into μG_1 = - 0.75703 °/h , and finally :

B1 - 3.2.1.10.3 - As per 3.2.1.6 update μT_o = 15.27752 °/h into μT_1 = 15.27764 °/h

B1 - 4.1 - First iteration on T

With T_{10} = 6.51066°, ϕ_1 = 60°00.99' and μT_1 = 15.27764 °/h, from **(2)** get T_{11} = 6.51204° , T_{12} = 6.51203°,
T_{13} = 6.51203° hence T_{20} = 6.51203°

B1 - 4.2. - First update on ϕ and µT

B1 - 4.2.1 - With T_{20}= 6.51203°, from **(1)** get : h_1 = 31°25.18' and (h_1 - $\mathbf{h_o}$) = 31°25.18' - **31°38.18'** = -13.00'

With (h_1 - $\mathbf{h_o}$) = -13.00', we need to decrease ϕ_1 in order to bring h_2 (next in **§B1 - 4.4**) closer to $\mathbf{h_o}$
Hence regula falsi ϕ_2 = ϕ_1 - (h_1 - $\mathbf{h_o}$) = 60°00.99' - (-13.00') = 59°47.99'

B1 - 4.2.2 - μG_2 = -0.75210 °/h and μT_2 = 15.27272 °/h

B1 - 4.3 - Second iteration on T

With T_{20}= 6.51203°, ϕ_2 = 59°47.99 ′ and μT_2 = 15.27272 °/h, from **(2)** get T_{21} = 6.45640° , T_{22} = 6.45692°, T_{23} = 6.45692°, hence T_{30}= 6.45692°

B1 - 4.4 - Second Update on ϕ and μT

B1 - 4.4.1 - With T_{30} = 6.45692°, from **(1)** get : h_2 = 31°38.29′ and (h_2 - $\mathbf{h_o}$) = 31°38.29′ - **31°38.18′** = +0.11′
With (h_2 - $\mathbf{h_o}$) = +0.11′ , we need to decrease ϕ_2 in order to bring h_3 (next in **§ B 4.6**) further from $\mathbf{h_o}$
Hence regula falsi ϕ_3 = ϕ_1 + (h_2 **-** $\mathbf{h_o}$) = 59°47.99′ - (+0.11′) = 59°48.09′.
B1 - 4.4.2 - μG_3 = -0.75214 °/h and μT_3 = 15.27276 °/h

B1 - 4.5 - Third iteration on T

With T_{30}= 6.45692°, ϕ_3 = 59°48.09′ and μT_3 = 15.27276 °/h, from **(2)** get T_{31} = 6.45737° , T_{32} = 6.45736°, T_{33} = 6.45736°hence T_{40}= 6.45736°

B1 - 4.6 - Third Update on ϕ and μT

With T_{40}= 6.45736°, from **(1)** get : h_3 = 31°38.18′ and (h_3 - $\mathbf{h_o}$) = 31°38.18′ - **31°38.18′** = 0.00′
Since (h_3 - $\mathbf{h_o}$) = 0.00′ , our iteration is then complete and (UT Mer. Pass - UT Culm) = $-T_{40}$ / μT_3 = -25m22.1s

B1 - 5 - FINAL POSITION FROM ITERATION METHOD, AND COMPARISON CHECKS WITH DATA FOR LAN METHODS

EXAMPLE B1 - 23 Nov 2023, UT culm = 06:19:44 Southerly Moon UL = 31°09.2′ Cmg/Smg = 153°/50 kts

FINAL POSITION OBTAINED BY THE ITERATION METHOD
Observer's position at UT Culmination: 06h19m44.0s / N 59°48.09′ / W 142°49.10′
ΔUTmp-c = UT Meridian passage - UT Culmination = -25m22.1 s
Observer's position at UT Meridian passage: 05h54m21.9s / N 60°06.93′ / W143°08.18′
Quick Check : At 05h54m21.9s MOON height UL = 30°56.082′ , Az = 179.99798° : quick check OK

B1 - Data for the Twin-altitude LAN Method
Difference between extreme heights: 20′

	6	31°09.2′		06:19:30.0
	UT	UL Heights		UT
5	06:00:07.0	31°01.3′	7	06:39:20.5
4	05:57:09.0	30°58.8′	8	06:42:13.0
3	05:54:11.5	30°55.9′	9	06:45:07.5
2	05:51:14.0	30°52.7′	10	06:48:02.0
1	05:48:17.5	30°49.1′	11	06:51:00.5

