Chapter 1 – Introduction

1.1 The Parts of a Slide Rule:

The following are the names we will be using throughout this book to describe the various parts of a Slide Rule.

1. Upper Body – or Stock
2. Lower Body – or Stock
3. Slide
4. Cursor
5. Hair Line

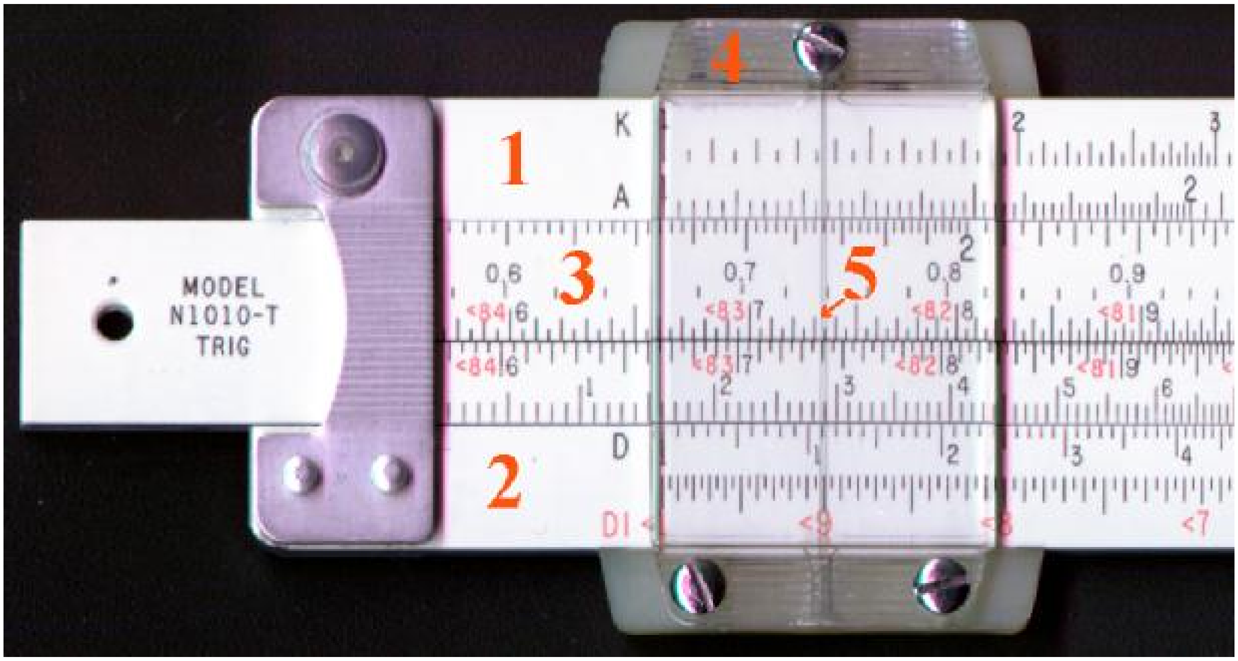


Fig 1-1

1.2 The C and D Scales

These two scales are the basic scales of the Slide Rule. The C scale is found on the lower edge of the slide and the D scale on the upper edge of the lower body. We will use the C and D scale for multiplication and division, and in later units we will see that all of the other scales on the Slide Rule are related directly to them.

For the first four units of this book we are going to forget all other scales and learn about the form and uses of the C and D scales.

The C and D scales are labeled from left to right beginning with the Left Index of 1 to the Right Index of 10. (Some Slide Rules may have the divisions extended past these values, and this is an advantage).

In Fig 1-2 you will notice (as on your Slide Rule) that the Primary Graduations (numbers) from 1 to 10 on the C and D scales are not evenly spaced. These scales are what we call logarithmic scales. That is, although the labeling is according to ordinary numbers the spacing or divisions between these are the logarithms of the numbers. You do not have to know about logarithms to use your Slide Rule, but if you have a knowledge of logarithms you will be able to understand how these scales work. If we check with a logarithm table we would find that: -

$\text{Log}_{10}1 = 0.0000$
$\text{Log}_{10}2 = 0.3010$
$\text{Log}_{10}3 = 0.4771$
$\text{Log}_{10}5 = 0.6990$
$\text{Log}_{10}10 = 1.0000$

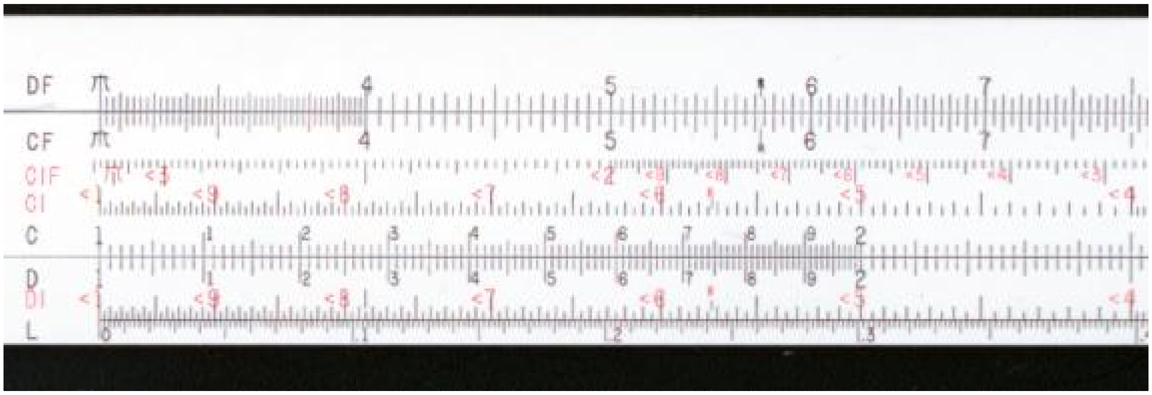


Fig 1-2

Thus when the numbers 1, 2, etc are placed on the C and D scales, their position is according to their logarithm. That is, 1 is no distance along, 2 is at the point .3010 of the distance along the scale, 3 is placed 0.4771 on the distance along the scale, and 10 is at the extreme end. This if your Slide Rule scale was exactly 10 inches long, you would find 2 at 3.01 inches, 3 at 4.771 inches, 5 at 6.99 inches and 10 at 10 inches.

1.3 Reading the C and D Scales

The upper scales in Fig 1-2 show the C and D scales divided into the Primary Graduations, that is 1, 2, 3, . . . 10. In the lower scales in Fig. 1-2 you will note that these have been further divided into tenths or Secondary Graduations. The graduations immediately after 2 thus reads 2.1, or the graduations immediately before 7 reads 6.9.

On your Slide Rule you will notice that there are still further subdivisions. These divisions are called the Tertiary Graduations and they vary along the scale as shown in fig. 1-3.

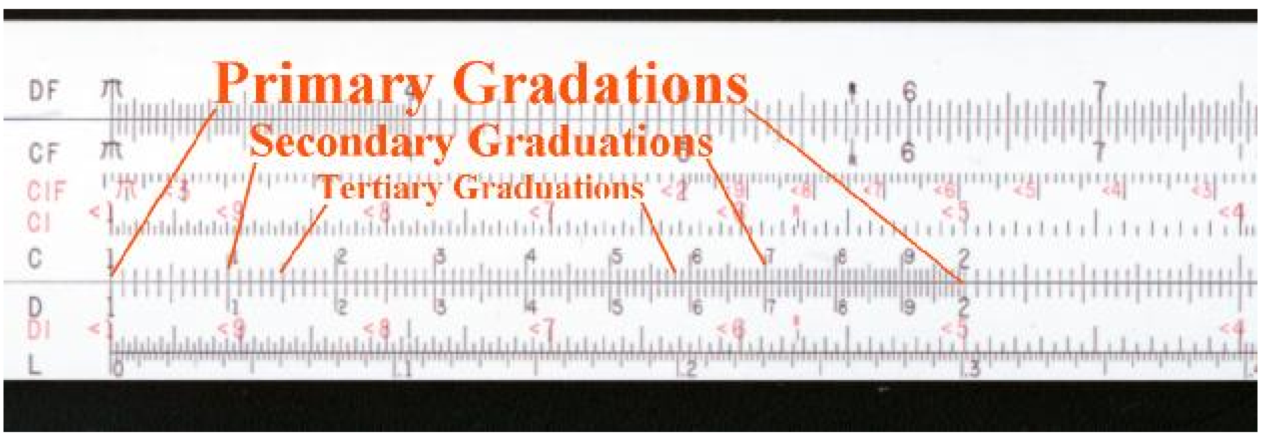


FIG 1-3

The C and D Scales are in Three Sections

- (i) From1 to 2 each small division is 0.01.
- (ii) From 2 to 4 each small division is 0.02.
- (iii) From 4 to 10 each small division is 0.05.

Thus the graduations immediately after 15 is 151, the graduation immediately before 3 is 298 and the graduation immediately following 7 is 705.

It is not just a case of learning off by heart these variations of scale divisions, but also practice at reading them. It is the same as reading a ruler with different divisions such as eighths, tenths, etc.

Even though the C and D scales are labeled from 1 to 10, this does not restrict us to this range of numbers. We can think of 1 as being 10 and 10 as being 100, then 3 is read as 30, 7 as 70, etc. Or if we take 1 as 100 and 10 as 1000 then 2 is read as 200, 4 as 400, etc. On the other hand if we take 1 at 0.1 and 10 as 1, then 5 is read as 0.5 and 9 as 0.9, etc. We treat .025, 2.5, 25, 250 etc. all as the same point on the C and D scales. The positioning of the decimal point for answers will be discussed at length in Chapter 2.

Exercise 1(a)

Find the following values on your C and D scales by using the Hair Line or Cursor.

(i) 114
(ii) 0.163
(iii) 19.8
(iv) 2700
(v) 0.04125
(vi) 5.75
(vii) 755
(viii) 0.009

1.4 Accuracy of the Slide Rule

With a ten inch Slide Rule we can generally read off three significant figures from the C and D scales. This is sometimes a little difficult at the right hand end, while at the left hand end sometimes we can manage four figures. Always try to estimate the value of the number you are reading when it does not fall exactly on a graduation.

Three significant figures means the error is about one part in one thousand of 0.1%. For all practical engineering purposes this is well within accepted limits. Of course a smaller Slide Rule is less accurate and a 20 inch one would give four significant figures or 0.01% error. Ten inch Slide Rules such as the Faber-Castle 2/N83 have ‘root scales’ (‘W scales’) which are 20 inch scales split and fitted on a 10 inch rule. These scales give the accuracy of four figure logarithm tables.

In later units we will note that by decreasing the number of moves required in a calculation, or by eliminating the need for reading a value off one scale and then finding it on another, we will increase our accuracy noticeably.

It is very important to avoid error due to Parallax. Because the hair line on the cursor is raised above the scales, error will result in reading values from an angle to either side. *Always read your Slide Rule from immediately above the hair line.*

Chapter 2 – Multiplication and Division (C and D Scale)

2.1 Adding with Uniform Scales

Fig. 2-1 shows how we could construct a simple calculating device to add numbers (e.g. two ordinary rules).

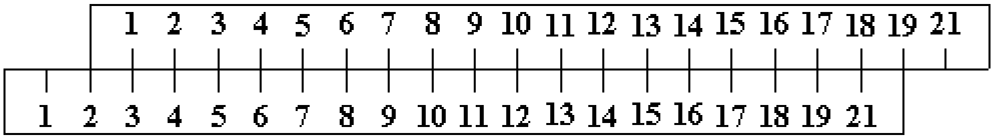


Fig 2-1

To calculate 2 + 3:

1. Find 2 on the lower scale.
2. Place the 0 (beginning) of the upper scale over the 2.
3. Move along the upper scale to 3 and read off the answer of 5 on the lower scale below the 3.

If we wish to find 2 + 7 we would follow the first two steps as above. Then for the third step go along the upper scale to 7 and read off 9 on the lower scale as the answer.

2.2 Simple Multiplication

The Slide Rule is designed to add or subtract lengths. In using the C and D scales, the lengths we add or subtract are logarithms of the numbers marked on the graduations. Thus when lengths are added the numbers are multiplied.

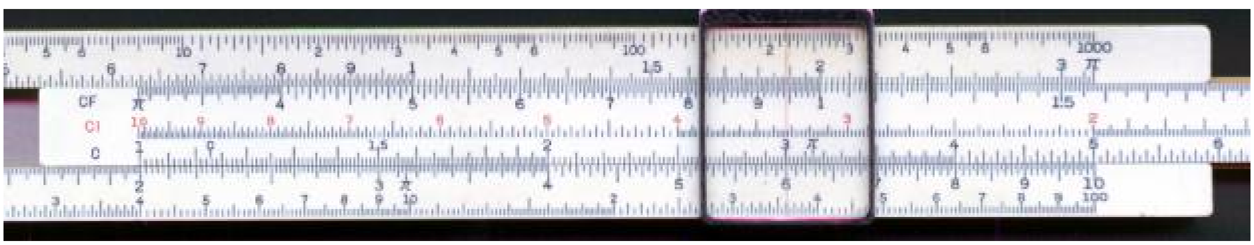


Fig 2-2

Example 1: 2 x 3 = 6 (Fig. 2-2)

1. Place the left index of the C scale over 2 on the D scale.
2. Set the hair line over 3 on the C scale.
3. Under the hair line read off 6 on the D scale as the answer.

Note: The hair line on the cursor may be used in the following ways for this method of multiplication.

(a) To mark a number on the D scale which does not fall exactly on a graduation, so that the index of the C scale may be set above it.
(b) To set over the second number on the C Scale so that the answer can be read off on the D scale.

Example 2: 1.14 x 1.32 = 1.505 (Fig. 2-3).

1. Place the left index of the C scale over 1.14 on the D scale.
2. Set the hair line over 1.32 on the C scale.
3. Under the hair line read off 1.505 on the D Scale as the answer.

Exercise 2(a)

Use the three steps as shown in the examples above to calculate the following:

(i)	1.5 x 4.7 =	(iv)	1.95 x 5.05 =
(ii)	2.2 x 2.4 =	(v)	7.6 1.25 =
(iii)	2.58 x 3.1 =	(vi)	6.88 x 1.09 =

Note: In parts (v) and (vi) of Exercise 2(a) it would be better to do the multiplication in the reverse order. (e.g. 1.25 x 7.6 instead of 7.6 x 1.25). This would mean much less movement of the slide, thus speeding up the calculation.

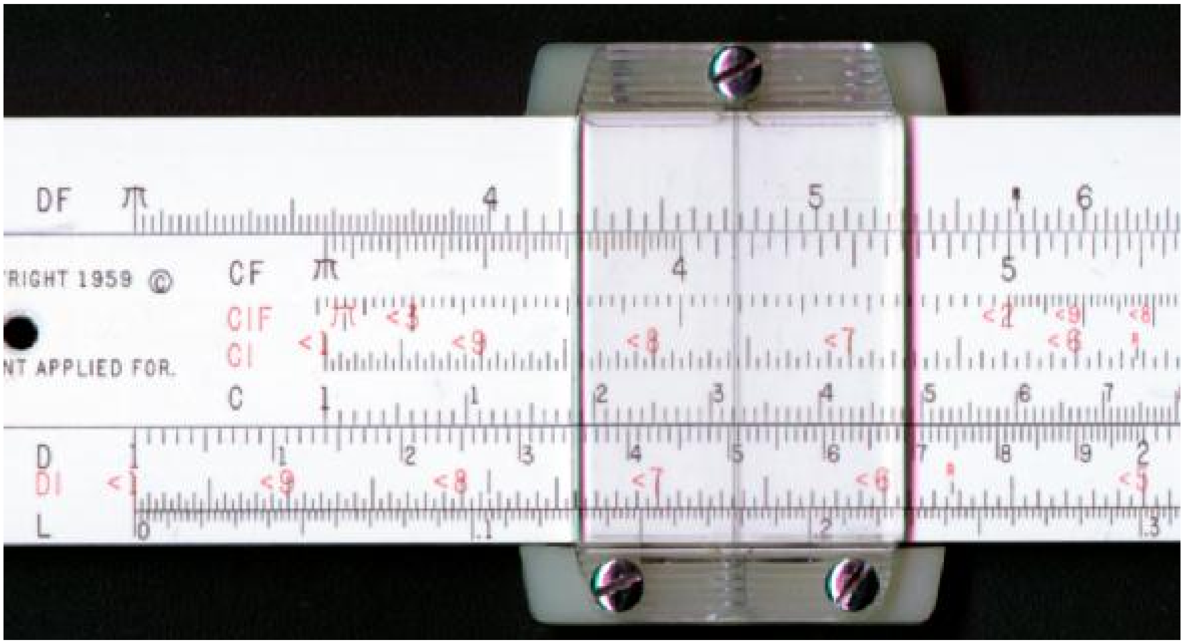


Fig 2-3

2.3 **Using the Right index for Multiplication**

Consider the product 9 x 8. If we follow the steps as used in the examples above, the 8 on the C scale falls hopelessly off the end of the D scale. The problem is overcome, by placing the right index of the C scale on the 9, instead of the left index.

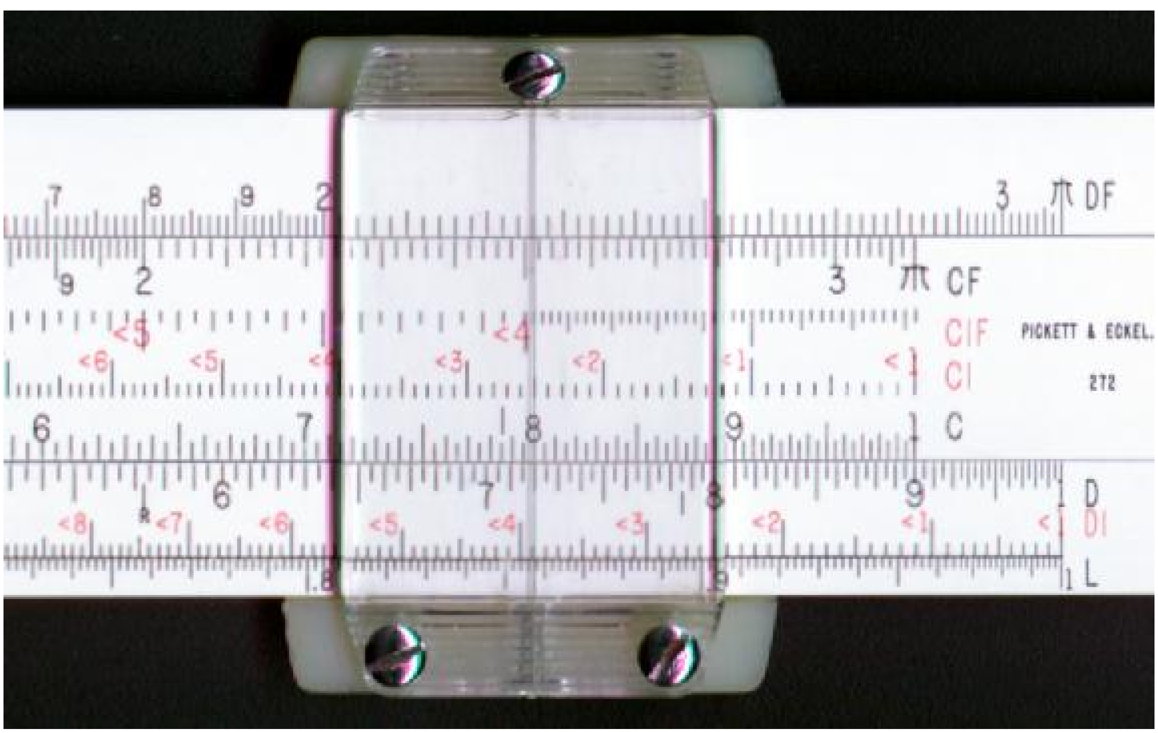


Fig 2-4

Example 1: 9 x 8 = 72 (Fig 2-4).

(i) Place the right index of the C scale over 9 on the D scale.

(ii) Set the hair line over 8 on the C scale.
(iii) Under the hair line read off 72 on the D scale as the answer.

Note:
- (a) This method works as it is equivalent to subtracting the logarithm of the reciprocal of 8, that is dividing by 1/8. (i.e. 9 ÷ 1/8 = 9 x 8).
- (b) The answer is read as 72, not 7.2. (The positioning of the decimal point will be treated in detail in 2.4).
- (c) Avoid excessive slide movement (e.g. for 2 x 7, work it as 7 x 2 thus bringing the right index of the C scale to 7 instead of right along to 2).

Example 2: 6.62 x 6.4 = 42.4
1. Place the right index of the C scale over 6.62 on the D Scale.
2. Set the hair line over 6.4 on the C scale.
3. Under the hair line read off 42.4 on the D scale as the answer.

Exercise 2(b)

(i)	5.7 x 6.5 =	(iv)	2.07 x 3.26 =
(ii)	6.9 x 3.9 =	(v)	1.85 x 7.4 =
(iii)	5.3 x 3.1 =	(vi)	3.24 x 0.56 =

2.4 Locating the Decimal Point

From the last set of exercises we can readily see that for multiplication on the C and D scales the Slide Rule does not give us the position of the decimal point. This we have to decide for ourselves. Do not try to learn a rule for locating the decimal point as this has been proved to be inadequate and confusing.

The Best method is to make a quick estimate of the answer. This will always give you the position of the decimal point and checked the magnitude of your answer too.

Example 1:

3.96 x 124.5 = ‘493’
(i.e. approx. 4 x 125 = 500)
therefore the answer is 493.0

Example 2:

488 x 0.283 = ‘1381’
(i.e. approx. 500 x .3 = 150)
therefore the answer is 138.1

Note: The other possibilities are 13.81 and 1,381 which are a long way off 150. So even if the approximation is fairly rough, it will not spoil this method.

Example 3:

0.46 x 72.3 x 52.2 = ‘1735’
(i.e. 0.5 x 70 x 50 = 0.5 x 3500 = 1750)
therefore the answer is 1735.0

When very large or very small numbers are involved, the above method can be streamlined by using standard form (or scientific notation). For example, we can express 2900 as 2.9×10^3 and 0.0012 as 1.2×10^{-3}. Thus 2900 could be approximated by 3×10^3 and by 1×10^{-3}. Always approximate 2.5, 2.6 etc. by 3 and 2.4, 2.3 etc. by 2.

Example 4:

640 x 0.00024 = ‘1538’
(i.e. approx. 6×10^2 X $2 \times 10^{-4} = 12 \times 10^{-2} = .12$)
therefore the answer is 0.1538

Example 5:

0.024 x 36 x 430 x 0.057 = ‘1538’

(i.e. approx. 2 x 10^{-2} X 4 X 10 X 4 x 10^{2} X 6 x 10^{-2} = 192 x 10-1 X = 19.2)

therefore the answer is 21.2

Exercise 2(c)

Locate the decimal point in the following:

(iv) 150 x 0.019 = ‘285’
(v) 0.0836 x 0.0042 = ‘351’
(vi) 1300 x 63 = ‘819’
(vii) 250 x 0.0025 = ‘625’
(viii) 0.125 x 0.5 = ‘625’
(ix) 304 x 75.6 = ‘230’
(x) 720 x 0.35 x 24.1 = ‘607’
(xi) 781 x 0.682 x 0.207 = 1102
(xii) 144 x 0.146 x 10.8 = ‘227’
(xiii) 0.03 x 0.81 x 0.362 = ‘880’

2.4 Continuous Multiplication

To Multiply three or more numbers together:

1. Multiply the first two together.
2. Hold this answer with the hair line on the cursor.
3. Multiply this answer by the next number. (The hair line acts as a memory, enabling us to hold a number for further use).

Example: 2 x 15 x 88 = 2640

Step 1

1. Place the left index of the C scale over the 2 on the D scale.
2. Set the hair line over 15 on the C Scale.
3. The hair line hold the answer on the D scale.

Step 2

4. Place the right index of the C scale under the hair line.
5. Reset the hair line over 88 on the C scale.
6. Under the hair line read off ‘264’ on the D scale as the answer.

To locate decimal point:

2 x 15 x 88 ≈ 2 x 20 x 90 ≈ 360

there for the answer is 2640

(≈ stands for “approximately equals”)

Exercise 2(d)

(i) 0.11 x 175 =
(ii) 50.2 x 31.8 =
(iii) 0.14 x 0.26 =
(iv) 25.6 x 142 =
(v) 18.3 x 0.031 =
(vi) 650 x 6.5 =
(vii) 0.945 x 61.5 =
(viii) 111 x 0.941 =
(ix) 2 x π =
(x) 40.25 x 51.5
(xi) 17 x 28 x 46 =
(xii) 0.18 x 104 x 0.043 =
(xiii) 62 x 1.2 x 0.47 =
(xiv) 36.2 x 38 x 0.042
(xv) 80.4 x 0.171 x 0.0316 =
(xvi) 2 x 0.31^2 =
(xvii) 52^3 =
(xviii) 29.2 x 4 x 75 x 132 x 72 =
(xix) 70^2 x 0.31^2 =
(xx) 20.25 x 22.5 x 0.0005 =

Chapter 3 – Division (C and D Scale)

3.1 Subtracting with Uniform Scales

Fig. 3.1 shows how we can subtract numbers using a pair of uniform scales (e.g. two ordinary rulers).

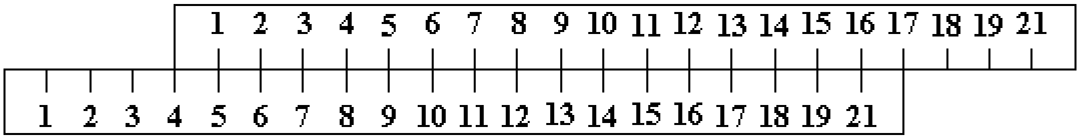


Fig 3-1

To Calculate 6 – 2:

1. Find 6 on the lower scale.
2. Place the 2 of the upper scale over 6.
3. the left index (i.e. the 0) of the upper scale indicates the answer as 4 on the lower scale.

3.2 Simple Division

When we subtract numbers on the C and D scales we have division.

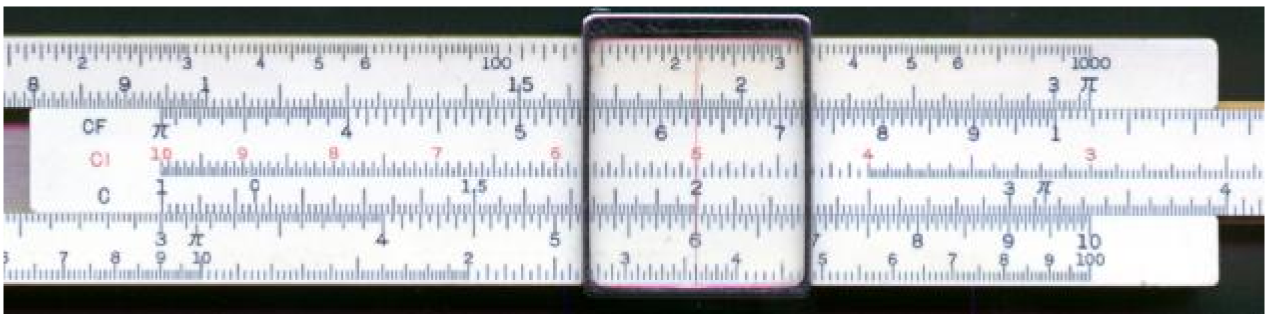


Fig 3-2

This is because the lengths we are subtracting are the logarithms of the numbers.

Example 1: 6 ÷ 2 = 3 (Fig. 3-2)

1. Set the hair line over 6 on the D scale.
2. Place the 2 of the C scale under the hair line.
3. Below the left index of the C scale read off the answer as 3 on the D scale.

Note: The hair line on the cursor may be used for division in the following ways.

(a) To mark the numerator (i.e. number we are dividing into) on the D scale if it does not fall exactly on a graduation, so that the denominator (i.e. number we are dividing by) on the C scale can be set above it.

(b) Then to set over the index on the C scale so that the answer can be located on easily on the D scale.

Important Points.

(a) When we set up division on the C and D scale it appears seemingly upside down. To calculate 6 ÷ 2 (i.e. $\frac{6}{2}$), we find 6 on the (lower) D scale and 2 is placed above it on the (upper) C scale, thus appearing on the Slide Rule as $\frac{2}{6}$.

(b) For division the answer is always indicated on the D scale by the index of the C scale. If the left index of the C scale runs off the end of the D scale, you will notice that the right index will come onto the D scale. Whichever index comes onto the scale, we can use that index to find the answer.

Example 2: 56 ÷ 7 = 8

1. Set the hair line over 56 on the D scale.
2. Place the 7 of the C scale under the hair line.
3. Below the right index of the C scale read off the answer as 8 on the D scale.

Exercise 3(a)

(i) $\frac{43}{5.5} =$

(ii) $\frac{5.7}{1.9} =$

(iii) $\frac{77}{35} =$

(iv) $675 \div 326 =$

(v) $196 \div 14 =$

(vi) $6.6 \div 14.2 =$

3.3 Locating the Decimal Point

The best method is to make a quick estimate of the answer. This can be accomplished by several different approaches.

Example 1:

$194 \div 4.15 =$ '467'

(i.e. approx. $200 \div 4 = 50$)

therefore the answer is 46.7

Standard form (or scientific notation) may be used when very large or vary small numbers are involved.

Example 2:

$56000 \div 750 =$ '746'

(i.e. approx. $(6 \times 10^4) \div (8 \times 10^2) = .75 \times 10^2$

Good general methods are:

(a) or large numbers divide both numbers by 10, 100, or 1000 etc. (whichever is applicable). That is, cancel corresponding zeros in both numerator and denominator (i.e. top and bottom).

Example 3:

$$\frac{47000}{3240} = \text{'145'}$$

(i.e. $\frac{50000}{3000} = \frac{50}{3} \approx 16$)

therefore the answer is 14.5

(b) For small numbers multiply both by 10, 100, 1000 etc., by moving the decimal point a certain number of places to the right as follows.

Example 4:

$$\frac{0.42}{0.061} = \text{'688'}$$

(i.e. approx. $\frac{0.4}{0.06} = \frac{40}{6} \approx 7$

therefore the answer is 6.88

Exercise 3(b)

Locate the decimal point for the following:

(i) $\frac{36}{4.1} =$ '878'

(ii) $\frac{75.9}{2.48} =$ '306'

(iii) $\dfrac{800}{0.243}$ = '362'

(iv) $\dfrac{0.23}{30.4}$ = '756'

(v) $\dfrac{261}{0.012}$ = '2175'

(vi) 9.42 ÷ 216 = '436'

(vii) 0.024 ÷ 0.08 = '300'

(viii) 520 ÷ 0.45 = '1155'

(ix) 0.084 ÷ 0.0025 = '336'

(x) 43500 ÷ 13.6 = '32

Note:

(a) When we divide by a number less than 1, the answer is always larger than the number we are dividing into.

(b) Unlike multiplication, with division we never run off the end of the D scale for the answer. Either the left or right index of the C scale will always be on the D scale.

3.4 Continuous Division

When dividing a number by 2 or more numbers, after each division, hold the answer on the D scale with the hair line and repeat the division process as many times as necessary. (For combined multiplication and division see Unit 4).

Exercise 3(c)

Miscellaneous Division.

(i) $\dfrac{360}{18}$ =

(ii) $\dfrac{4800}{0.6}$ =

(iii) $\dfrac{12.25}{35}$ =

(iv) $\dfrac{1}{8}$ =

(v) $\dfrac{43.75}{0.0304}$ =

(vi) 3025 ÷ 55 =

(vii) 1925 ÷ 17.5 =

(viii) π ÷ 2 =

(ix) π ÷ 6 =

(x) 93 ÷ 9600 =

(xi) $\dfrac{219}{17x28}$ =

(xii) $\dfrac{35}{0.12x0.47}$ =

(xiii) $\dfrac{805}{104x0.043}$ =

(xiv) $\dfrac{1406}{52^2}$ =

(xv) $\dfrac{19.22}{31^2}$ =

(xvi) 0.00593 ÷ 2.66 =

(xvii) 0.00207 ÷ 0.000523 =

(xviii) 36400 ÷ 26 =

(xix) 20.25 ÷ 0.00045 =

(xx) 0.001035 ÷ 111 =

Chapter 4 – Combined Operation on the C and D Scale

4.1 Alternate Division and Multiplication

For a problem such as $\frac{414x2.62}{545}$, if we multiply 414 by 2.62 and then divide by 545, the following steps would be required.

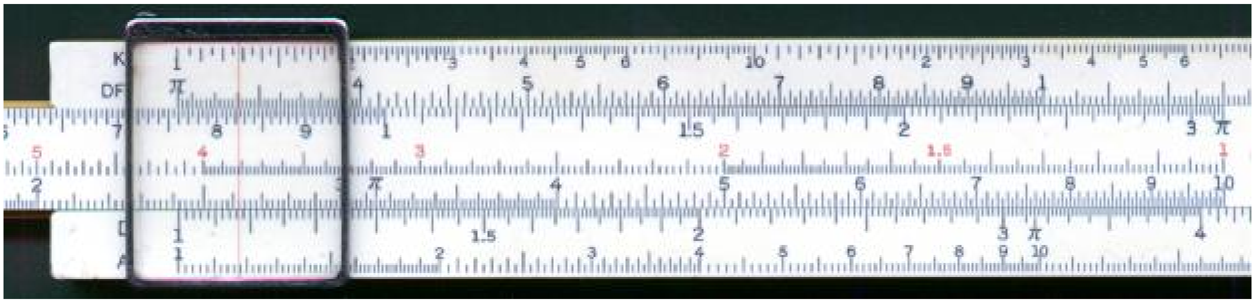


Fig 4-1

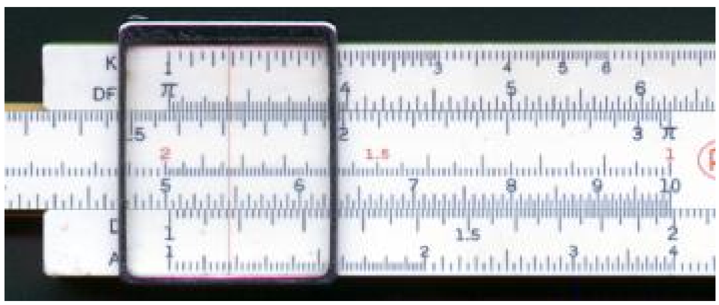


Fig 4-2

1. Place the right index on the C scale over 414 on the D scale. (Fig 4-1)
2. Set the hair line over 2.62 on the C scale. (Fig 4-1)
3. The hair line hold the answer on the D scale.
4. Move the slide so that 545 on the C scale is placed below the hair line (i.e. to divide the previous answer by 545). (Fig 4-2)
5. Below the right index of the C scale read off the answer as ‘199’.
6. (i.e. $\frac{414x2.62}{545} \approx \frac{400x3}{500} \approx 2$)
7. therefore the answer is 1.99

Note: By using the procedure above, we would have to hold the answer once with the hair line and move the slide twice. This could be reduced to only one move of the slide, with no need to hold the answer with the hair line, if we were first to divide 414 by 545, and then multiply by 2.62.

This saving is because the answer from a division is marked on the D scale by the left or right index of the C scale, thus allowing us to then multiply without moving the slide.