B1 - Position by the Twin-altitude LAN Method
With a corrective factor applied to both ΔUTmp-c
and -μφΔUTmp-c : "k" = $(15/\mu T)^2$ = 0.9646
D≃+1°39′ μD=+16.7′/h φ ≃ +59°50′ μφ = -44.6′/h
μT=15.273°/h **ΔUTmp-c = -26m24s**
-0.5|(μφ-μD)ΔUTmp-c| = -13.5′
UTculm = 06:19:40 *k*ΔUTmp-c = -25m28s
UTmp = 05:54:12 -0.5|(μφ-μD)*k*ΔUTmp-c|= -13.0′
hULculm=31°09.2′ hoocculm=31°38.5′ hoomp=31°25.5′
Position at UTmp : N60°06.5′ / W 143°05.8′

"*STEP 2*" PERFORMANCE :
Error in UT Culm : - 4s Error in H culm : 0.0′
"*STEP 3*" and OVERALL POSITION PERFORMANCE
Error in *k*ΔUTmp-c : +5.9 s

When using "k" : Position Error = 1.3 NM
When NOT using "k" : Position Error = 8.0 NM

B1 - Data for other LAN Methods
Difference between extreme heights : 25′

	UT	UL Heights		UT	UL Heights
1	05:45:00.0	30°44.7′	10	06:30:00.0	31°07.0′
2	05:50:00.0	30°51.2′	11	06:35:00.0	31°04.4′
3	05:55:00.0	30°56.7′	12	06:40:00.0	31°00.8′
4	06:00:00.0	31°01.2′	13	06:45:00.0	30°56.1′
5	06:05:00.0	31°04.8′	14	06:50:00.0	30°50.4′
6	06:10:00.0	31°07.3′	15	06:55:00.0	30°43.6′
7	06:15:00.0	31°08.7′	16		
8	06:20:00.0	31°09.2′	17		
9	06:25:00.0	31°08.6′	18		

B1 - Position by a 6th order LAN Method
UTculm = 6h19m44.7s ΔUTmp-c = -25m22.0s
Position : UTmp = 5h54m22.6s / N60°06.7′ / W143°08.3′
SDEV=0.2 NM , Valid interval width +/- 1h20m @ UTculm ,
i.e. Valid Observation time span: (04:59-07:40) εT′/εh′≤4.9
Longitude Error Uncertainty (1σ) = 0.9′

"*STEP 2*" PERFORMANCE :
Error in UT Culm : -0.7 s Error in H culm : 0.0′

"*STEP 3*" and OVERALL POSITION PERFORMANCE
Error in *k*ΔUTmp-c : -0.1 s

Actual Position Error : 0.2 NM

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Example B2 : **FAST MOVING Observer, Northerly Culmination with UL h = 31°**

B2 - 23 Nov 2023, TT-UT = +69.2 s, HoE = 15', Temperature = 50° F, QFE = 29.83", UT culm = 06:19:44, Northerly Moon culminating at UL h = 31°09.2' seen from an Observer moving at 50 kts with Cmg at 153°

NOTE : The following steps refer to the identically numbered earlier precepts

Here steps 3.2.1.1 and 3.2.2.2 are identical for both examples A and B.

B2 - 3.2.1.1 - From own Software accurate to +/- 4", get for the Moon:

GHA=149°16.54' , µRA=0.5205°/h , µGHA=14.52061°/h , Dec=+1°39.17' , µD=0.2790°/h , Parallax=59.20'

B2 - From the ***Éphémérides Nautiques (2D) Tables***, let's perform a preliminary estimate of $\mathbf{h_0}$:
Dip = -3.7' , LL Correction (-Refraction, +Parallax +SD) = +65.2' , UL D Correction = -32.2'
Total Correction = +33.0' , hence **h_0 provisional value : h_{00} = 31°38.5'**

B2 - 3.2.1.4 - From h_{00} get regula falsi ϕ_0 = D + (90° - h_{00}) = -56°42.33', i.e. a Southerly Latitude

B2 - 3.2.1.5. - **$\mu\phi$ = (Smg/60) sin Cmg = -0.74251 °/h**

B2 - 3.2.1.6 - μG_0 °/h = - (Smg/60) sin Cmg / cos ϕ_0 = -0.37833 * 60 NM /h = - 0.68919 °/h

B2 - 3.2.1.7 - μT_0 °/h = (15.0410684 - µRA - μG_0) °/h = (µGHA - μG_0) °/h = 15.20980 °/h

B2 - 3.2.1.8 - From T_{00} = 0° , ϕ_0 = -56°42.33' and μT_0 = 15.20980 °/h from **(2)** get T_{01} = -5.98093° ,
T_{02} = -5.95745°, T_{03} = -5.95763°, T_{04} = -5.95763° hence T_{10}= -5.95763° . With G_0 = (GHA - T_{10}) get:

Observer's 1st Approximate Position at : ϕ_0 = -56°42.33', G_0 = W 155°14.00'

B2 - 3.2.1.9 - Through a Marcq Saint Hilaire Line of Position yielding +11.6 NM / 007.0° , the 3D correction onto Moon UL height gives **h_0 = 31°38.18'**, (vs. earlier 31°38.5') to be kept from now on.