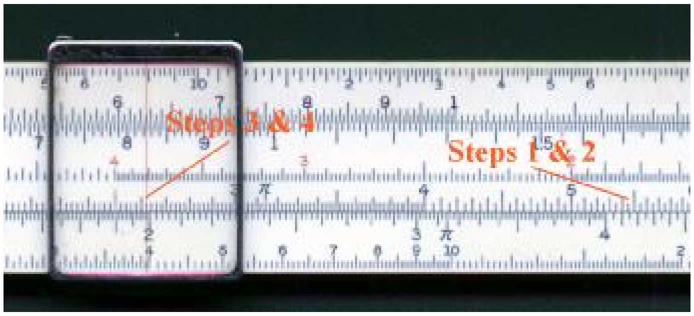


Fig 4-3

Example 1: $\frac{414x2.62}{545} = 1.99$ (Fig 4-3)

1. Set the hair line over 414 on the D scale.
2. Place the 545 of the C scale under the hair line. (Right index on the C scale marks the answer of this division).
3. Reset the hair line over the 2.62 on the C scale.
4. Under the hair line read off 1.99 on the D scale as the answer.

The hair line is only used for convenience of reading values, and the slide is positioned once for the two calculations. These savings, apart from anything else, will increase accuracy and speed.

Example 2: $\frac{107.5x30.6x125}{18.5x216} = 102.9$

1. Set the hair line over 107.5 on the D scale.
2. Place the 18.5 on the C scale under the hair line.
3. Reset the hair line over the 30.6 on the C scale.
4. Place the 216 of the C scale under the hair line.
5. Reset the hair line over the 125 on the C scale.
6. Under the hair line read off '1029' on the D scale as the answer.

(i.e. approx. $\frac{100x30x100}{20x200} = \frac{300}{4} \approx 80$
therefore the answer is 102.9

Note: The above method necessitated one move of the slide and one holding of the answer with the hair line. To multiply the three numbers in the numerator and then divide by the two numbers in the denominator would mean four moves of the slide, instead of one. Thus the three progressive answers would have to be held instead of only one, as in example 2. For the alternating division and multiplication, the only time extra moves may be required is when we run off the end of the scale for multiplication.

4.2 Locating the Decimal Point
The procedure outlined in 2.4 and 3.3 covers all that is required for combined operations.

Example:

$\frac{560x0.032x95}{0.73x410}$ = '569'

(i.e. approx. $\frac{600x0.03x100}{1x400} = \frac{6x10^2x3x10^{-1}x10^2}{4x10^2} = \frac{18}{4} \approx 4$)
therefore the answer is 5.69

Exercise 4(a)

Locate the decimal points for the following:

(i) $\dfrac{370x143}{18}$ = '294'

(ii) $\dfrac{0.68x16.3}{0.041x23.2}$ = '1165'

(iii) $\dfrac{564x134x413}{232x1745}$ = '771'

(iv) $\dfrac{1}{26.4x3.44x0.511}$ = '215'

(v) $\dfrac{19.5x135x744}{220x0.62x5.1}$ = '281

4.3 Miscellaneous Problems

Exercise 4(b)

(i) $\dfrac{35x23}{46}$

(ii) $\dfrac{12.78}{61}$

(iii) $\dfrac{0.21x36}{0.125}$

(iv) $\dfrac{0.0835x445}{96}$

(v) $\dfrac{1.4^2}{0.475}$

(vi) $\dfrac{64.5x468}{0.374}$

(vii) $\dfrac{64x2.5}{48x0.15}$

(viii) $\dfrac{230x560}{19x840}$

(ix) $\dfrac{220x0.36x4.3}{0.15x95}$

(x) $\dfrac{2.8x76x11}{0.37x0.51}$

(xi) $\dfrac{2500x0.052}{0.37x46}$

(xii) $\dfrac{235}{11.5x2.9x3.22}$

(xiii) $\dfrac{5.72^2 x314}{7.66}$

(xiv) $\dfrac{23.4x964x183.5}{48.2x38.2x103.6}$

Exercise 4(c)

(i) Find V if r=3.6cm and h =7.1cm, given that $V=\pi r^2 H$

(ii) Find F if $F = \dfrac{MV^2}{gr}$, given M=145.0, V=42, g=9.8 and r=3.65.

(iii) If $P = R\dfrac{(t-1)}{(t+1)}$ find P when R=64.2 and t=1.54.

(iv) Simple Interest = $\frac{PRT}{100}$. Find the interest if P=\$245 at R=6.25% for T=5 Years.

(v) Surface area, A, of a cylinder, is given by $A=2\pi r(h+r)$. Find A if r=7.3cm and h=12.2cm.

Chapter 5 – Squares and Square Roots (A and B or X^2 Scales)

5.1 The form of the A and B Scales.
On most Slide Rules the B scale is on the top edge of the slide and the A scale on the bottom edge of the upper stock or body of the Rule. The A and B scales are labeled from 1 to 100. They consist of two parts, 1 to 10 and 10 to 100, each actually a half size replica of the C and D.

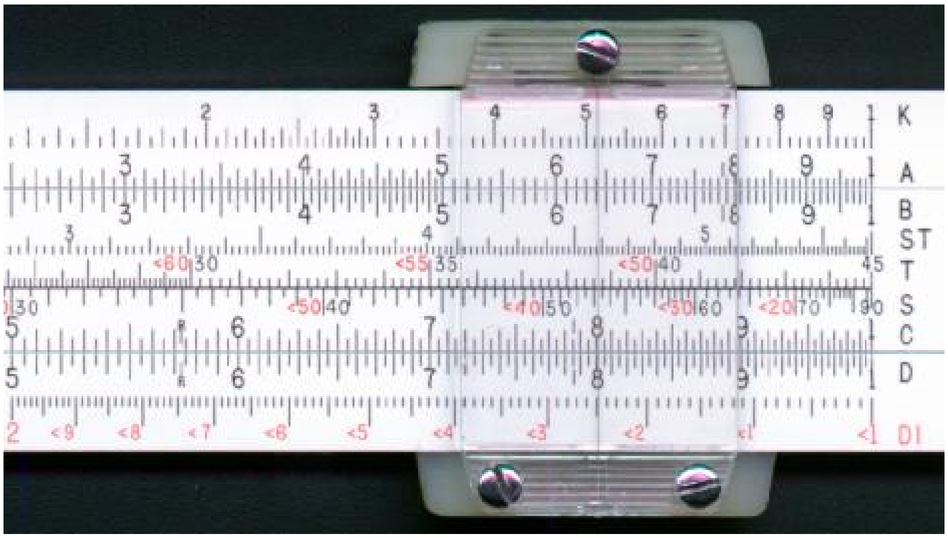


Fig 5-1

Example 1: $8^2 = 64$ (Fig. 5-1)

1. Set the hair line over 8 on the D scale.
2. Under the hair line read off 64 on the A scale as the Answer.

Note: It is advisable to use the A and D scales together, or the B and C scales together. That is, use a pair of scales that are either both on the body of the Slide Rule, or both on the slide.

5.2 Squares

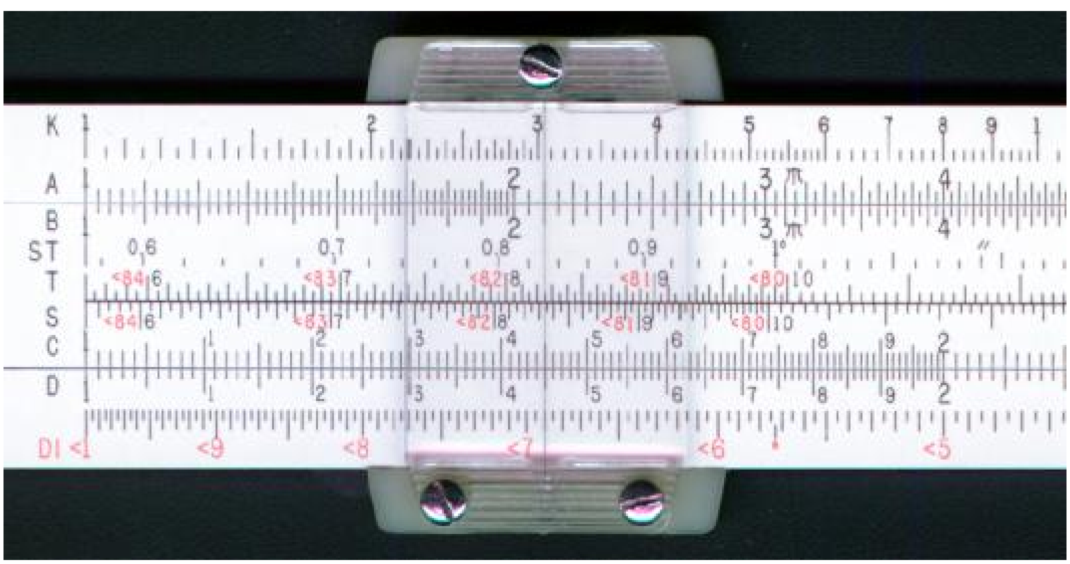


Fig 5-2

Example 2: $14.5^2 = 210$ (Fig. 5-2)

1. Set the hair line over 14.5 and the D scale.
2. Under the hair line read off 210 on the A scale as the answer. (i.e. approx. 152 = 225)
3. Therefore the answer is 210.

Exercise 5(a)

(i)	$7^2 =$	(iv)	$0.9^2 =$
(ii)	$5.5^2 =$	(v)	$11.3^2 =$
(iii)	$7.65^2 =$	(vi)	$1.414^2 =$

5.3 Locating the Decimal Point for Squares

An approximate answer is the best method, and quite often standard form (scientific notation) is a great help.

Example 1:

$35^2 =$ '1225'
(i.e. approx. $40^2 = 1600$)
there for the answer is 1225.0.

Example 2:

$197^2 =$ '389'
(i.e. approx. $200^2 = 40{,}000$
or $(2 \times 10^2)^2 = 4 \times 10^4$)
Therefore the answer is 38,900.

Note: On squaring numbers larger than 1, the answer is always greater that the original number. The opposite is the case for numbers less than 1. The following examples show us that on squaring a number less than 1, the answer is always smaller than the original number.

Example 3:

$0.31^2 =$ '96'
(i.e. approx. $0.3^2 = 0.09$)
therefore the answer is 0.096.

Example 4:

$0.0085^2 =$ '7225'
(i.e. approx. $0.009^2 = (9 \times 10^{-3})^2 = 81 \times 10^{-6} = .000081$)
therefore the answer is 0.00007225

5.4 Miscellaneous Squares

Exercise 5(b)

(i)	65^2	(v)	0.00022^2
(ii)	207^2	(vi)	30.25^2
(iii)	0.084^2	(vii)	$(5.4 \times 10^3)^2$
(iv)	0.123^2	(viii)	$(2.83 \times 10^{-2})^2$

5.5 Square Roots (Numbers between 1 and 100)

These are read directly by finding the number on the A and B scales, and with the aid of the hair line its square root is immediately below on the C and D scales.

Example 1: $\sqrt{56.25} = 7.5$ (Fig. 5-3)

1. Set the hair line over 56.25 on the A scale.
2. Under the hair line read off 7.5 on the D scale as the answer.

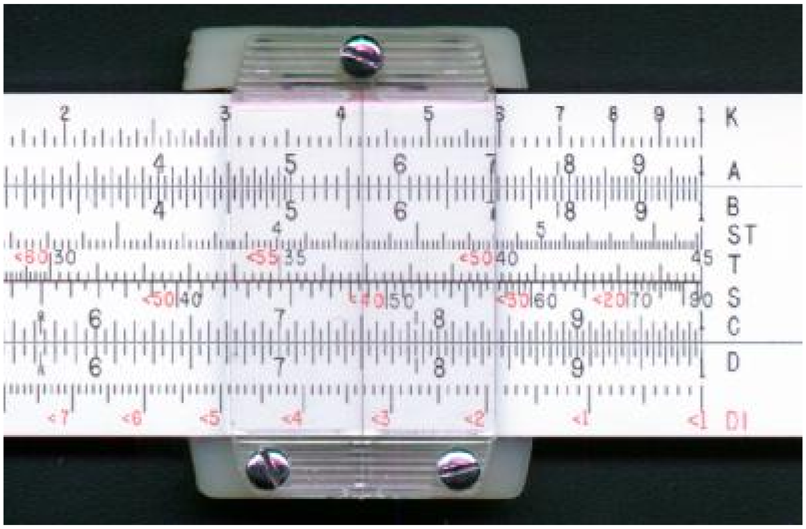


Fig 5-3

Example 2: $\sqrt{5.625}$ =2.37 Set the hair line over 5.625 on the A scale.

1. Under the hair line read off 2.37 on the D scale as the answer.

Note:

(a) In contrast to previous calculations on the C and D scales where we took 56.25 and 5.625 at the same point, on the A and B scales these two numbers are separate points.

(b) When we take the square root of the numbers between 1 and 100, there is no difficulty in locating the decimal point, as there square roots range from 1 to 10 and are read off as shown on the C and D scale graduations.

Exercise 5(c)

(i)	$\sqrt{6.25} =$	(iii)	$\sqrt{3} =$
(ii)	$\sqrt{93.5} =$	(iv)	$\sqrt{1.125} =$

5.6 Square Roots (Numbers greater than 100)

The difficulty for numbers greater than 100, is knowing where to locate them on the A and B scales. In previous calculations 5.625, 56.25, 562.5, 0.5625, etc. have all been placed at the same point on the C and D scales, but now the question arises, where to locate 562.5 on the A and B scales to obtain its square root. It is on 5.625 or 56.25? The best method to decide this, also find the decimal point, is as follows:

Example 1:

$$\sqrt{562.5} = 23.7$$

$$\sqrt{562.5} = \sqrt{5.625x100}$$

$$= \sqrt{5.625}x\sqrt{100}$$

$$= \sqrt{5.625}x10$$

(The $\sqrt{5.625}$ is found as shown by Example 2 in 5.5)

$$\sqrt{562.5} = 2.37 \text{ x } 10$$

$$= 2.37$$

Note: We could break 562.5 up into any two factors (each smaller than 100) and find their individual square roots. Then multiply these together to give the square root of 562.5. This would present difficulties and extra steps, where as when we make 100 as one of the two factors, the other factor is immediately obtained by moving the decimal point two places to the left. The location on the A and B scale is established, and also there is no need for any extra work to decide the position of the decimal point in the answer.

Example 2:

$\sqrt{272}$ =16.5

$\sqrt{272} = \sqrt{2.72x100}$

= 1.65 x 10

therefore the answer is 16.5

(The $\sqrt{2.72}$ is found as outlined in the Examples in 5.5)

Exercise 5(d)

(i) $\sqrt{324} =$

(ii) $\sqrt{960} =$

(iii) $\sqrt{425} =$

(iv) $\sqrt{1000} =$

Example 3:

$\sqrt{5625}$ =75

$\sqrt{5625} = \sqrt{56.25x100}$ =7.5 x 10

Therefore the answer is 75

Example 4:

$\sqrt{56250}$ =237

$\sqrt{56250} = \sqrt{5.625x10,000}$ =2.37 x 100

Therefore the answer is 237

Note: For numbers greater than 10,000 always break them up with one of the factors at 10,000. We do not use 1000 as a factor because it does not have a neat square root.

Exercise 5(e)

(i) $\sqrt{2,000}$

(ii) $\sqrt{20,000}$

(iii) $\sqrt{2,720}$

(iv) $\sqrt{82,000}$

5.7 Square Roots (Numbers less than 1)

We must have numbers between 1 and 100 to use the A and B scale. In the case of numbers less than 1, we do not break the number up into a factor multiplied by 100, 10,000, etc.; but as a number over 100, 10,000 etc.

Example 1:

$\sqrt{0.5625}$ =0.75

$$\sqrt{0.5625} = \sqrt{\frac{56.25}{100}} = \frac{7.5}{10}$$

Therefore the answer is 0.75

Note:

(a) we do not express $\sqrt{0.5625} = \sqrt{\frac{5.625}{10}}$ as this would mean dividing by $\sqrt{10}$, which is not a simple value.

(b) It is good to remember that the square root of a fraction less than one is always large that the original number (e.g. $\sqrt{0.64}$ =0.8).

Example 2:

$\sqrt{0.05625}$ =0.237

$$\sqrt{0.05625} = \sqrt{\frac{5.625}{100}} = \frac{2.37}{10}$$

Therefore the answer is 0.237

Example 3:

$\sqrt{0.005625}$ =0.075

$$\sqrt{0.005625} = \sqrt{\frac{56.25}{10,000}} = \frac{7.5}{100}$$

Therefore the answer is 0.075

(In each of the above examples $\sqrt{5.625}$ and $\sqrt{56.25}$ are obtained in the usual way.)

Exercise 5(f)

(i) $\sqrt{0.9}$

(ii) $\sqrt{0.06}$

(iii) $\sqrt{0.143}$

(iv) $\sqrt{0.0025}$

5.8 Miscellaneous Problems

Exercise 5(g)

(i) 35^2 =

(ii) 265^2 =

(iii) 0.34^2 =

(iv) 5260^2 =

(v) $\sqrt{61.6}$ =

(vi) $\sqrt{0.4}$ =

(vii) $\sqrt{0.0076}$ =

(viii) $\sqrt{496}$ =

(ix) 21^2 x $\sqrt{130}$ =

(x) $\sqrt{36.5^2 x 6.3}$ =

(xi) $\sqrt{81.5}$ ÷14.2 =

(xii) $\sqrt{6.2^2 + 5.7^2}$ =

(xiii) $\sqrt{375} x 0.6^2$ =

(xiv) $\sqrt{1000} x 3.6$ =

(xv) $\sqrt{6} + \sqrt{8}$ =

(xvi) $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}$ =

Chapter 6 – Cube and Cube Roots (K Scale)

6.1 The Form of the K Scale

The K scale is labeled from 1 to 1,000. It consists of three parts, 1 to 10, 10 to 100 and 100 to 1,000, each a third size replica of the C and D scales. Hence the accuracy with which the K scale can be read is very much less than the C and D scales.

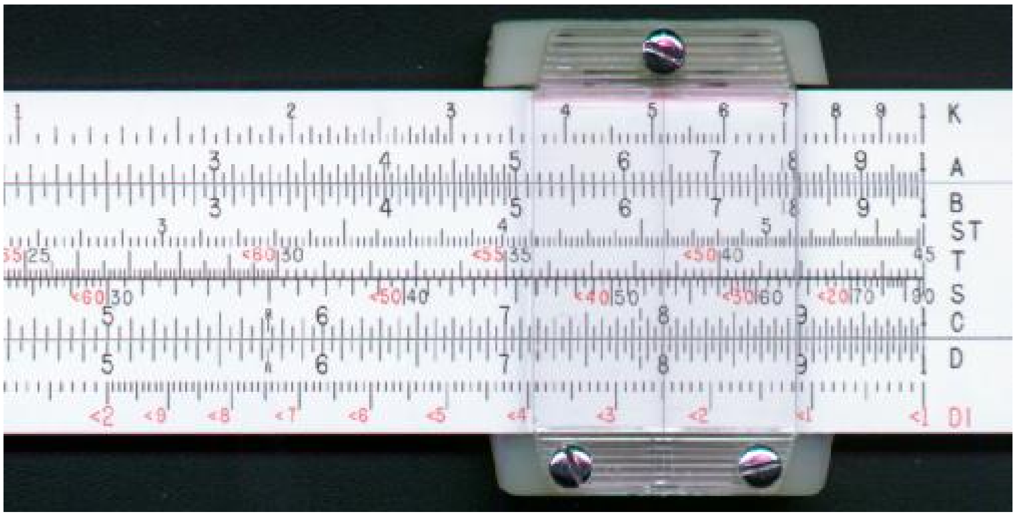


Fig 6-1

6.2 Cubes

Example 1: $8^3 = 512$ (Fig 6-1)

1. Set the hair line over 8 on the D scale.
2. Under the hair line read off 512 on the K scale as the answer.

Note: It is advisable to use the D scale in combination with the K scale, as using the C scale with the K scale will lead to errors if the slide is slightly displaced.

Example 2: $123^3 = 1{,}860{,}000$

1. Set the hair line over 123 on the D scale.
2. Under the hair line read off '186' on the K scale as the answer.
   $(12^3 = (1.23 \text{ x } 10^2)^3$
   $= 1.86 \text{ x } 10^6$
   $= 1{,}860{,}000$

Example 3: $0.378^3 = 0.0054$

1. Set the hair line over 378 on the D scale.
2. Under the hair line read off '54' on the K scale as the answer.
   $(0.378^3 = (3.78 \text{ x } 10^{-1})^3$
   $= 54 \text{ x } 10^{-3})$
   $= 0.0054$

Note:

(a) The cube of numbers between 1 and 10 is read directly off the K scale as numbers between 1 and 1,000.
(b) For numbers greater than 1,000, express the number in standard form (scientific notation) as in example 2 above. Notice that $1.23^3 = 1.86$, as read directly off the K scale.
(c) For numbers less than 1, express the number in standard form (scientific notation) as in example 3 above. Notice that $3.78^3 = 54$ as read directly off the K scale.

Exercise 6(a)

(i)	$6.5^3 =$	(v)	$165^3 =$
(ii)	$25^3 =$	(vi)	$0.063^3 =$
(iii)	$0.425^3 =$	(vii)	$0.705^3 =$
(iv)	$93.5^3 =$	(viii)	$206^3 =$

6.3 Cube Roots (Numbers between 1 and 1,000)

These are read directly by finding the number on the K scale, and with the aid of the hair line, its cube root is immediately below on the D scale.

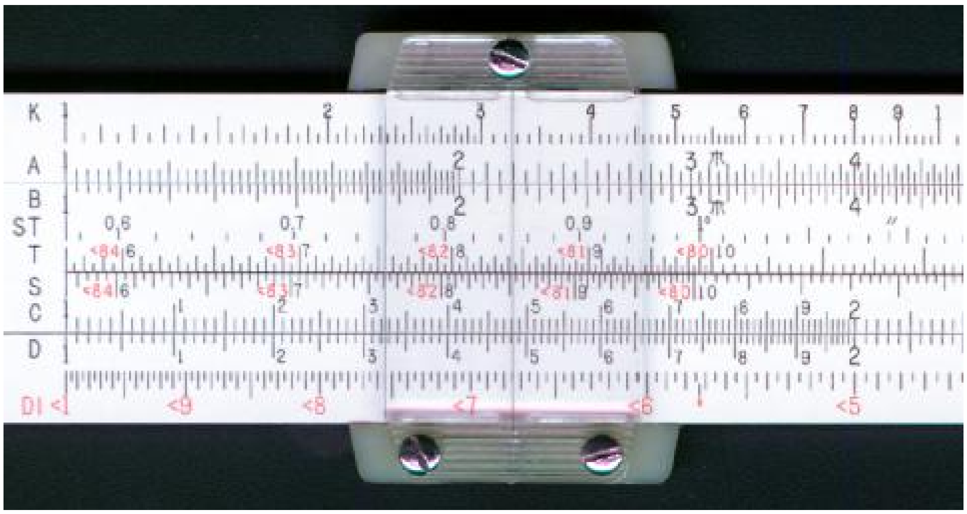


Fig 6-2

Example 1: $\sqrt[3]{3.25} = 1.48$ (Fig. 6-2)

1. Set the hair line over 3.25 on the K scale.
2. Under the hair line read off 1.48 on the D scale as the answer.

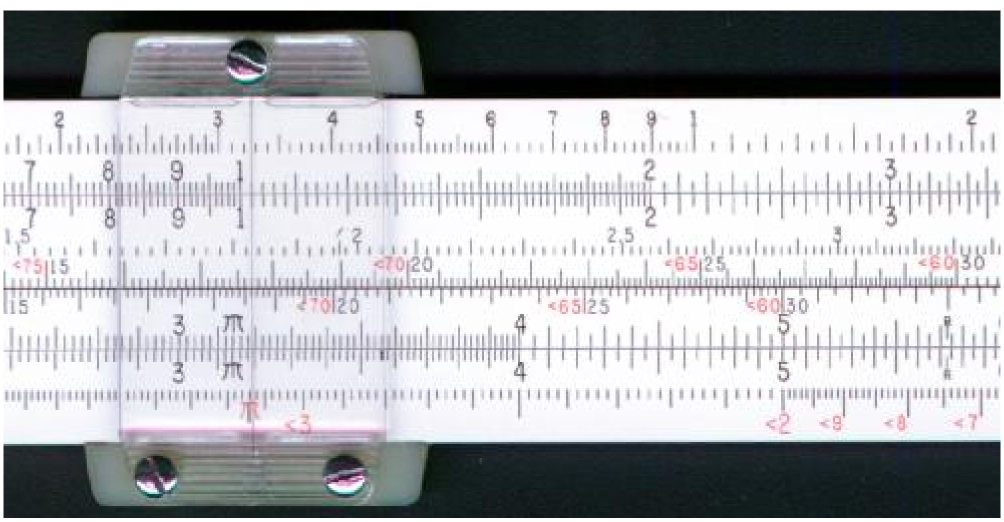


Fig 6-3

Example 2: $\sqrt[3]{32.5} = 3.19$ (Fig 6-3)

1. Set the hair line over 32.5 on the K scale.
2. Under the hair line read off 3.19 on the D scale as the Answer.

Example 3: $\sqrt[3]{325} = 6.875$ (Fig. 6-4)

1. Set the hair line over 325 on the K scale.
2. Under the hair line read off 6.875 on the D scale as the Answer.

Note:

(a) Note that 3.25, 32.5 and 325 are each at different points on the K scale.
(b) For numbers between 1 and 1,000, there is no difficulty in locating the decimal point, as their cube roots lie between 1 and 10, and are read directly off the D scale.

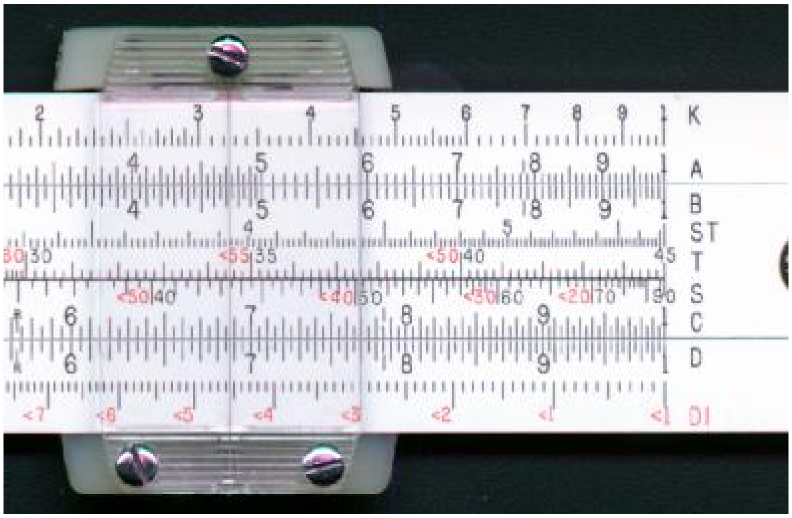


Fig 6-4

Exercise 6(b)

(i)	$\sqrt[3]{64} =$	(iv)	$\sqrt[3]{9.4} =$
(ii)	$\sqrt[3]{2.2} =$	(v)	$\sqrt[3]{94} =$
(iii)	$\sqrt[3]{132} =$	(vi)	$\sqrt[3]{940} =$

6.4 Cube Root (Numbers greater than 1,000)

For numbers such as 3,250, 32,500, etc. the difficulty is to decide where to locate them on the K scale to obtain its cube root. The following procedure will allow us to locate the number on the K scale and also automatically give us the position of the decimal point.

Example 1:

$$\sqrt[3]{3250} = 14.8$$

$$(\sqrt[3]{3250} = \sqrt[3]{3.25x1000} = \sqrt[3]{3.25}x10$$

The $\sqrt[3]{3.25}$ is found as shown in Example 1 in 6.3)

Therefore $\sqrt[3]{3250} = 1.48x10 = 14.8$

Example 2:

$$\sqrt[3]{32500} = 31.9$$

$$(\sqrt[3]{32500} = \sqrt[3]{32.5x1,000} = \sqrt[3]{32.5}x10$$

The $\sqrt[3]{32.5}$ is found as shown in Example 1 in 6.3)

Therefore $\sqrt[3]{32500} = 3.19x10 = 31.9$

Note: For cube roots of numbers greater than 1,000, we break the numbers up into factors, one of which is 1,000. We do not use 10 or 100, as these do not have simple cube roots, where as 1,000 has a cube root of 10.

Exercise (6c)

(i) $\sqrt[3]{1,200}$

(ii) $\sqrt[3]{94,000}$

(iii) $\sqrt[3]{10,000}$

(iv) $\sqrt[3]{132,000}$

6.5 Cube Roots (Numbers less than 1)

For numbers less than 1, we express them as a fraction over 1,000, or if the number is less than 0.001, as a fraction over 1,000,000.

Example 1:

$$\sqrt[3]{0.325} = 0.6875$$

$$(\sqrt[3]{0.325} = \sqrt[3]{\frac{325}{1,000}}$$

$$= \frac{6.875}{10}$$

Therefore the answer is 0.6875

Example 2:

$$\sqrt[3]{0.0325} = 0.319$$

$$(\sqrt[3]{0.0325} = \sqrt[3]{\frac{32.5}{1,000}}$$

$$= \frac{3.19}{10}$$

Therefore the answer is 0.319

Example 3:

$$\sqrt[3]{0.00325} = 0.148$$

$$(\sqrt[3]{0.00325} = \sqrt[3]{\frac{3.25}{1,000}}$$

$$= \frac{1.48}{10}$$

Therefore the answer is 0.148

(In each of the above examples $\sqrt[3]{325}$, $\sqrt[3]{32.5}$, $\sqrt[3]{3.25}$ is obtained in the usual way.)

Exercise 6(d)

(i) $\sqrt[3]{0.8}$

(ii) $\sqrt[3]{0.09}$

(iii) $\sqrt[3]{0.132}$

(iv) $\sqrt[3]{0.001}$

(v) $\sqrt[3]{0.0615}$

(vi) $\sqrt[3]{0.0094}$

6.6 Miscellaneous Problems

Exercise 6(e)

(i) $63^3 =$

(ii) $174^3 =$

(iii) $0.16^3 =$

(iv) $0.073^3 =$

(v) $\sqrt[3]{16.5} =$

(vi) $\sqrt[3]{980} =$

(vii) $\sqrt[3]{10,800} =$

(viii) $\sqrt[3]{0.0875} =$

(ix) $14.3^3 + 21.6^3 =$

(x) $\sqrt[3]{631 x 14.6^3} =$

(xi) $\sqrt[3]{73} - \sqrt[3]{20.25} =$

(xii) $\sqrt[3]{36 x 1.95^3} =$

Chapter 7 – Inverted (Reciprocal) Scale (CI)

7.1 The Form of the CI Scale
The CI scale is identical with the C scale except that the CI scale reads from right to left for this reason great care should be taken in reading the CI scale. Note, by the term 'reciprocal of a number 'N' we mean $\frac{1}{N}$.

7.2 Reciprocals (Numbers between 1 and 10)
For numbers between 1 and 10 on the C scale, its reciprocal is read directly off the CI scale as a number between 1 and 0.01. We can also find reciprocal by working from the CI scale to the C scale.

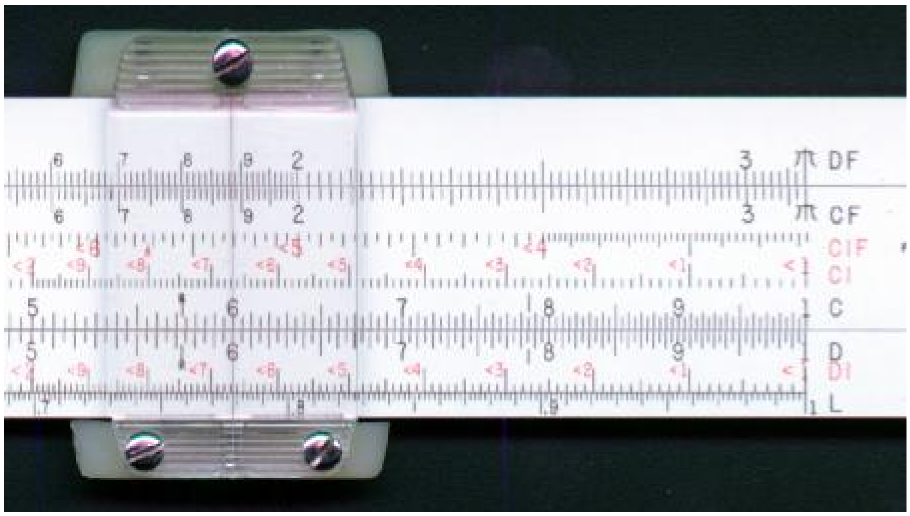


Fig 7-1

Example 1: $\frac{1}{6} = 0.167$ (Fig. 7-1)

1. Set the hair line over 6 on the C scale.
2. Under the hair line read off 0.167 on the CI scale.

   or

1. Set the hair line over 6 on the CI scale.
2. Under the hair line read off 0.167 on the C scale.

Example 2: $\frac{1}{2.44} = 0.41$

1. Set the hair line over 2.44 on the C (or CI) scale.
2. Under the hair line read off 0.41 on the CI (or C) scale.

Exercise 7(a)

(i) $\frac{1}{8}$ (iii) $\frac{1}{3.76}$ (iv) $\frac{1}{7.4}$

(ii) $\frac{1}{1.5}$

7.3 Reciprocals (Numbers outside the range 1 to 10)
For numbers less than 1, their reciprocals will always be larger than 1.
e.g.

$$\frac{1}{0.2} = 5$$
$$\frac{1}{0.02} = 50$$
$$\frac{1}{0.002} = 500$$

For numbers greater than 10, their reciprocals will always be smaller than 0.1.
e.g.

$$\frac{1}{20} = 0.05$$
$$\frac{1}{200} = 0.005$$

Example 1:

$$\frac{1}{0.042} = 23.8$$

1. Set the hair line over 42 on the C (or CI) scale.
2. Under the hair line read off ‘238’ on the CI (or C) scale as the answer.

To locate the decimal point, the following procedure is possibly the easiest

Note: $\frac{1}{0.042} = \frac{100}{4.2} = 100x0.238$

(As the reciprocal of a number between 1 and 10 is always between 1 and 0.1.) The answer is therefore 23.8.

Example 2: $\frac{1}{420} = 0.00238$

1. Set the hair line over 420 on the C (or CI) scale.
2. Under the hair line read off ‘238’ on the CI (or C) scale as the answer.

Note: $\frac{1}{420} = \frac{1}{4.2x100} = 0.238x\frac{1}{100}$

Therefore the answer is 0.00238.

Note: Some modern Slide Rules have a DI scale located on the body. This scale can be used in conjunction with the D scale to obtain recprocals.

Exercise 7(b)

(i) $\frac{1}{2.6} =$

(ii) $\frac{1}{26} =$

(iii) $\frac{1}{.26} =$

(iv) $\frac{1}{1.11} =$

(v) $\frac{1}{0.625} =$

(vi) $\frac{1}{262} =$

(vii) $\frac{1}{0.0575} =$

(viii) $\frac{1}{0.0018} =$

(ix) $\frac{1}{0.000645} =$

(x) $\frac{1}{1740} =$

7.4 Multiplication (CI and D Scales)
Note that instead of multiplying 2 by 7, we could divide by the reciprocal of 7.

i.e. $2x7 = 2 \div \frac{1}{7}$

Example 1: $2x7 = 14$ (Fig. 7-2)
1. Set the hair line over 2 on the D scale.
2. Place the 7 of CI scale under the hair line.
3. Below the left index of the C scale read off 14 on the D scale as the answer.