B2 - 3.2.1.10 - From **h_0 = 31°38.18'**

B2 - 3.2.1.10.1 - As per 3.2.1.4 update ϕ_0 = -56°42.33' into regula falsi ϕ_1 = -56°42.65' , and :

B2 - 3.2.1.10.2 - As per 3.2.1.5 update μG_0 = - 0.38499 °/h into μG_1 = - 0.68929 °/h , and finally :

B2 - 3.2.1.10.3 - As per 3.2.1.6 update μT_0 = 14.90560 °/h into μT_1 = 15.20990 °/h

B2 - 4.1 - First iteration on T

With T_{10}= -5.95763°, ϕ_1 = -56°42.65' and μT_1 = 15.20990 °/h, from **(2)** get T_{11} = -5.95879 ° , T_{12} = -5.95878 °,
T_{13} = -5.95878 hence T_{20} = -5.95878 °

B2 - 4.2. - First update on ϕ and µT

B2 - 4.2.1 - With T_{20}= -5.95878 °, from **(1)** get : h_1 = 31°26.22' and (h_1 - $\mathbf{h_0}$) = 31°26.22' - **31°38.18'** = -11.96'

With (h_1 - $\mathbf{h_0}$) = -11.96', we need to increase ϕ_1 in order to bring h_2 (next in **§ B2 - 4.4**) closer to $\mathbf{h_0}$

Hence regula falsi ϕ_2 = ϕ_1 - (h_1 - $\mathbf{h_0}$) = -56°42.65' - (-11.96') = -56°30.70'

B2 - 4.2.2 - μG_2 = -0.68566 °/h and μT_2 = 15.20627 °/h

B2 - 4.3 - Second iteration on T

With T_{20} = -5.95878 °, ϕ_2 = -56°30.70'and μT_2 = 15.206278 °/h, from **(2)** get T_{21} = -5.91594° , T_{22} = -5.91627°, T_{23} = -5.91627° Hence T_{30}= -5.91627°

B2 - 4.4 - Second Update on ϕ and µT

B2 - 4.4.1 - With T_{30}= -5.91627° from **(1)** get : h_2 = 31°38.26' and (h_2 - h_o) = 31°38.26' - **31°38.18'** = +0.08'
With (h_2 - h_o) = +0.08', we need to decrease ϕ_1 in order to bring h_3 (next in **§ B2 - 4.6**) closer to h_o
Hence regula falsi ϕ_3 = ϕ_2 - (h_2 - h_o) = -56°30.70' - (0.08') = -56°30.78'
B2 - 4.4.2 - μG_3 = -0.68568 °/h and μT_3 = 15.20630 °/h

B2 - 4.5 - Third iteration on T

With T_{30}= -5.91627°, ϕ_3 = -56°30.70' and μT_3 = 15.20630 °/h, from **(2)** get T_{31} = -5.91656° , T_{32} = -5.91656°, T_{33} = -5.916565°, hence T_{40}= -5.91656°

B2 - 4.6 - Third Update on ϕ and µT

B2 - 4.6.1 - With T_{40}= -5.91656° from **(1)** get : h_4 = 31°38.18' and (h_4 - h_o) = 31°38.18' - **31°38.18'** = -0.00'
Since (h_4 - h_o) = 0.00' , our iteration is then complete. (UT Mer. Pass - UT Culm) = $-T_{40}$ / μT_3 = +23m20.7s

B2 - 5 - FINAL POSITION FROM ITERATION METHOD, AND COMPARISON CHECKS WITH DATA FOR LAN METHODS

EXAMPLE B2 - 23 Nov 2023, UT culm = 06:19:44 Northerly Moon UL = 31°09.2' Cmg/Smg = 153°/50 kts

FINAL POSITION OBTAINED BY THE ITERATION METHOD

Observer's position at UT Culmination: 06h19m44.0s / S 56°30.78' / W 155°11.53'

ΔUT_{mp-c} = UT Meridian passage - UT Culmination = +23m20.7 s

Observer's position at UT Meridian passage: 06h43m04.7s / S 56°48.11' / W154°55.53'

Quick Check : At 06h43m04.7s MOON height UL = 30°57.130' , Az = 359.99895° : quick check OK

B2 - Data for the Twin-altitude LAN Method

Difference between extreme heights: 34'

	6	31°09.2'		06:20:00.0
	UT	UL Heights		UT
5	05:50:53.5	30°50.6'	7	06:48:50.0
4	05:48:17.0	30°47.1'	8	06:51:23.0
3	05:45:43.0	30°43.4'	9	06:53:57.0
2	05:43:17.5	30°39.6'	10	06:56:24.5
1	05:40:22.0	30°34.7'	11	06:59:19.0

B2 - Position by the Twin-altitude LAN Method with a corrective factor applied to both ΔUT_{mp-c} and $-\mu\phi\Delta UT_{mp-c}$: "k" = $(15/\mu T)^2$ = 0.9731