Note: When we place the 7 of the CI scale under the hair line (step 2 above), this brings 0.1428 on the C scale immediately above the 2 on the D scale. Thus we are dividing 2 by 0.1428.

(i.e. $2 \div 0.1428 = 2 \div \frac{1}{7} = 2 \times 7$)

Example 2: 4.15 x 1.35 = 5.6
1. Set the hair line over 4.15 on the D scale.
2. Place the 1.35 of the CI scale under the hair line.
3. Below the right index of the C scale read off 5.6 on the D scale as the answer.

Note: Using the D and CI scale to multiply, we never run off the end of the scale for the answer as we did when using the C and D scales. The answer is always found on the D scale under the left or right index of the C scale.

Exercise 7(c)

(i)	1.5 x 4.7 =	(iv)	1.95 x 5.05 =
(ii)	2.2 x 2.4 =	(v)	7.6 x 1.25 =
(iii)	2.258 x 3.1 =	(vi)	6.88 x 1.09 =

7.5 Division (CI and D scale)
Instead of dividing, say, 108 by 7.5, we could simply multiply by the reciprocal of 7.5.

i.e. $108 \div 7.5 = 108 \times \frac{1}{7.5}$

Example 1: 108 ÷ 7.5 = 14.4
1. Place the left index of the C scale over 108 on the D scale.
2. Set the hair line over 7.5 on the CI scale.
3. Under the hair line read off 14.4 on the D scale as the answer.

Note: In the above procedure we have effectively multiplies 108 by 0.1335, or $\frac{1}{7}$, (i.e. the value on the C scale under the hair line.)

$$(108 \times 0.1335 = 108 \times \frac{1}{7.5} = 108 \div 7.5)$$

Example 2: 96 ÷ 149 = 0.644
1. Place the right index of the C scale over 96 on the D scale.
2. Set the hair line over 149 on the CI scale.
3. Under the hair line read off 0.644 on the D scale as the answer.

Note: When we divide with the CI and D scales, sometimes we use the left index (example 1 above), while on other occasions we use the right index (example 2 above). This is dictated by the numbers involved, and if one index does not bring the numbers we are dividing by onto the scale, the other index will.

Exercise 7(d)

(i)	43 ÷ 5.5 =	(iv)	675 ÷ 326 =
(ii)	5.7 ÷ 1.9 =	(v)	196 ÷ 14 =
(iii)	77 ÷ 35 =	(vi)	6.6 ÷ 14.2 =

Chapter 8 – Folded Scales (CD, DF, and CIF)

8.1 The Form of the Folded Scales

The CF and DF scales are identical with the C and D scale, except they are displaced by a factor of π. The CF and DF scales could be constructed by cutting the C and D scales at π (i.e. 3.1416 . . .) and joining the two secitions with the left and right indices (i.e. 1 and 10) together. The CIF is the inverted or reciprocal scale in relation to the CF scale.

8.2 Multiplicaiton and Division (CF, CIF, DF scales)

If either index of the C scale is set over a number on the D scale, then the index ("1") of the CF scale wil be opposite the same number on the DF scale. This allows us to either use the CF and DF scales by themselves for multiplication, or in combination the C and D scales.

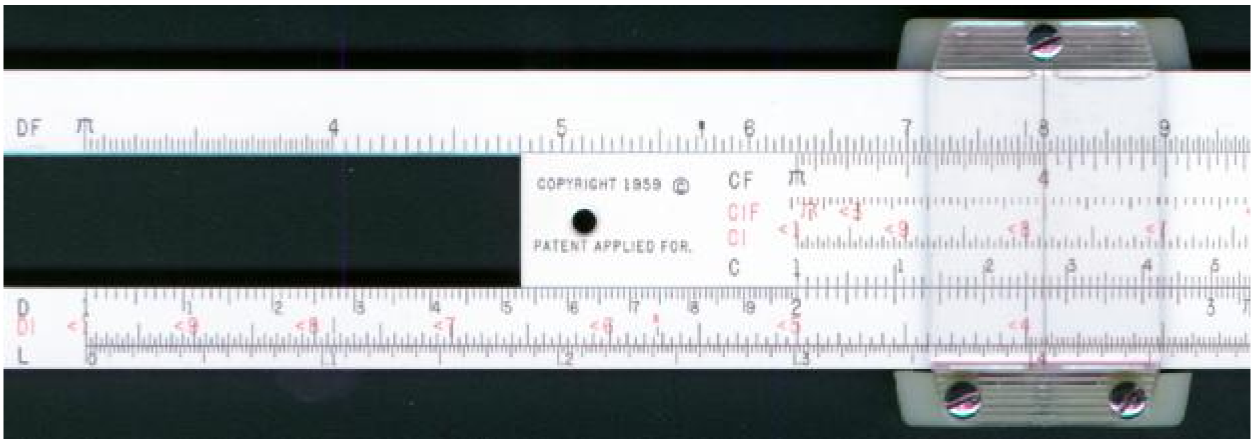


Fig 8-1

Example 1: 2 x 4 = 8 (Fig. 8-1)

1. Place the left index of the C scale over the 2 on the D scale.
2. Set the hair line over the 4 on the CF scale.
3. Under the hair line read off 8 on the DF scale as the answer.

Example 2: 3 x 9 = 27

To calculate this problem on the C and D scales alone, you will note it would require that the right index of the C scale to be set over the 3 on the D scale. Thus if we wished to place the left index of the C scale over the 3 on the D scale, 9 of the C scale would be off the end of the D scale. However, using the CF and DF scales, we could do the following.

1. Place the left index of the C scale over the 3 on the D scale.
2. Set the hair line over 9 on the CF scale.
3. Under the hair line read off 27 on the DF scale as the answer.

Exercise 8(a)

Use the left index of the C scale and read the answer off the DF scale for the following.

(i)	2 x 8 =	(iii)	3.1 x 7.5 =
(ii)	16 x 9 =	(iv)	27.4 x 6.1 =

To multiply using the CF and DF scales only:

Example: 9 x 8 = 72

1. Place the index of the CF scale on the 9 of the DF scale.
2. Set the hair line over the 8 on the CF scale.
3. Under the hair line read off 72 on the DF scale as the answer.

The above multiplication can be done also by using the CIF and DF scales in exactly the same was as previously with the CI and D scales.

1. Set the hair line over 9 on the DF scale.
2. Place the 8 of the CIF scale under the hair line.
3. Above the index of the CF scale, read off 72 on the DF scale as the answer.

To divide using the CF and DF scales only:

Example: 56 ÷ 7 = 8

1. Set the hair line over 56 on the DF scale.
2. Place the 7 of the CF scale under the hair line.
3. Above the index of the CF scale, read off 8 as the answer on the DF scale.

The above division can also be done using the CIF and D scales with the same procedure as used previously with the CI and D scales. (fig 8-6)

1. Place the index of the CF scale under the 56 on the DF scale.
2. Set the hair line over the 7 on the CIF scale.
3. Under the hair line read off 8 on the DF scale as the answer.

Note:

(a) For practicing the above multiplication and division, use the exercises in units 2 and 3.

(b) For combinations of these methods of multiplication and division, with those previously used, see unit 10.

8.3 Multiplication and Division by π.

Because the CF and DF scales are displaced by π in respect to the C and D scales, we can very easily multiply or divide by π as follows.

Example 1: 1.45 x π = 4.56

1. Set the hair line over 1.45 on the D scale.
2. Under the hair line read off 4.56 on the DF scale as the answer.

Conversely, for 4.56 ÷ π = 1.45 we could:

1. Set the hair line over 4.56 on the DF scale.
2. Under the hair line read off 1.45 on the D scale as the answer.

Example 2: 1.41^2 x π = 6.15

1. Place the left index of the C scale over 1.4 on the D scale.
2. Set the hair line over 1.4 on the C scale. (note: This would give 1.4^2 on the D scale under the hair line).
3. Under the hair line read off 6.15 on the DF scale as the answer.

Note: We have multiplied 1.4^2 by π in reading the answer of the DF scale instead of the D scale.

Example 3: $\frac{8.3}{6.85x\pi} = 0.386$

1. Set the hair line over 8.3 on the DF scale. (Note: under the hair line on the D scale we have 8.3 ÷ π).
2. Place the 6.85 of the C scale under the hair line.
3. Below the right index of the C scale read off 0.386 on the D scale as the answer.

Exercise 8(b)

(i) 3.6 x π =

(ii) 7.9 ÷ π =

(iii) $\frac{4.8x\pi}{2.6}$ =

(iv) $\frac{\pi x8.3}{17.1}$ =

(v) π X 1.5^2 =

(vi) π X 6.3 X 5.4 =

(vii) $\frac{24.6}{12.68x\pi}$ =

(viii) $\frac{17.2}{\pi x7.45}$ =

8.4 Miscellaneous Problems

Exercise 8(c)

(i) Find the circumferences of circles with the following diameters: 2.6, 45.5 and 15.2.

(ii) Find the diameter of circles with the following circumferences: 6.6, 15.6 and 88.

(iii) Find the area, A, of a circle, when r = 4.1cm, for $A = \pi r^2$.

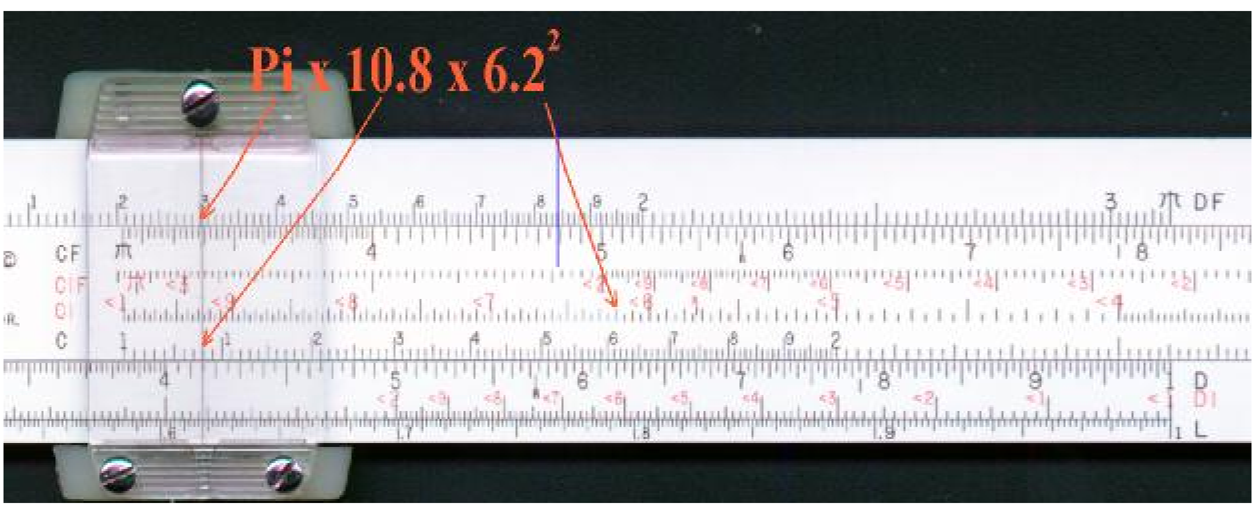
(iv) The volume of a cylinder is given by $V = \pi r^2 h$. Find V for r = 6.2 and h = 10.8.
(Hint: This can be done with one setting of the slide by using CI, C, D and DF scales.)

(v) The area of an ellipse is A = a x b x π. Fin for a = 3.2 and b = 2.6.

(vi) For the length of an arc S = $\frac{r\pi\theta}{180}$, calculate S for θ = 35.2° and r = 26.2.

(vii) The area of a sector of a circle, A, is given by A = $\frac{r^2\pi\theta}{2x180}$. Find the area if r =6.3 and θ = 48°.

Answer to 8(c)(vi)



Chapter 9 – Percentages and Ratio and Proportion

9.1 Percentage
Percentage problems generally amount simply to a multiplication or a division.

Example 1:
Express 5 as a percentage of 6.
(e.g. = $\frac{5}{6}x\frac{100\%}{1} = 83.4\%$)

1. Set the hair lineover 5 on the D scale.
2. Place the 6 of the C scale under the hair line.
3. Read off 83.4% on the D scale as the answer.

Example 2:
Find 17% of $400
(e.g. = $\frac{17}{100}x400 = \$68$)
Note 17% = $\frac{17}{100}$ and "of" is taken as meaning multiplication. Thus we just multiply 17 by 4 to calculate the above.

Example 3: Find the quantity of which 52 is 13%.
(e.g. Let 52 = 13% of A

$$52 = \frac{13}{100}xA$$

$$\therefore A = \frac{52x100}{13}$$

= 400)

Note in this case we simply divide 52 by 13.

Exercise 9(a)
Express as a Percentage:

(i) $\frac{3}{5}$

(ii) $\frac{1}{16}$

(iii) $\frac{4}{300}$

(iv) $\frac{11}{6}$

(v)

Find the value of:

(vi) 5% of $25
(vii) 22½% of 180
(viii) 14% of $250
(ix) 85% of 132

Find the quantity of which:

(x) 3 is 12½%
(xi) 66 is 85%
(xii) 31 is 62%
(xiii) 146 is 125%

(xiv) A man buys goods costing $9, and he pays a deposit of 15%. How much does he still owe?
(xv) The population of a town in 1971 was 26,800, which 11% less than it was in 1961. What was the population in 1961?

(xvi) A man's salary was $5440 in 1969 and was increased by 15% in 1970. If he pays 12½% in taxes, how much tax did he pay in 1970?

9.2 Ratio and Proportion

Definitions

(i) The *ratio* of two numbers A and B is the quotient $\frac{A}{B}$, sometimes expressed A:B.

(ii) A proportion is the statement of equality of two ratios, e.g. $\frac{A}{B} = \frac{C}{D}$.

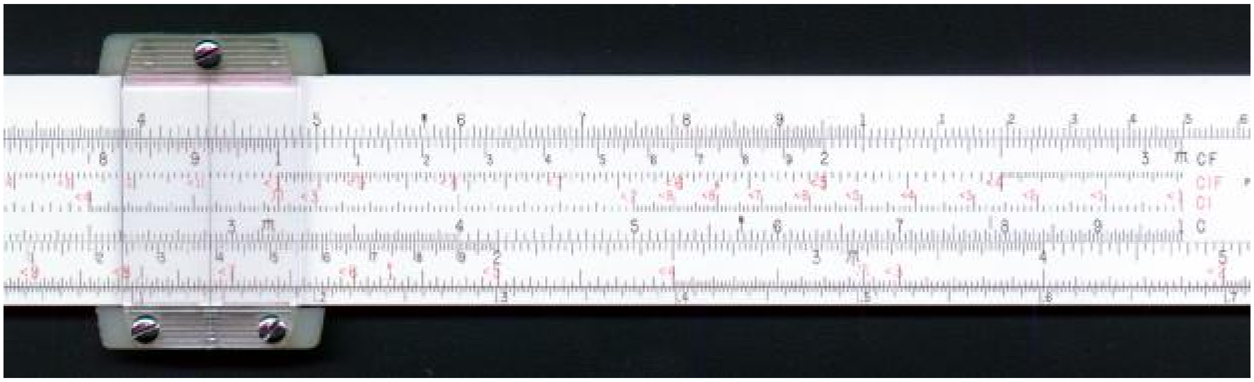


Fig 9-1

Example 1: Find x for $\frac{x}{29.2} = \frac{1.26}{8.43}$ (fig. 9-1)

This could be done by rearranging the proportions as $x = \frac{1.26x29.2}{8.43}$ and then calculating the right hand side in the usual way.

A better method would be as follows: (See Fig. 9-1)

1. Set the hair line over 1.26 on the DF scale.
2. Place 8.43 of the C scale under the hair line.
3. Reset the hair line over 29.2 on the C scale.
4. Under the hair line read off 4.36 on the DF scale as the answer.

Note: The above method could be applied in solving an equation of the form $\frac{29.2}{x} = \frac{8.43}{1.26}$ as this can be written as $\frac{x}{29.2} = \frac{1.26}{8.43}$.

Example 2: Find x and y if $\frac{45.1}{73} = \frac{2.8}{x} = \frac{y}{15.5}$

Both x and y can be found with only one move of the slide, as follows:

1. Set the hair line over 73 on the D scale.
2. Place 45.1 of the scale below the hair line.
3. Reset the hair line over 3.3 on the C scale, and under the hair line read of x = 4.52 on the D scale.
4. Reset the hair line over 15.5 on the DF scale, and under the hair line read off y = 9.6 on the CF scale.

Note:

(a) Once the ratio is set up on the C and D scales, we can reset the hair line and read off any number of values for the numerators or denominators of equivalent fractions.
(b) The only time this will be impossible, is when we run off the end of the scale. This occurs when the numerator and denominator of the given ratio are at opposite ends of the C and D scales (e.g. $\frac{11.6}{9.3}$ or $\frac{87}{119}$). This is overcome as shown in example 3.

Example 3: Find x and y if $\frac{92}{143} = \frac{x}{66} = \frac{183}{y}$

1. Set the hair line over 143 on the DF scale.
2. Place 92 of the CF scale below the hair line.
3. Reset the hair line over 66 on the DF scale, and under the hair line read off x = 42.5 on the CF scale.
4. Reset the hair line over 183 on the CF scale, and under the hair line read off y = 248 on the DF scale.