D≃+1°39' µD=+16.7'/h ϕ ≃ -56°30' µϕ = -44.6'/h

µT=15.206°/h **ΔUT_{mp-c} = +24m02s**

-0.5|(µϕ-µD)ΔUT_{mp-c}| = -12.3'

UT_{culm} = 06:19:51 ***k*ΔUT_{mp-c} = +23m23s**

UT_{mp} = 06:43:14 -0.5|(µϕ-µD)*k*ΔUT_{mp-c}|= -11.9'

hUL_{culm}=31°09.2' $h_{oo culm}$=31°38.5' $h_{oo mp}$=31°26.6'

Position at UT_{mp} : S 56°47.7' / W 154°57.8'

***"STEP 2"* PERFORMANCE :**

Error in UT Culm : -9 s Error in H culm : 0.0'

***"STEP 3"* and OVERALL POSITION PERFORMANCE**

Error in *k*ΔUT_{mp-c} : -2 s

When using "k" : Position Error = 1.3 NM

When NOT using "k" : Position Error = 6.4 NM

B2 - Data for other LAN Methods

Difference between extreme heights : 34'

	UT	UL Heights		UT	UL Heights
1	05:40:22.0	30°34.7'	10	06:56:24.5	30°39.6'
2	05:43:17.5	30°39.6'	11	06:59:19.0	30°34.7'
3	05:45:43.0	30°43.4'	12	*This example carries*	
4	05:48:17.0	30°47.1'	13	*the same data as the*	
5	05:50:53.5	30°50.6'	14	*Twin-Altitude Lan Method*	
6	06:20:00.0	31°09.2'	15	*running on the left side*	
7	06:48:50.0	30°50.5'	16	*of this page*	
8	06:51:23.0	30°47.1'	17		
9	06:53:57.0	30°43.4'	18		

B2 - Position by a 6th order LAN Method

UT_{culm} = 6h19m43.7s ΔUT_{mp-c} = +23m20.9s

Position : UT_{mp} = 06h43m04.6s / S56°48.2' / W154°55.5'

SDEV=0.2 NM , Valid interval width +/- 1h17m @ UT_{culm} ,

i.e. Valid Observation time span: (05:02-07:37) εT'/εh'≤4.8

Longitude Error Uncertainty (1σ) = 0.8'

***"STEP 2"* PERFORMANCE :**

Error in UT Culm : 0.3 s Error in H culm : 0.0'

***"STEP 3"* and OVERALL POSITION PERFORMANCE**

Error in *k*ΔUT_{mp-c} : -0.2s

Actual Position Error : 0.1 NM

****

B2 - 6 : Example B2 FULL CHECKS

From the Final position and time, the following Moon UL heights are obtained through reverse-engineering:

B2 - 6.1 - Moon Position at UT Culmination = 06h19m44s

h UL = 31°09.194' (*vs. UL benchmark at **31°09.2'***) / Az = 006.95159 °

B2 - 6.2 - Check for actual culmination occurring within +/- 30 SECONDS of time around Culmination time

UT culm - 30s = 06h19m14s / **31°09.188'** **UT Culm / 31°09.194'** **UT Culm + 30s** = 06h20m14s / **31°09.188'**

B2 - 6.3 - Check for actual culmination occurring within +/- 10 SECONDS of time around Culmination time

UT culm - 10s = 06h19m34s **/ 31°09.193'** **UT Culm / 31°09.194'** **UT Culm + 30s** = 06h19m54s / **31°09.193'** *Bingo !*

****

Example B3 : FAST MOVING Observer, Southerly Culmination with UL h = 81°

B3 - 23 Nov 2023, TT-UT = +69.2 s, HoE = 15', Temperature = 50° F, QFE = 29.83", UT culm = 06:19:44, Southerly Moon culminating at UL h = 81°09.2' seen from an Observer moving at 50 kts with Cmg at 153°

NOTE : The following steps refer to the identically numbered earlier precepts

Here steps 3.2.1.1 and 3.2.2.2 are identical for both examples A and B.