Exercise 9(b)
Find the number in the following:

(i) $\frac{x}{32.7} = \frac{423}{67.7}$

(ii) $\frac{1.09}{4.93} = \frac{x}{18.18}$

(iii) $\frac{8.42}{x} = \frac{21.3}{3.72}$

(iv) $\frac{5.12}{65.6} = \frac{x}{3.45} = \frac{y}{176}$

(v) $\frac{32.7}{14.25} = \frac{x}{4.65} = \frac{21.2}{y}$

(vi) $\frac{13.6}{74.5} = \frac{x}{6.24} = \frac{42}{z}$

(vii) $\frac{0.394}{59} = \frac{R_1}{6.4} = \frac{R_2}{26.7} = \frac{R_3}{1.35}$

(viii) $\frac{47.7}{7.16} = \frac{38.2}{V_1} = \frac{275}{V_2} = \frac{1050}{V_3}$

Complete the following tables:

(ix) $\frac{Feet}{Meters} = \frac{3.281}{1} = \frac{625}{\quad} = \frac{83.6}{\quad} = \frac{2560}{\quad}$

(x) $$\frac{Kg}{lb.} = \frac{1}{2.204} = \frac{6.52}{\quad} = \frac{\quad}{9.48} = \frac{73.5}{\quad}$$

(xi) $$\frac{Miles/hr.}{Km/hr.} = \frac{1}{1.609} = \frac{63.2}{\quad} = \frac{\quad}{15.6} = \frac{49.3}{\quad}$$

(xii) $$\frac{sq.in.}{sq.cm} = \frac{1}{6.45} = \frac{\quad}{11.2} = \frac{63.8}{\quad} = \frac{\quad}{258}$$

(xiii) For R = 1.34T, find the corresponding values of R for T = 1.25, 3.5 and 7.25.

(xiv) For y = 5.2/x, find the corresponding values of y for x = 1.5, 3.4 and 7.6.

Chapter 10 – Combined Operations on C, D, CI, DI, CD, DF, CIF, A, B, BI, K and K' Scales

In this unit we will show how the order of operations and the selection of scales can greatly reduce the moves required. The effect of this is to increase accuracy, speed and the range of problems you can handle on your Slide Rule.

Note: The A and B scales or the K and K' scales (if you Slide Rule has a K' Scale), can be used for multiplication and division. On the A and B scales the 1, 10 , 100 can be read as an index, while the K and K' scales 1, 10, 100, or 1,000 can be used as an index in calculations. The BI scale can be used in conjunction the A and B scales in the same way as the CI with the C and D scales, or the CIF with the CF and DF scales.

10.1 Simple Combinations or Roots, Powers and Reciprocals.

Example 1: $\dfrac{1}{29.6^2} = 0.00114$

1. Set the hair line over 29.6 on the CI (or DI) scale. (Note under the hair line on the C (or D) scale we have the reciprocal of 29.6, but we need not read this).
2. Under the hair line read off '144' on the B (or A) scale as the answer.

(i.e. approx = $\dfrac{1}{30^2} = \left(\dfrac{1}{30}\right)^2$
$= (.03)^2$
$= 0.0009$)
Therefore the answer is 0.00114

Note: We find the reciprocal of 29.6 first and then the square. This is possible because $\dfrac{1}{29.6^2} = \left(\dfrac{1}{29.6}\right)^2$.

If we were to use the reverse order of operations, it would necessitate reading the square of 29.6 off the A or B scale and transferring this value onto the C or CI scale to obtain the reciprocal, which of course is unsatisfactory.

Example 2: $\dfrac{1}{29.6^3} = 0.0000386$

1. Set the hair line over 29.6 on the CI (or DI) scale.
2. Under the hair line read off 0.0000386 on the K (or K') scale as the answer.

Example 3: $\dfrac{1}{\sqrt{29.6}} = 0.1838$

1. Set the hair line over 29.6 on the B (or A scale). (Note: under the hair line on the C (or D) scale we have the square root of 29.6, but we need not read this).
2. Under the hair line read off 0.1838 on the CI (or DI) scale as the answer.

Example 4: $\dfrac{1}{3\sqrt{29.6}} = 0.3235$

1. Set the hair line over 29.6 on the K (or K') scale.
2. Under the hair line read off 0.3235 on the CI (or DI scale) as the answer.

Example 5: $3\sqrt{29.6^2} = 9.57$

1. Set the hair line over 29.6 on the K (or K') scale. (Note: under the hair line on the C (or D) scale we have the cube root of 29.6, but we need not read this.)
2. Under the hair line read off 9.57 on the A (or B) scale as the answer.

Example 6: $\sqrt{29.6^3} = 161$

1. Set the hair line over 29.6 on the A (or B) scale.
2. Under the hair line read off 161 on the K (or K') scale as the answer.

Example 7: $29.6^4 = 768{,}000$

1. Set the hair line over 29.6 on the D scale.
2. Place the 29.6 on the CI scale below the hair line.
3. Above the right index of the B scale read off 768,000 on the A scale as the answer. (Note: the left index of the B scale will sometimes be used instead of the right index.)

OR

1. Set the hair line over 29.6 on the K scale.
2. Place the 29.6 of the CI scale below the hair line.
3. Above the right index of the CI scale read off 768,000 on the K scale as the answer.

Example 8: $29.6^5 = 22{,}750{,}000$

1. Set the hair line over 2.6 on the D scale.
2. Place the 29.6 of the CI scale below the hair line.
3. Reset the hair line over 29.6 on the B scale.
4. Under the hair line read off 22,750,000 on the A scale as the answer.

Example 9: $29.6^6 = 673{,}000{,}000$

1. Place the left index of the C scale over the 29.6 on the D scale.
2. Set the hair line over 29.6 on the C scale.
3. Under the hair line read off 673,000,000 on the K scale as the answer.

Example 10: $\dfrac{1}{29.6^4} = 0.0000013$

1. Place the 29.6 of the C scale over the left index of the D scale.
2. Set the hair line over 29.6 on the CI scale.
3. Under the hair line read off 0.0000013 on the A scale as the answer.

Note: The last two steps in the above example could be done as follows:

2. Set the hair line over 29.6 on the D scale.
3. Under the hair line read off 0.0000013 on the BI scale as the answer.

Exercise 10(a)

(i) $\dfrac{1}{6.8^2}$

(ii) $\dfrac{1}{0.47^2}$

(iii) $\dfrac{1}{34^2}$

(iv) $\dfrac{1}{1.4^2}$

(v) $\dfrac{1}{\sqrt{70}}$

(vi) $\dfrac{1}{\sqrt{5.4}}$

(vii) $\dfrac{1}{\sqrt[3]{143}}$

(viii) $\dfrac{1}{3\sqrt{0.54}}$

(ix) $\sqrt{13.5^3}$

(x) $\sqrt{61^3}$

(xi) $\sqrt[3]{96.1^2}$

(xii) $\sqrt[3]{0.85^2}$

(xiii) 2.3^4

(xiv) 49.5^4

(xv) 6.7^5

(xvi) 19.5^5

(xvii) 0.9^6

(xviii) 4.1^6

(xix) $\dfrac{1}{8.3^4}$ (xx) $\dfrac{1}{0.674^4}$

10.2 Continued Multiplication and Division

In this section we will combine the basic methods of multiplication and division of Units 2, 3, 4 and 8. You may find it helpful to review these briefly before proceeding with this section.

Example 1: 12.4 x 8.4 x 0.157 = 16.35

1. Set the hair line over 12.4 on the D scale.
2. Place the 8.4 of the CI scale under the hair line.

Note: the progressive answer is marked on the D scale by the left index of the C scale.

3. Reset the hair line over the 0.157 on the C scale.
4. Under the hair line read off 16.35 on the D scale as the answer.

Example 2: 2.32 x 60.5 ÷ 0.082 = 1710

1. Set the hair line over 2.32 on the D scale.
2. Place the 60.6 on the CI scale below the hair line.

Note: the progressive answer is marked on the D scale by the left index of the C scale.

3. Reset the hair line over the 0.082 on the CI scale.
4. Under the hair line read off 1710 on the D scale as the answer.

Example 3: 7.5 ÷ 4.8 x 30.4 = 47.5

1. Set the hair line over 7.5 on the D scale.
2. Place the 4.8 on the C scale under the hair line.

Note the progressive answer is marked on the D scale by the left index of the C scale.

3. Reset the hair line over 30.4 on the C scale.
4. Under the hair line read off 47.5 on the D scale as the answer.

Example 4:36.6 ÷ 0.71 ÷ 2.26 = 22.8

1. Set the hair line over 36.6 on the D scale.
2. Place the 0.71 on the C scale under the hair line.

Note the progressive answer is marked on the D scale by the right index of the C scale.

3. Reset the hair line over 2.26 on the CI scale.
4. Under the hair line read off 22.8 on the D scale as the answer.

Note: In combination multiplication and division rules are:

(a) When multiplication comes first, always do it by using the D and CI scales.
(b) When division comes first, always do it by using the C and D scales.

The reason for this is that when we multiply by the CI and D scales, or divide by the C and D scales, the answer is always marked on the D scale by the left or right index of the C scale, thus allowing a further multiplication or division without moving the slide. Hence, if the second operation is multiplication, we use the C and D scales, and if the second operation is a division we sue the CI and D scales. (Check these points carefully in the four examples above.)

Example 5: 3.35 x 47 ÷ 25.9 ÷ 41 x 8.85 = 1.312

1. Set the hair line over 3.35 on the D scale.
2. Place the 47 of the CI scale under the hair line.
3. Reset the hair line over 25.9 on the CI scale.
4. Place the 41 of the C scale under the hair line.
5. Reset the hair line over 8.85 of the CF scale.
6. Under the hair line read off 1.38 on the DF scale as the answer.

(Note: in the above example we switched to the Folded scales for the last multiplication, as 8.85 on the C scale was off the end of the D scale. Otherwise it would have required the slide to have been moved.)

Exercise 10(b)

(i) 43 x 5.1 x 0.12 =
(ii) 52 x 720 x 0.041 =
(iii) 2.4 x 32 ÷ 1.94 =
(iv) 156 x 6.57 ÷ 92.1 =
(v) 19 ÷ 6.4 ÷ 15 =
(vi) 16.4 ÷ 3.14 ÷ 2.75 =
(vii) 11 ÷ 73 ÷ 830 =
(viii) 38 ÷ 5.2 x 4.75 =
(ix) 6.4 x 3.5 x 18 x 0.14 =
(x) 2.46 x 52.3 ÷ 25.2 ÷ 3.6 =
(xi) 2.46 x 52.3 ÷ 25.2 x 3.6 =
(xii) 6.3 ÷ 1.3 x 2.3 x 4.9 =
(xiii) 70.6 ÷ 4.32 ÷ 15.25 ÷ 1.16 =
(xiv) 39.4 ÷ 1.73 ÷ 12.4 x 8.66 =
(xv) 8.3 x 3.9 x 20 ÷ 16 =
(xvi) 28 ÷ 1.3 x 5.6 ÷ 1.5 =

10.3 Multiplication and Division of Roots, Powers and Reciprocals

There are too many possible combinations for us to cover ever type, but the following table gives many possibilities. Note that often care must be taken in setting numbers on the A, B and K scales, as we will recall for example $\sqrt{2}$ and $\sqrt{20}$ are located at different positions on the A and B scale. Thus, numbers must be located on the A, B and K scales according to the rules given in Units 5 and 6. In some of the following, with certain numbers the answer may run off the end of the scale. In such cases it will be necessare to reset the slide unless the C and D scales are involved, when it will be possible to us the CF and DF scales and avoid a further movement of the slide. (In the following, 'H.L." stands for Hair Line.)

Example	Set HL Over	Under HL Place	Reset HL over	Under HL answer
ab^2	a on A scale	Index of B scale	b on C scale	on A scale
a^2b	a D	Index C	b B	A
ab^3	a K	Index B	b C	K
ab^4	a A b D	b CI a BI	b C b C	A A
a^2b^2	a D a D	Index C b CI	b C Index C	A A
a^2b^3	a D	b CI	b B	A
a^2b^2c	c A a D	a CI c BI	b C b C	A A
a^3b^3c	c K	a CI	b C	K
$\frac{a}{b^2}$	a A a A	b C Index B	Index C b CI	A A
$\frac{a}{b^3}$	a K	b C	Index C	K
$\frac{a^2}{b^3}$	a D a D	b C b B	b BI b CI	A A
$\frac{a^2}{b}$	a D	b B	Index B	A
$\frac{a^3}{b}$	a D a D	b B b K'	a B index K'	A K
$\frac{a^3}{b^2}$	a A	a CI	b CI	A
$\frac{ab^2}{c^2}$	b D	c C	a B	A

$\dfrac{a}{b^2c^2}$	a A	b C	c CI	A
$\dfrac{a}{bc^2}$	a A	b B	c CI	A
$\dfrac{1}{ab}$	Index D	a C	b CI	D
$\dfrac{1}{ab^2}$	Index A	a B	b CI	A
$\dfrac{1}{a^2b^2}$	Index D	a C	b CI	A
$\dfrac{1}{a^2b^3}$	a K	b C	a CI	K
$\sqrt{ab}$	a A	Index B	b B	D
$\sqrt{abc}$	a A	b BI	c B	D
$a\sqrt{b}$	a D	Index C	b B	D
$a^2\sqrt{b}$	b A	a CI	a C	D
$\sqrt{\dfrac{a}{b}}$	a A	b B	Index B	D
$\dfrac{\sqrt{a}}{b}$	a A	b C	Index C	D
$\dfrac{a}{\sqrt{b}}$	a D	b B	Index C	D
$\sqrt{\dfrac{ab}{c}}$	a A	c B	b B	D
$\sqrt{\dfrac{a}{bc}}$	a A	b B	c BI	D
$\dfrac{a\sqrt{b}}{c}$	a A	b B	c BI	D
$\dfrac{\sqrt{a}}{bc}$	a A	b C	c C	D
$\dfrac{ab}{\sqrt{c}}$	a D	c B	b C	D
$ab\sqrt{c}$	c A	b CI	a C	D
$a\sqrt{bc}$	b A	a CI	c B	D
$a\sqrt[3]{b}$	b K	a CI	Index CI	D
$\dfrac{\sqrt[3]{a}}{b}$	a K	b C	Index C	D

$\dfrac{a}{\sqrt[3]{b}}$	b K	a C	Index D	D
$\dfrac{\sqrt{a}\sqrt[3]{b}}{c}$	b K	c C	a B	D
$\dfrac{a\sqrt[3]{b}}{\sqrt{c}}$	b K	c B	a C	D
$\dfrac{1}{\sqrt{ab}}$	Index A a A	a B Index B	b A b B	CI DI
$\dfrac{1}{a\sqrt{b}}$	b A Index D	a CI a C	Index D b BI	C D
$a\pi\sqrt{b}$	b A	a CI	Index C	DF
$\dfrac{a\pi}{\sqrt{b}}$	a D	b B	Index C	DF
$\sqrt{a\pi}$	π A	Index B	a B	D
$\dfrac{a\sqrt{b}}{\pi}$	b A	a CI	Index CF	D
$\dfrac{a}{\pi\sqrt{b}}$	a D	b B	Index CF	D
$a\sqrt{\dfrac{b}{\pi}}$	b A	π B	a CF	DF

Exercise 10(c)

(i) $47.9\sqrt{1.46}$

(ii) $2.17x8.63\sqrt{0.562}$

(iii) $5.75\sqrt[3]{69.8}$

(iv) $\dfrac{\sqrt{6.45}}{8.27}$

(v) $\dfrac{67.3}{\sqrt{123.4}}$

(vi) $\dfrac{8.47\sqrt{63.7}}{14.64}$

(vii) $\dfrac{15.7x6.37}{\sqrt{127}}$

(viii) $\dfrac{\sqrt{436}}{2.08x5.63}$

(ix) $\dfrac{1}{42.7\sqrt{2.63}}$

(x) $2.46\sqrt{6.23x4.97}$

(xi) $\dfrac{17.22}{\sqrt{31.9x20.6}}$

(xii) $\dfrac{\sqrt{673}}{4.22\sqrt{51.5}}$

(xiii) $40.2\sqrt{12.25}\sqrt[3]{106}$

(xiv) $\sqrt{32.6}\sqrt[3]{24.6}$

(xv) $27x3.4^2$

(xvi) $\dfrac{26.4^2}{31.2}$

(xvii) $\dfrac{256}{12.7^2}$

(xviii) $\dfrac{31.9x16.3^2}{2.57}$

(xix) $\dfrac{1}{\sqrt{6.3x13.1}}$

(xx) $\dfrac{1}{4.2\sqrt{6.1}}$

(xxi) $\dfrac{1}{6.7^2\,x3.1^3}$

(xxii) $\sqrt{61\pi}$

(xxiii) $\dfrac{61.4}{\pi\sqrt{19.7}}$

(xxiv) $17.2\sqrt{\dfrac{29}{\pi}}$

(xxv)

10.4 Miscellaneous Problems
Exercise 10(d)

(i) $\left(\dfrac{24x31.6}{68}\right)^2 =$

(ii) $11.2\left(1+\dfrac{39.4}{85}\right)^2 =$

(iii) $\dfrac{2.9x3.4^2}{6.1^2-4.4^2} =$

(iv) $\dfrac{\sqrt{21}-\sqrt{15.4}}{\pi} =$

(v) $2\left(\dfrac{29.8}{111}-\dfrac{3.21}{94.7}\right) =$

(vi) $14.2\sqrt[3]{6.4+1.96^2} =$

(vii) $\dfrac{\sqrt{6.9^2-1}}{\sqrt{2.7^2+1}} =$

(viii) $\dfrac{2.3^4\pi}{132} =$

(ix) $\dfrac{2500x3.4x32}{3x10^6\,x\pi(1.7^4-1.4^4)} =$

(x) $15x0.43^2\,x1.19^4 =$

(xi) $\sqrt{13(13-6.4)(13-9.9)(13-9.7)}$

(xii) $2\pi\sqrt{\dfrac{87.6}{980}} =$

(xiii) $32^3-\dfrac{(61-24)^2}{7.3} =$

Chapter 11 – Sine and Cosine (S and ST scale)

On the S scale the graduations in black are for sines. Most Slide Rules have graduations in red for Cosines, these reading from right to left.

11.1 Sine (S scale – for angles between 5°44' and 90°

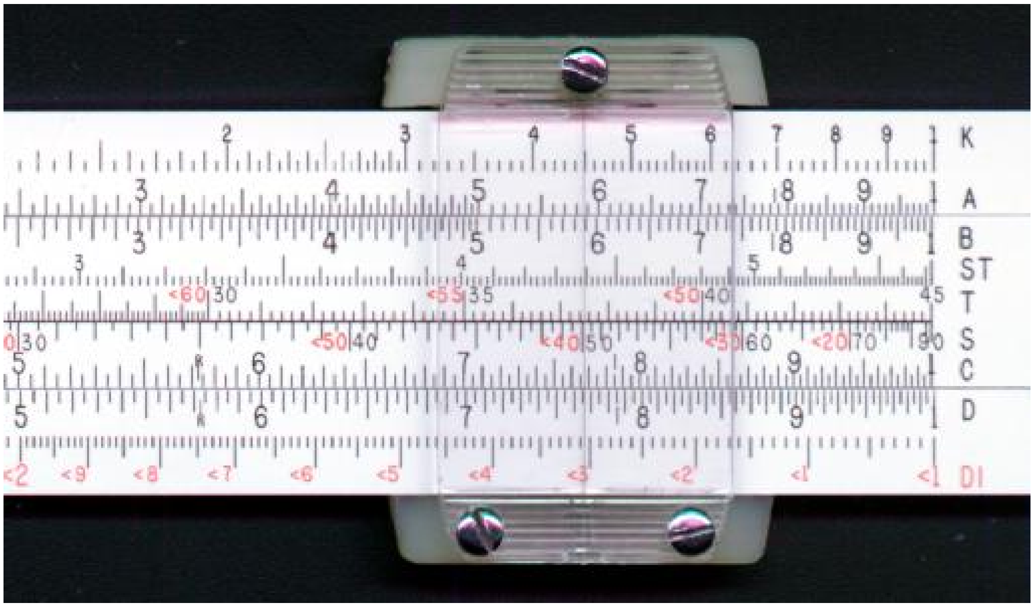


Fig 11-1

Example: sin 50° = 0.766 (Fig. 11-1)

1. Set the hair line over 50° (in black) on the S scale.
2. Under the hair line read off 0.766 on the D scale as the answer.

Note:

(a) The sines of angles between 5°44' and 90° are read directly off the D scale as values between 0.1 and 1.

(b) The subdivisions on the S scale (i.e. graduations between the degree marks) on most rules allow us to express a fraction of a degree as either a decimal of a degree or so many minutes. That is, there are 10, 5, or 2 subdivisions.

(c) There are two limitations in using the S scale:
   a. We cannot obtain the sines of angles less than 5°44'. (See 11.2 for the use of the ST scales for the sines of small angles.)
   b. The lack of accuracy for angles close to 90° (see Unit 13 for better method involving P scale for sines of angles close to 90°.)

(d) For angles greater than 90°, convert them to an equivalent expression involving and angle less than 90°, as is the usual practice.
   e.g. sin 150° = sin (180°-30) = sin 30°
   sin 240° = sin(180°+60) = - sin 60°
   sin 315° = sin(360°-45°) = - sin 45°

(e) If your Slide Rule has on S' scale on the slide, this is the same scale as the S, and is best used in conjunction with the C' scale (which is also on the slide).

(f) If the value of the sine of an angle is known, then the angle can be obtained by:
   a. Setting the hair line over the 'value' on the D scale.
   b. Under the hair line read off the angle on the S scale.
   (This of course is evaluating inverse trigonometrical functions, e.g. $\sin^{-1} 0.866 = 60°$)

Exercise 11(a)

(i)	sin 21° =	(v)	sin 120° =
(ii)	sin 36° 36' =	(vi)	sin 300° =
(iii)	sin 56° 30' =	(vii)	sin x = 0.274
(iv)	sin 7° 21' =	(viii)	sin x = 0.855

11.2 Sine (ST scale – for angles less then 5°44'

For sines of angles between 0.573° (or 34') and 5.74° (or 5°44') we find the angle on the ST scale and read the sine of the angle off the D scale as a value between 0.01 and 0.1. For the sines of angles between 0.0573° and 0.573° we find the angles on the ST scale, reading the graduations not as 0.9, 1.5, 5.5, etc. but as 0.09, 0.15, 0.55 etc. (i.e. $\frac{1}{10}$ of their marked value). The sine of such angles is read off the D scale as a value between 0.001 and 0.01. This can be repeated for even smaller angles if desired. For more detailed explanation and further uses of the ST scale for very small angles, see Unit 14. In using the ST scales note that minutes have to be expressed as a decimal fration of a degree (i.e. by dividing by 60).

Example: sin 0.82° = 0.0143 (Fig. 11-2)

1. Set the hair line over 0.82° on the ST scale.
2. Under the hair line read off 0.0143 on the D scale as the answer.

Note: If we required sin 0.082°, we would locate the hair line as above, and the answer would be then read off as 0.00143.

Exercise 11(b)

(i)	sin 4°31' =	(iv)	sin 30' =
(ii)	sin 2.3° =	(v)	sin 0.2° =
(iii)	sin 0.79° =	(vi)	sin 9' =

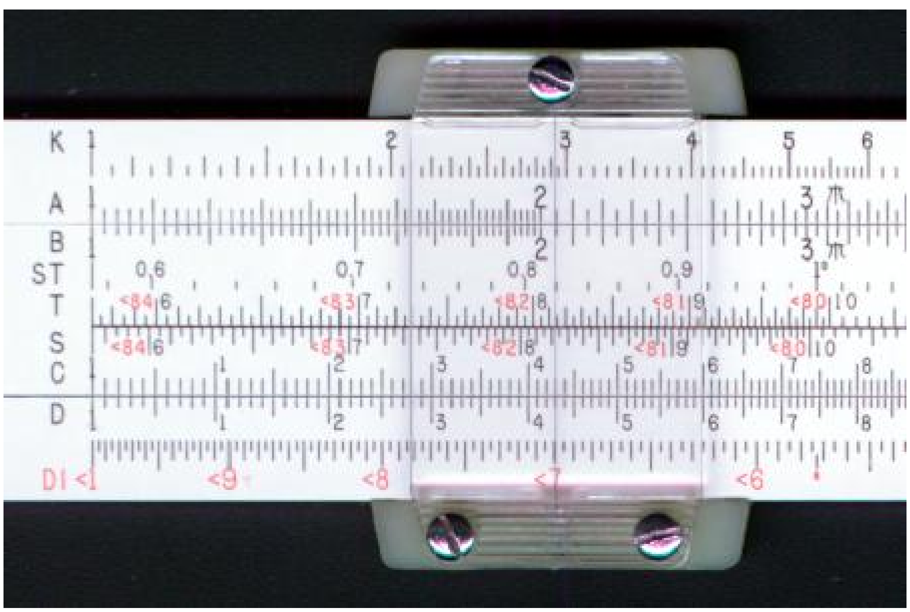


Fig 11-2

11.3 Cosine (S and ST scale)

As cos θ = sin (90 - θ) (e.g. cos 60 = sin 30, etc.) we find the cosine of an angle by looking up the sine of its complement. To facilitate this, the angles for cosines are usually marked on the S scale in red.

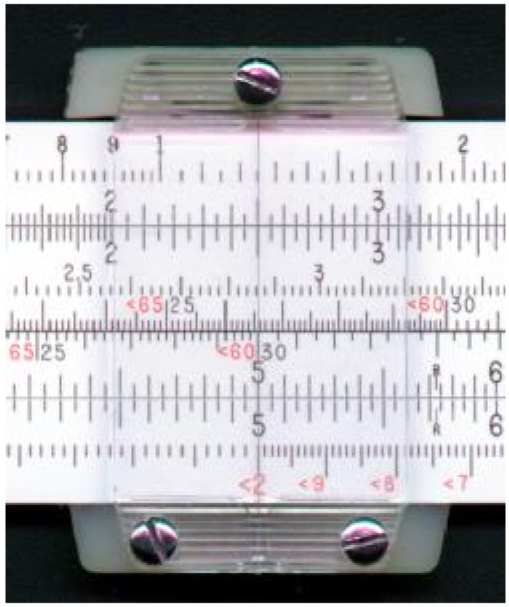


Fig 11-3

Example 1: Cos 42° = 0.743 (Fig 11-3)
1. Set the hair line over the 42° (for red graduations) on the S scale.
2. Under the hair line read off 0.743 on the D scale as the answer.

Note: The S scale allows us to obtain cosines of angles between 0° and 84°16’. For angles greater than 84°16’ we can use the ST scales as follows: --

Example 2: cos 88° = 0.0349
1. Set the hair line over 2° on the ST scale. (as cos 88° = sin 2°).
2. Under the hair line read off 0.0349 on the D scale as the answer.

Exercise 11(c)

(i)	cos 30° =	(v)	cos 89° =
(ii)	cos 79° =	(vi)	cos 89°30’ =
(iii)	cos 65.4° =	(vii)	cos x = 0.4
(iv)	cos 85°30’ =	(viii)	cos x = 0.04

11.4 Cosecant and Secant

We recall that:

$$\text{cosec } \theta = \frac{1}{\sin\theta} \quad \text{and sec } \theta = \frac{1}{\cos\theta}$$

Thus, to find cosecant or secant we look them up the same as for sine and cosine (using S or ST scales) We read the answer off on the reciprocal scales DI or CI instead of the D or C scale.

Example 1: cosec 15° = 3.86
(i) Set the hair line over 15° on the S scale.
(ii) Under the hair line read off 3.86 on the DI (or CI) scale as the answer.

Example 2: sec 59° = 1.94
1. Set the hair line over 59° (for red graduations) on the S scale.
2. Under the hair line read off 1.94 on the DI (or CI) scale as the answer.

Note: As the value of sine and cosine is always less than 1, the value of their reciprocals cosecant and secant will always be greater than 1.

Exercise 11(d)

(i)	cosec 45° =	(iv)	sec 82° =
(ii)	cosec 24°12’ =	(v)	sec 22° =
(iii)	cosec 3.5° =	(vi)	sec 2°30’ =

11.5 Multiplication and Division with Sines and Cosines

The following table gives a few possible calculations involving sines and cosines. It will be observed that the S’ scale on the slide is extremely handy to use in combination with the usual S scale on the body of the Slide Rule. If the answer runs off the end D scale make use of the DF scale for the answer. Also note for small angles the ST scale would be used where the table indicates the S scale. In the following ‘H.L.” stands for hair line.)

Example	Set HL Over	Under HL Place	Reset HL over	Under HL answer
a sin θ	θ on S scale	Index of C scale	a on C scale	on D scale
a cos θ	θ(red) S	Index C	a C	D
(a sin θ)^2	θ S	Index C	a C	A
(a cos θ)^2	θ(red) S	Index C	a C	A
$\dfrac{a}{\sin\theta}$	θ S a C	a C θ S’	Index D Index C	C D
$\dfrac{a}{\cos\theta}$	θ(red) S	a C	Index D	C
$\dfrac{\sin\theta}{a}$	θ S	a C	Index C	D
$\dfrac{a\sin\theta}{b}$	θ S	b C	a C	D
$ab\cos\theta$	θ(red) S	a CI	b C	D
$\dfrac{1}{\sin^2\theta}$	θ S θ S	index C	Index A	B BI
$\sin\theta\sin\phi$	θ S	Index C	φ S’	D
$(\cos\theta\cos\phi)^2$	θ(red) S	Index C	φ(red) S’	A
$\dfrac{\sin\theta}{\sin\phi}$	θ S	φ S’	Index C	D
$\dfrac{\cos^2\theta}{\cos^2\phi}$	θ(red) S	φ(red) S’	Index C	A
$\pi\sin\theta$	θ S			DF
$\dfrac{\sin\theta}{\pi}$	θ S	π C	Index C	D
$\pi\cos^2\theta$	θ(red) S	Index B	π C	A
$\sqrt{\pi}\sin\theta$	θ S	Index C	π B	D
$\dfrac{\sin\theta\sin\phi}{\sin\alpha}$	θ S	α S’	φ S’	D

Exercise 11(e)

(i) $3.4 \sin 27° =$

(ii) $2.7 \cos 60° =$

(iii) $(3.4 \sin 27°)^2 =$

(iv) $\dfrac{1.4}{\sin 56°} =$

(v) $\dfrac{6.4}{\cos 74°30'} =$

(vi) $\dfrac{5.46 \sin 69°}{\sin 12°} =$

(vii) $\sin 14° \text{ x } \sin 26° =$

(viii) $(\cos 59° \text{ x } \cos 75°)^2 =$

(ix) $\left(\dfrac{\sin 70°}{\sin 21°}\right)^2 =$

(x) $\dfrac{1}{\cos^2 36°30'} =$

(xi) $\pi \sin 21°20' =$

(xii) $\dfrac{\cos 76°45'}{\pi}$

(xiii) $\pi \sin^2 81° =$

(xiv) $\sqrt{\pi} \sin 32°12' =$

(xv) $\dfrac{\sin 29° x \sin 34°}{\sin 36°} =$

(xvi) $\dfrac{\cos 67° x \cos 53°}{0.031} =$

Chapter 12 – Tangent (T, T_1, T_2 and ST scales)

It is an advantage to have two tangent scales (T_1, and T_2) on your Slide Rule, instead of just a single tangent scale (T). The T_1 and T scales are identical and used for angles between 5°44' and 45°, while the T_2 scale allows us to read directly angles greater the 45°. On the tangent scales the graduations in black are for tangents and those in read are for co-tangents, the latter reading from right to left.

12.1 Tangent (T_1 or T scale – for angles between 5°44' and 45°).

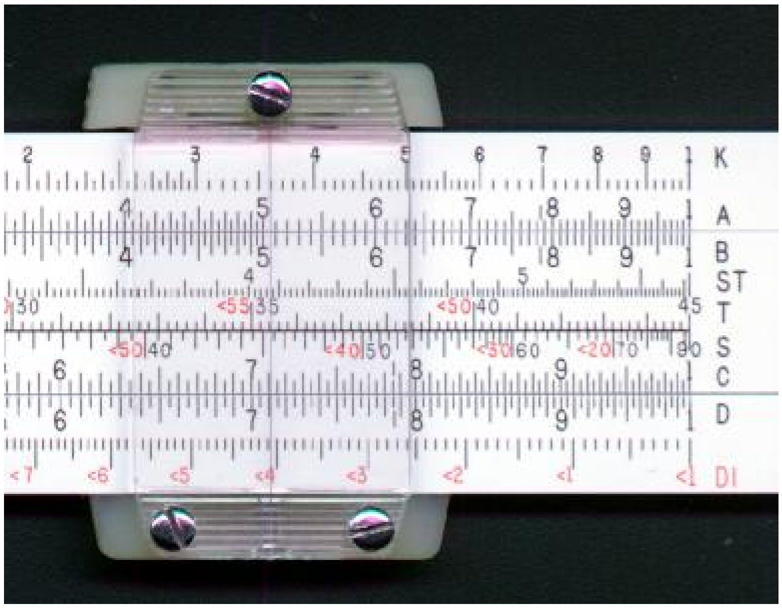


Fig 12-1

Example: Tan 35°24' = 0.71 (Fig 12-1)

1. Set the hair line over 35°24' on the T_1 (or T) scale.
2. Under the hair line read off 0.71 on the D scale as the answer.

Exercise 12(a)

(i) tan 31° = (iii) tan 15°36' =

(ii) tan 44°30' = (iv) tan 8.7° =

12.2 Tangent (ST scale – for angles less than 5°44')

Less than 5 or 6 degrees, the sines and tangents of angles are the same at least to three figures of accuracy. Thus, the same scale will do for both (hence we call it the ST scale), and we find tangents of angles less than 5°44' in exactly the same way as we did for sine. (see fig 11.2)

Example 1: tan 4°12' = 0.0733

First convert 4°12' to 4.2°

1. Set the hair line over 4.2° on the ST scale.
2. Under the hair line read off 0.0733 on the D scale as the answer.

Note: For angels less than 0.573° (34') we use the procedures as outlined in 11.2.

Example 2: tan 0.42° = 0.00733

1. Set the hair line over 0.42 on the ST scale (i.e. at the point actually marked 4.2°).
2. Under the hair line read off 0.00733 on the D scale as the answer.

Exercise 12(b)

(i) tan 4°31’ =
(ii) tan 2.3° =
(iii) tan 0.79° =
(iv) tan 30’ =
(v) tan 0.2° =
(vi) tan 9’ =
(vii)

12.3 Tangent (T_2 and T scale – for angels between 45° and 84°18’).

1. Using the T_2 Scale

With the T_2 scale the tangents of angles in this range can be directly read off as follows:

Example 1: tan 52° = 1.28

1. Set the hair line over 52° on the T_2 scale.
2. Under the hair line read off 1.28 on the D scale as the answer.

Note: The tangents of angles between 45° and 84°18’ are number between 1 and 10, hence, tan 52° is read off on the D scale as 1.28.

2. Using the T or T_1 scale.

For a slide Rule without a T_2 scale, we can use the T scale because the Tan θ = $\dfrac{1}{\tan(90-\theta)}$. (Note that the T scale is identical with the T_1 scale.) This relationship can be proved using the fact that tan θ = cot(90-θ) and tan θ = $\dfrac{1}{\cot\theta}$

$$\text{i.e. } \tan\theta = \frac{1}{\cot\theta} = \frac{1}{\tan(90-\theta)}$$

Thus to find tan 52° we use the complement of 52° (i.e. 38°), and read the answer on the DI (or CI) scale instead of the D scale.

Example 2: tan 52° = 1.28

1. Set the hair line over 38° (the complement of 52°) on the T (or T_1) scale.
2. Under the hair line read off 1.28 on the DI (CI) scale as the answer.