B3 - 3.2.1.1 - From own Software accurate to +/- 4", get for the Moon:

GHA=149°16.54' , µRA=0.5205°/h , µGHA=14.52061°/h , Dec=+1°39.17' , µD=0.2790°/h , Parallax=59.20'

B3 - 3.2.1.2 - From the ***Éphémérides Nautiques (2D) Tables***, let's perform a preliminary estimate of h_o :
Dip = -3.7' , LL Correction (-Refraction, +Parallax +SD) = +25.3' , UL D Correction = -32.3'
Total Correction = -10.7' hence **h_o provisional value : h_{oo} = 80°58.5'**

B3 - 3.2.1.4 - From h_{oo} get regula falsi ϕ_o = D + (90° - h_{oo}) = +10°40.67', i.e. a Northerly Latitude

B3 - 3.2.1.5. - **$\mu\phi$ = (Smg/60) sin Cmg = -0.74251 °/h**

B3 - 3.2.1.6 - μG_o °/h = - (Smg/60) sin Cmg / cos ϕ_o = -0.37833 * 60 NM /h = - 0.38499 °/h

B3 - 3.2.1.7 - μT_o °/h = (15.0410684 - µRA - μG_o) °/h = (µGHA - μG_o) °/h = 14.90560 °/h

B3 - 3.2.1.8 - From T_{oo} = 0° , ϕ_o = +10°40.67' and μT_o = 14.90560 °/h from **(2)** get T_{01} = 0.62704° ,
T_{02} = 0.62701°, hence T_{10}= 0.62701° . With G_o = (GHA - T_{10}) get:

Observer's 1st Approximate Position at : ϕ_o = 10°40.67', G_o = W 148°38.92'

B3 - 3.2.1.9 - Through a Marcq Saint Hilaire sight yielding +1.0 NM / 184.0° , the 3D correction onto Moon UL height gives **h_o = 80°58.23'**, (vs. earlier 80°58.5') to be kept from now on.

B3 - 3.2.1.10 - From **h_o = 80°58.23'**

B3 - 3.2.1.10.1 - As per 3.2.1.4 update ϕ_o = +10°40.47' into regula falsi ϕ_1 =+10°40.94' , and :

B3 - 3.2.1.10.2 - As per 3.2.1.5 update μG_o = - 0.38499 °/h into μG_1 = - 0.38500 °/h , and finally :

B3 - 3.2.1.10.3 - As per 3.2.1.6 update μT_o = 14.90560 °/h into μT_1 = 14.90561 °/h

B3 - 4.1 - First iteration on T

With T_{10}= 0.62677°, ϕ_1 =+10°40.94' and μT_1 = 14.90561 °/h, from **(2)** get T_{11} = 0.62733 ° , T_{12} = 0.62733°,
hence T_{20} = 0.62733°

B3 - 4.2. - First update on ϕ and µT

B3 - 4.2.1 - With T_{20}= 0.62733°, from **(1)** get : h_1 = 80°56.94' and (h_1 - **h_o**) = 80°56.94' - **80°58.23'** = -1.29'

With (h_1 - **h_o**) = -1.29', we need to decrease ϕ_1 in order to bring h_2 (next in **§ B3 - 4.4**) closer to **h_o**

Hence regula falsi ϕ_2 = ϕ_1 + (h_1 **-** **h_o**) = +10°40.93' + (-1.29') = 10°39.65'

B3 - 4.2.2 - μG_2 = -0.38497 °/h and μT_2 = 14.90558 °/h

B3 - 4.3 - Second iteration on T

With T_{20} = 0.62733°, ϕ_2 = 10°39.65'and μT_2 = 14.90558 °/h, from **(2)** get T_{21} = 0.62581° , T_{22} = 0.62581°,
Hence T_{30}= 0.62581°

B3 - 4.4 - Second Update on ϕ and μT

B3 - 4.4.1 - With T_{30}= 0.62581° from **(1)** get : h_2 = 80°58.23' and (h_2 - h_o) = 80°58.23' - **80°58.23'** = +0.01'
With (h_2 - h_o) = +0.01', we need to decrease ϕ_1 in order to bring h_3 (next in **§ B3 - 4.6**) closer to h_o
Hence regula falsi $\phi_3 = \phi_2$ - (h_2 - h_o) = 10°39.65' - (0.01') = 10°39.65'
B3 - 4.4.2 - μG_3 = -0.38497 °/h and μT_3 = 14.90558 °/h

B3 - 4.5 - Third iteration on T

With T_{30}= 0.62581°, ϕ_3 = 10°39.65' and μT_3 = 14.90588 °/h, from **(2)** get T_{31} = 0.62581° , hence T_{40}= 0.62581°

B3 - 4.6 - Third Update on ϕ and μT

B3 - 4.6.1 - With T_{40}= 0.62581° from **(1)** get : h_3 = 80°58.23' and (h_3 - h_o) = 80°58.23' - **80°58.23'** = 0.00'
Since (h_3 - h_o) = 0.00' , our iteration is then complete. (UT Mer. Pass - UT Culm) = $-T_{40} / \mu T_3$ = -2m31.1s

B3 - 5 - FINAL POSITION FROM ITERATION METHOD, AND COMPARISON CHECKS WITH DATA FOR LAN METHODS

EXAMPLE B3 - 23 Nov 2023, UT culm = 06:19:44 Southerly Moon UL = 81°09.2' Cmg/Smg = 153°/50 kts

FINAL POSITION OBTAINED BY THE ITERATION METHOD

Observer's position at UT Culmination: 06h19m44.0s / N 10°39.65' / W 148°38.99'

ΔUT_{mp-c} = UT Meridian passage - UT Culmination = -2m31.1 s

Observer's position at UT Meridian passage: 06h17m12.9s / N 10°41.52' / W148°39.96'

Quick Check : At 06h17m12.9s MOON height UL = 81°07.891' , Az = 180.00043° : quick check OK

B3 - Data for the Twin-altitude LAN Method

Difference between extreme heights : 37'

	4	81°09.2'		06:20:00.0
	UT	UL Heights		UT
3	06:09:57.0	80°49.8'	5	06:29:30.0
2	06:08:03.5	80°41.7'	6	06:31:23.5
1	06:06:05.5	80°32.0'	7	06:33:21.0

B3 - Position by the Twin-altitude LAN Method with a corrective factor applied to both ΔUT_{mp-c} and $-\mu\phi\Delta UT_{mp-c}$: "k" = $(15/\mu T)^2$ = 1.0127

D≃+1°39' μD=+16.7'/h ϕ ≃ +10°30' μϕ = -44.6'/h
μT=14.906°/h **ΔUT_{mp-c} = -2m27s**
-0.5|(μϕ-μD)ΔUT_{mp-c}| = -1.3'
UT_{culm} = 06:19:43 *k*ΔUT_{mp-c} = -2m28s
UT_{mp} = 06:17:15 -0.5|(μϕ-μD)*k*ΔUT_{mp-c}| = -1.3'
hUL_{culm}=81°09.2' $h_{o}culm$=80°58.5' $h_{o}omp$=80°57.2'
Position at UT_{mp} : N 10°41.3' / W 148°40.5'

"*STEP 2*" PERFORMANCE :
Error in UT Culm : 1 s Error in H culm : 0.0'
"*STEP 3*" and OVERALL POSITION PERFORMANCE
Error in *k*ΔUT_{mp-c} : -3 s

When using "k" : Position Error = 0.6 NM
When NOT using "k" : Position Error = 0.6 NM

B3 - Data for other LAN Methods

Difference between extreme heights : 55'

	UT	UL Heights		UT	UL Heights
1	6:03:03.0	80°14.4'	10	6:21:39.0	81°08.4'
2	6:05:06.5	80°26.6'	11	6:23:42.0	81°06.0'
3	6:07:07.5	80°37.3'	12	6:25:51.0	81°01.5'
4	6:09:08.5	80°46.5'	13	6:27:53.0	80°55.6'
5	6:11:11.0	80°54.3'	14	6:29:59.0	80°47.9'
6	6:13:17.0	81°00.7'	15		
7	6:15:25.0	81°05.4'	16		
8	6:17:32.0	81°08.2'	17		
9	6:19:34.5	81°09.2'	18		

B3 - Position by a 6th order LAN Method

UT_{culm} = 06h19m45.8s ΔUT_{mp-c} = -2m31.2s
Position : UT_{mp} = 06h17m14.6s / N10°41.5' / W148°40.4'
SDEV=0.1 NM , Valid interval width +/- 0h14m @ UT_{culm} ,
i.e. Valid Observation time span : (06:06 - 06:33) εT'/εh'≤2.8
Longitude Error Uncertainty (1σ) = 0.4'

"*STEP 2*" PERFORMANCE :
Error in UT Culm : -1.8 s Error in H culm : 0.0'

"*STEP 3*" and OVERALL POSITION PERFORMANCE
Error in *k*ΔUT_{mp-c} : 0.1 s

Actual Position Error : 0.3 NM

****

B3 - 6 : Example B3 FULL CHECKS

From the Final position and time, the following Moon UL heights are obtained through reverse-engineering:

B3 - 6.1 - Moon Position at UT Culmination = 06h19m44s

h UL = 81°09.200' (*vs. h UL benchmark at **81°09.2'***) / Az = 183.98875 °

B3 - 6.2 - CHECK FOR ACTUAL UL CULMINATION OVER +/- 30 SECONDS of time around Culmination time

UT culm - 30s = 06h19m14s / **81°09.148'** **UT Culm / 81°09.200'** **UT Culm + 30s** = 06h20m14s / **81°09.148'**

B3 - 6.3 - Check for actual culmination occurring within +/- 10 SECONDS of time around Culmination time

UT culm - 10s = 06h19m34s **/ 81°09.194'** **UT Culm / 81°09.200'** **UT Culm + 10s** = 06h19m54s / **81°09.194'**

B3 - 6.4 - Check for actual culmination occurring within … *+/- 4 SECONDS !!* of time around Culmination time

UT culm - 4s = 06h19m40s **/ 81°09.199'** **UT Culm / 81°09.200'** **UT Culm + 4s** = 06h19m48s / **81°09.199'**

Bingo !

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Example B4 : **FAST MOVING Observer, Northerly Culmination with UL h = 81°**

B4 - 23 Nov 2023, TT-UT = +69.2 s, HoE = 15', Temperature = 50° F, QFE = 29.83", UT culm = 06:19:44, Northerly Moon culminating at UL h = 81°09.2' seen from an Observer moving at 50 kts with Cmg at 153°

NOTE : The following steps refer to the identically numbered earlier precepts

Here steps 3.2.1.1 and 3.2.2.2 are identical for both examples A and B.

B4 - 3.2.1.1 - From own Software accurate to +/- 4", get for the Moon:
GHA=149°16.54' , µRA=0.5205°/h , µGHA=14.52061°/h , Dec=+1°39.17' , µD=0.2790°/h , Parallax=59.20'

B4 - 3.2.1.2 - From the ***Éphémérides Nautiques (2D) Tables***, let's perform a preliminary estimate of $\mathbf{h_o}$:
Dip = -3.7' , LL Correction (-Refraction, +Parallax +SD) = +25.3' , UL D Correction = -32.3'
Total Correction = -10.7' hence **h_o provisional value : h_{oo} = 80°58.5'**

B4 - 3.2.1.4 - From h_{oo} get regula falsi ϕ_o = D + (90° - h_{oo}) = -7°22.33', i.e. a Southerly Latitude

B4 - 3.2.1.5. - **µϕ = (Smg/60) sin Cmg = -0.74251 °/h**

B4 - 3.2.1.6 - μG_o °/h = - (Smg/60) sin Cmg / cos ϕ_o = -0.37833 * 60 NM /h = - 0.38148 °/h

B4 - 3.2.1.7 - μT_o °/h = (15.0410684 - µRA - μG_o) °/h = (µGHA - μG_o) °/h = 14.90209 °/h

B4 - 3.2.1.8 - From T_{oo} = 0° , ϕ_o = -7°22.33' and μT_o = 14.90209 °/h from **(2)** get T_{01} = -0.62147° ,
T_{02} = -0.62144°, T_{03} = -0.62144°, hence T_{10}= -0.62144° . With G_o = (GHA - T_{10}) get:

Observer's 1st Approximate Position at : ϕ_o = -7°22.33', G_o = W 149°53.83'

B4 - 3.2.1.9 - Through a Marcq Saint Hilaire sight yielding +1.0 NM / 004.0° , the 3D correction onto Moon UL height gives **h_o = 80°58.25'**, (vs. earlier 80°58.5') to be kept from now on.

B4 - 3.2.1.10 - From **h_o = 80°58.25'**

B4 - 3.2.1.10.1 - As per 3.2.1.4 update ϕ_o = -7°22.33' into regula falsi ϕ_1 = -7°22.58' , and :

B4 - 3.2.1.10.2 - As per 3.2.1.5 update μG_o = - 0.38148 °/h into μG_1 = - 0.38148 °/h , and finally :

B4 - 3.2.1.10.3 - As per 3.2.1.6 update μT_o = 14.90209 °/h into μT_1 = 14.90210 °/h

B4 - 4.1 - First iteration on T

With T_{10}= -0.62144°, ϕ_1 = - 7°22.58' and μT_1 = 14.90210 °/h, from **(2)** get T_{11} = -0.62173 ° , T_{12} = -0.62173°,
hence T_{20} = -0.62173°

B4 - 4.2. - First update on ϕ and µT

B4 - 4.2.1 - With T_{20}= -0.62173°, from **(1)** get : h_1 = 80°56.97' and (h_1 - $\mathbf{h_o}$) = 80°56.94' - **80°58.25'** = -1.28'

With (h_1 - $\mathbf{h_o}$) = -1.28', we need to increase ϕ_1 in order to bring h_2 (next in **§ B4 - 4.4**) closer to $\mathbf{h_o}$

Hence regula falsi ϕ_2 = ϕ_1 - (h_1 - $\mathbf{h_o}$) = -7°22.58' + (-1.28') = -7°21.31'

B4 - 4.2.2 - μG_2 = -0.38146 °/h and μT_2 = 14.90208 °/h

B4 - 4.3 - Second iteration on T

With T_{20} = -0.62173°, ϕ_2 = -7°21.31'and μT_2 = 14.90208 °/h, from **(2)** get T_{21} = -0.62025° , T_{22} = -0.62025°,
Hence T_{30}= -0.62025°

B4 - 4.4 - Second Update on ϕ and μT

B4 - 4.4.1 - With T_{30}= -0.62025° from **(1)** get : h_2 = 80°58.25' and (h_2 - h_o) = 80°58.23' - **80°58.25'** = 0.01'
With (h_3 - h_o) = +0.01', we need to decrease ϕ_1 in order to bring h_3 (next in **§ B4 - 4.6**) closer to h_o
Hence regula falsi ϕ_3 = ϕ_2 - (h_2 - h_o) = -7°21.31' - (0.01') = 10°39.65'
B4 - 4.4.2 - μG_3 = -0.38146 °/h and μT_3 = 14.90208 °/h

B4 - 4.5 - Third iteration on T

With T_{30}= -0.62025°, ϕ_3 = 10°39.65' and μT_3 = 14.90208 °/h, from **(2)** get T_{31} = -0.62025° , hence T_{40}= -0.62025°

B4 - 4.6 - Third Update on ϕ and μT

B4 - 4.6.1 - With T_{40}= 0.62525° from **(1)** get : h_3 = 80°58.25' and (h_3 - h_o) = 80°58.25' - **80°58.25'** = 0.00'
Since (h_2 - h_o) = 0.00' , our iteration is then complete. (UT Mer. Pass - UT Culm) = $-T_{30}$ / μT_2 = +2m29.8s

B4 - 5 - FINAL POSITION FROM ITERATION METHOD, AND COMPARISON CHECKS WITH DATA FOR LAN METHODS

EXAMPLE B4 - 23 Nov 2023, UT culm = 06:19:44 Northerly Moon UL = 81°09.2' Cmg/Smg = 153°/50 kts

FINAL POSITION OBTAINED BY THE ITERATION METHOD

Observer's position at UT Culmination: 06h19m44.0s / S 07°21.31' / W 149°53.76'

ΔUT_{mp-c} = UT Meridian passage - UT Culmination = +2m29.8 s

Observer's position at UT Meridian passage: 06h22m13.8s / S 07°23.16' / W149°52.80'

Quick Check : At 06h22m13.8s MOON height UL = 81°07.898' , Az = 0.00245° : quick check OK

B4 - Data for the Twin-altitude LAN Method

Difference between extreme heights: 35'

	4	81°09.2'		06:19:30.0
	UT	UL Heights		UT
3	06:10:30.0	80°51.7'	5	06:28:59.0
2	06.08:31.0	80°43.5'	6	06:30:59.5
1	06:06:30.5	80°33.8'	7	06:33:00.0

B4 - Position by the Twin-altitude LAN Method with a corrective factor applied to both ΔUT_{mp-c} and $-\mu\phi\Delta UT_{mp-c}$: "k" = $(15/\mu T)^2$ = 1.0132

D≃+1°39' μD=+16.7'/h ϕ ≃ -07°30' μϕ = -44.6'/h
μT=14.902°/h **ΔUT_{mp-c} = +2m30s**
-0.5|(μϕ-μD)ΔUT_{mp-c}| = -1.3'
UT_{culm} = 06:19:45 **$k\Delta UT_{mp-c}$ = +2m32s**
UT_{mp} = 06:22:17 -0.5|(μϕ-μD)$k\Delta UT_{mp-c}$| = -1.3'
hUL_{culm}=81°09.2' h_{oculm}=80°58.5' h_{oomp}=80°57.2'
Position at UT_{mp} : S 07°22.9' / W 149°53.6'

"*STEP 2*" PERFORMANCE :
Error in UT Culm : -1 s Error in H culm : 0.0'
"*STEP 3*" and OVERALL POSITION PERFORMANCE
Error in $k\Delta UT_{mp-c}$: -2 s

When using "k" : Position Error = 0.8 NM
When NOT using "k" : Position Error = 0.8 NM

B4 - Data for other LAN Methods

Difference between extreme heights : 38'

	UT	UL Heights		UT	UL Heights
1	06:06:03.0	80°31.4'	10	06:24.35.0	81°04.3'
2	06:08:06.0	80°41.7'	11	06:26:42.0	80°59.2
3	06:10:10.5	80°50.5'	12	06:28:47.5	80°52.4'
4	06:12:14.0	80°57.6'	13	06:30:49.5	80°44.1'
5	06:14:17.0	81°03.0'	14	06:32:53.0	80°34.3'
6	06:16:20.5	81°06.8'	15		
7	06:18:25.0	81°08.8'	16		
8	06:20:27.0	81°09.1'	17		
9	06:22:29.5	81°07.6'	18		

B4 - Position by a 6th order LAN Method

UT_{culm} = 06h19m44.2s ΔUT_{mp-c} = +2m29.9s
Position : UT_{mp} = 06h22m14.1s / S07°23.2' / W149°52.9'
SDEV=0.1 NM , Valid interval width +/- 0h13m @ UT_{culm} ,
i.e. Valid Observation time span : (06:06 - 06:33) εT'/εh'≤2.8
Longitude Error Uncertainty (1σ) = 0.2'

"*STEP 2*" PERFORMANCE :
Error in UT Culm : -0.2 s Error in H culm : 0.0'

"*STEP 3*" and OVERALL POSITION PERFORMANCE
Error in $k\Delta UT_{mp-c}$: -0.1 s

Actual Position Error : 0.1 NM

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