Exercise 12(c)

(i) tan 60° =
(ii) tan 70° =
(iii) tan 63°6’ =
(iv) tan 82.5° =
(v) tan 47°45’ =
(vi) tan 76° =
(vii)

12.4 Tangent (ST scale – for angles greater than 84°18’).

Using $\tan\theta = \dfrac{1}{\tan(90-\theta)}$ we can obtain the tangents of angles greater than 84°18’ by finding their compliments on the ST scale and reading their value on the DI (or CI) scale.)

Example: Tan 89.17° = 69.

1. Set the hair line over 0.83 (the compliment of 89.17°) on the ST scale.
2. Under the hair line read off 69 on the DI (or CI) scale as the answer.

Note: The tangents of angles between 84°18’ and 89.427° lie between 10 and 100.

Exercise 12(d)

(i) tan 85° =

(ii) tam 87°30’ =

(iii) tan 89.1° =

(iv) tan 88°45’ =

(v)

12.5 Cotangents

As $\cot\theta = \dfrac{1}{\tan\theta}$ we can find the cotangents of an angle by following the same procedures as we did for the tangents of the angle. If the tangent is red off the D (or C) scale the cotangent will be read off the DI (or CI) scale and visa-versa.

(Note: for small angels the cotangents are large, while the cotangents for angles near 90° are small.)

Example 1: Cot 1° = 57.3.

1. Set the hair line over 1° on the ST scale.
2. Under the hair line read off 57.3 on the DI (or CI) scale as the answer.

Example 2: cot 39°48’ = 1.2
(Express 39°48’ as 39.8°).

1. Set the hair line over 39.8° on the T_1 (or T) scale.
2. Under the hair line read off 1.2 on the DI (or CI) scale as the answer.

Example 3: cot 89° = 0.1746

Set the hair line on 1° on the ST scale.
Under the hair line read off 0.1746 on the D (or C) scale as the answer.

Exercise 12(e)

(i) cot 37° =

(ii) cot 71° =

(iii) cot 4°30’ =

(iv) cot 87° =

(v) cot 61°20’ =

(vi) cot 44’ =

(vii) cot 22°12’ =

(viii) cot 89°6’ =

12.6 Multiplication and Division with Tangents

The following table gives a few possible calculations involving tangents of angles up to 84°18’ using the ST, T_1 and T_2 scales located on the body of the Slide Rule. If your slide rule has only the ST and T scale these methods would have to be varied for angels greater 45°.
Note: In the following table T stand for whichever is appropriate of the ST, T_1 or T_2 scales (according to the size of the angle).

Exercise 12(f)

(i) 2.6 tan43° =

(ii) $\dfrac{\tan 52°}{0.45}$ =

(iii) 1.19 tan 4°30’ =

(iv) $(6.3 \tan 17.2°)^2$ =

(v) $6.3 \tan^2 17.2°$ =

(vi) $\dfrac{1}{\tan^2 81°30'}$ =

(vii) $\pi \tan 39° 24'$ =

(viii) $\dfrac{\tan 67°36'}{\pi}$ =

(ix) $\sqrt{\pi}\,\tan 3.5°$ =

(x) $\dfrac{\tan^2 23°30'}{\pi}$ =

(In the following H.L. Stands for hair line.)

Example	Set HL Over	Under HL Place	Reset HL over	Under HL answer
a tanθ	θ on T scale	Index of C scale	A on C scale	On D scale
$\frac{\tan\theta}{a}$	θ T	a C	Index C	D
$\frac{a}{\tan\theta}$	θ T	a C	Index D	C
$(a\tan\theta)^2$	θ T	Index C	a C	A
$a\tan^2\theta$	θ T	Index C	a B	A
$\frac{1}{\tan^2\theta}$	θ T	Index C	Index A	B
$\pi\tan\theta$	θ T			DF
$\frac{\tan\theta}{\pi}$	θ T	π C	Index C	D
$\sqrt{\pi}\tan\theta$	θ T	Index C	π B	D
$\frac{\tan^2\theta}{\pi}$	θ T	π B	Index B	A

Chapter 13 – Pythagorean (P) Scale

13.1 The Form of the P Scale
The P scale is an inverted scale reading from right to left, hence the graduations are in red. The P scale is related to the D scale such that for a number "x" on the D scale, immediately below it on the P scale we have $\sqrt{1-x^2}$. This of course is only valid for $-1 \le x \le 1$ (i.e. $|x| \le 1$), as we cannot have the square root of a negative number.

13.2 Calculating $\sqrt{1-x^2}$ (P and D scales)
(Note we must have $-1 \le x \le 1$ i.e. $|x| \le 1$ for $\sqrt{1-x^2}$ to have a real value.)

Example 1: $\sqrt{1-0.06^2} = 0.8$
1. Set the hair line over 0.6 on the D scale.
2. Under the hair line read off 0.8 on the P scale as the answer.

Example 2: $\sqrt{1-0.08^2} = 0.6$
1. Set the hair line over 0.8 on the D scale.
2. Under the hair line read off 0.6 on the P scale as the answer.

Note: If $y = \sqrt{1-x^2}$ then $x = \sqrt{1-y^2}$, thus to find $\sqrt{1-x^2}$ we could either find x on the D scale and read $\sqrt{1-x^2}$ off the P scale, or find x on the P scale and read $\sqrt{1-x^2}$ off the D scale.

Exercise 13(a)

(i)	$\sqrt{1-0.2^2}$ =	(iii)	$\sqrt{1-0.955^2}$ =
(ii)	$\sqrt{1-0.43^2}$ =	(iv)	$\sqrt{1-0.119^2}$ =

13.3 Converting Sines to Cosines (and vise versa)
From the relationship $\sin^2\theta + \cos^2 = 1$ we can express:

(i) $\sin\theta = \sqrt{1-\cos^2\theta}$

(ii) $\cos\theta = \sqrt{1-\sin^2\theta}$

Thus, given the value of sin θ we can read off directly the value of cos θ, and vise versa.

Example: sin 60° = 0.866 then cos 60° = 0.5
1. Set the hair line over 0.866 (i.e. sin 60°) on the D scale.
2. Under the hair line read off 0.5 (i.e. cos 60°) on the P scale as the answer.

Exercise 13(b)
- (i) if sin 35°48' = 0.585, then cos 35°48' =
- (ii) if sin 90° = 1.000, then cos 90° =
- (iii) if cos 70° = 0.342, then sin 70° =
- (iv) if cos 81°42' = 0.1445, then sin 81°42 =

13.4 Sines of large angels and Cosines of small angels
For sines of large angles (i.e. in the region 80 to 90) working from the S scale is very inaccurate, as you can see from a glance at this region S scale.

Take for example sin 84°, the best we could estimate using the S and D scales would be 0.994. It would be impossible to make any more accurate estimation if the questing was 84°20'. A better method is as follows:

Example: sin 84°6' = 0.9947

1. Set the hair line over 84°6' (i.e. in the red graduations) on the S scale. (84°6' in red graduation is the same as 5°54' in black).
2. Under the hair line read off 0.09947 on the P scale as the answer.

The same situation arises for cosines of small angels. Therefore, using the fact cos 5°54' = sin 84°6' we have:

Example: sin 5°54' = 0.9947

1. Set the hair line over 5°54' (in black) on the S scale.
2. Under the hair line read off 0.09947 on the P scale as the answer.

Note:

(a) For angles in red on the S scale, the P scale gives us the sine.
(b) For the angle in black on the S scale, the P scale gives us the cosine.

Exercise 13(c)

(i)	sin 61° =	(iv)	cos 27°48' =
(ii)	sin 78°30' =	(v)	cos 14°6' =
(iii)	83°24' =	(vi)	cos 47°48' =

13.5 Square Roots (numbers just less than 1, 100, etc.)

The square root of numbers a little less than 1, 100, etc, can be obtained using the D and P scales to a greater degree of accuracy than in the conventional way, with the D (or C) and A (or B) scales.

Example 1: $\sqrt{0.911} = 0.9545$ (Fig 13.4)

Express $\sqrt{0.911} = \sqrt{1-0.089}$

$$= \sqrt{1-0.298^2}$$

Note: We must have the form $\sqrt{1-x^2}$ to use the P scale. Thus we subtract 0.911 from 1 to obtain 0.089, and express 0.081 = $(0.298)^2$ using the A and D scales.

Once we subtract 0.911 from 1, to obtain 0.089 the procedure is as follows:

1. Set the hair line over 0.089 (i.e. at 8.9) on the A scale.
2. Under the hair line read off 0.9545 on the P scale as the answer.

Example 2: $\sqrt{0.9755} = 0.9877$

Express $\sqrt{0.9755} = \sqrt{1-0.0245}$

$$= \sqrt{1-0.298^2}$$

1. Set the hair line over 0.0245 (i.e. at 2.45) on the A scale.
2. Under the hair line read off 0.9877 on the P scale as the answer.

Note: For $\sqrt{97.55}$ we would express it as:

$$\sqrt{100x0.9755} = 10\sqrt{0.9755}$$

and obtain $\sqrt{0.9755}$ as in example 2

i.e. = 10x0.9877

therefore = 9.877

Exercise 13(d)

(i) $\sqrt{0.95}$ =
(ii) $\sqrt{92.5}$ =
(iii) $\sqrt{0.86}$ =
(iv) $\sqrt{0.69}$ =
(v) $\sqrt{76}$ =
(vi) $\sqrt{9826}$ =

13.6 The Difference of Two Squares ($\sqrt{x^2 - y^2}$ or $x^2 - y^2$)

This is the form often encountered when using Pythagoras' Theorem to find the third side of a right triangle.
We note that:

$$\sqrt{x^2 - y^2} = \sqrt{x^2\left(1-\frac{y^2}{x^2}\right)}$$

$$= x\sqrt{1-\left(\frac{y}{x}\right)^2}$$

Thus, if we calculate $\frac{y}{x}$ using the C and D scales and transfer the result onto the P scale, on the D scale we have $\sqrt{1-\left(\frac{y}{x}\right)^2}$. Then we could easily multiply by x to obtain $x\sqrt{1-\left(\frac{y}{x}\right)^2}$ (i.e. $\sqrt{x^2 - y^2}$.

This answer would be read off the D scale, thus to obtain $x^2 - y^2$ we would read the answer off the A scale.

Example: $\sqrt{4.3^2 - 3.62^2} = 2.32$

Express $\sqrt{4.3^2 - 3.62^2} = 4.3\sqrt{1-\left(\frac{3.62}{4.3}\right)^2}$

$$= 4.3\sqrt{1-0.842^2}$$

(evaluate $\frac{3.62}{4.3} = 0.842$ in any of the usual ways.)

1. Set the hair line over 0.842 on the P scale. (The $= \sqrt{1-0.842^2}$ is on the D scale under the hair line.)
2. Place the right index of the C scale under the hair line.
3. Reset the hair line over 4.3 on the C scale.
4. Under the hair line read off 2.32 on the D scale as the answer.

Note: If instead of $\sqrt{4.3^2 - 3.62^2}$ we required $(4.3^2 - 3.62^2)$ on the A scale as the answer.

Exercise 13(e)

(i) $\sqrt{8^2 - 6^2}$ =
(ii) $\sqrt{91^2 - 83.5^2}$ =
(iii) $\sqrt{8.7^2 - 6.9^2}$ =
(iv) $13.3^2 - 11.1^2$ =
(v) $105^2 - 98^2$ =
(vi) $0.45^2 - 0.39^2$ =

13.7 Further Application of the P scale

1. To calculate the ordinates of an ellipse $\frac{x}{a}+\frac{y}{b}=1$. Transpose the equation to $y=\pm b\sqrt{1-\left(\frac{x}{a}\right)^2}$ and work from the P to D scale as in 13.6.
2. The following tables give a few other uses of the P scale.

Example	Set the H.L. over	Under the H.L. answer
$1-x^2$	x on P scale	on A scale
$\frac{1}{1-x^2}$	x P	BI
$\sqrt{(1-x^2)^3}$	x P	K
$\frac{1}{\sqrt{1-x^2}}$	x P	D (or CI)
$\sqrt{1-x}$	x A	P
$\sqrt{1-\frac{1}{x}}$	x BI	P
$\sqrt{1-\frac{1}{x^2}}$	x CI	P
$\sqrt{1-x^{\frac{2}{3}}}$	x K	P

Example	Set HL Over	Under HL Place	Reset HL over	Under HL answer
$a\sqrt{1-x^2}$	Index of D scale	a on C scale	x on P scale	on C scale
$\frac{a}{\sqrt{1-x^2}}$	Index DI	a CI	x P	CI
$\frac{\sqrt{1-x^2}}{a}$	x P	a C	Index C	D

Exercise 13(f)

(i) $1-0.24^2 =$

(ii) $\frac{1}{1-0.75^2} =$

(iii) $\sqrt{\left(1-0.9^2\right)^3} =$

(iv) $\frac{1}{\sqrt{1-0.43^2}} =$

(v) $\sqrt{1-0.76} =$

(vi) $\sqrt{1-\frac{1}{3.5}} =$

(vii) $\sqrt{1-\frac{1}{1.2^2}} =$

(viii) $\sqrt{1-0.83^{\frac{2}{3}}} =$

(ix) $13.1\sqrt{1-0.36^2} =$

(x) $\frac{2.1}{\sqrt{1-0.87^2}} =$

(xi) $\frac{\sqrt{1-0.17^2}}{3.95} =$

(xii) $$\frac{2.6\sqrt{1-0.46^2}}{5.8} =$$

Chapter 14 – Radians

14.1 Basic Relationships

When working with trigonometrical functions we often find the need to convert an angle in degrees to radians and vise verse.

Recall: 1 radian = $\frac{180}{\pi}$ degrees = 57.3°
(or π radians = 180 degrees)

To convert x° to radians we have:

$$x^\circ = x\left(\frac{\pi}{180}\right)\text{radians}$$

and to covert x radians o degrees we have:

$$\text{x radians } = \left[x\left(\frac{180}{\pi}\right)\right]^\circ$$

14.2 Converting Using C and D scales

As degrees are converted to radians by multiplying the angle in degrees by $\frac{\pi}{180} = 0.01745$, many Slide Rule have a mark labeled δ (or with some other symbol) at '1745' on the C and D scales (also CF and DF scales). Thus, if we set the index of the C scale above the δ on the D scale we have its radian equivalent on the D scale.

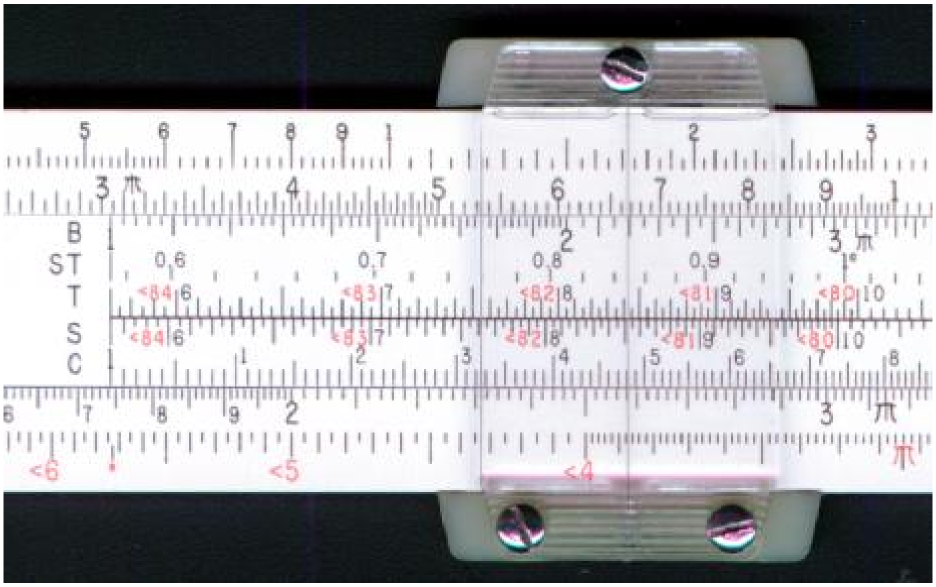


Fig 14-1

Example: 14.8° = 0.258 radians (Fig. 14-1)

1. Set the hair line over δ on the D scale.
2. Place the left index of the C scale under the hair line.
3. Reset the hair line over 14.8 on the C scale.
4. Under the hair line read off 0.258 on the D scale as the answer.

Note:

(a) The above setting of the Slide Rule would also give us 1.48° = 0.025 radians, 148° = 2.58 radians, etc. This is because the relationship between degrees and radians is a simple linear one. (e.g. 57.3° = 1 radian ∴ 114.6° = 2 radians, etc.)

(b) For the C and D scales set as above (from steps 1 and 2), if the angle in degrees on the C scale takes us off the end of the D scale, we find the angle in degrees on the CF scale and hence its value in radians on the DF scale. For example, observe on Fig. 14-1 under the hair line, 46.5° on the CD scale gives 0.81 radians on the DF scale.

(c) To convert from radians to degrees, we find the radian value on the D or DF scale and its degree equivalent is thus read off the C or CF scale.

Exercise 14(a)
Convert to Radians:

(i)	24° =	(iv)	125° =
(ii)	53°30' =	(v)	1.25° =
(iii)	69°24' =	(vi)	250° =

Convert to Degrees:

(vii)	1 radian =	(x)	0.065 radians =
(viii)	0.8 radians =	(xi)	6.5 radians =
(ix)	3.8 radians =	(xii)	1.3 radians =

14.3 Converting using the ST Scale

For small angles (i.e. below about 5° or 6°), the sine, tangent and radian value of an angle are all same to at least three figures. Thus, for an angle in degrees on the ST scale, its radians equivalent is read directly off the D scale. The actual graduations on the ST scale are only from 0.574° to 5.74°, 57.4° to 574°, 0.0574 to 0.574°, etc. This is because we have a linear relationship between degrees and radians, which is of course not so for an angle in degrees and its sine, cosine, tangent, etc.

Example 75° = 1.31 radians (Fig 14-2)

1. Set the hair line over 75 (marked as 0.75) on the ST scale.
2. Under the hair line read off 1.31 on the D scale as the answer.

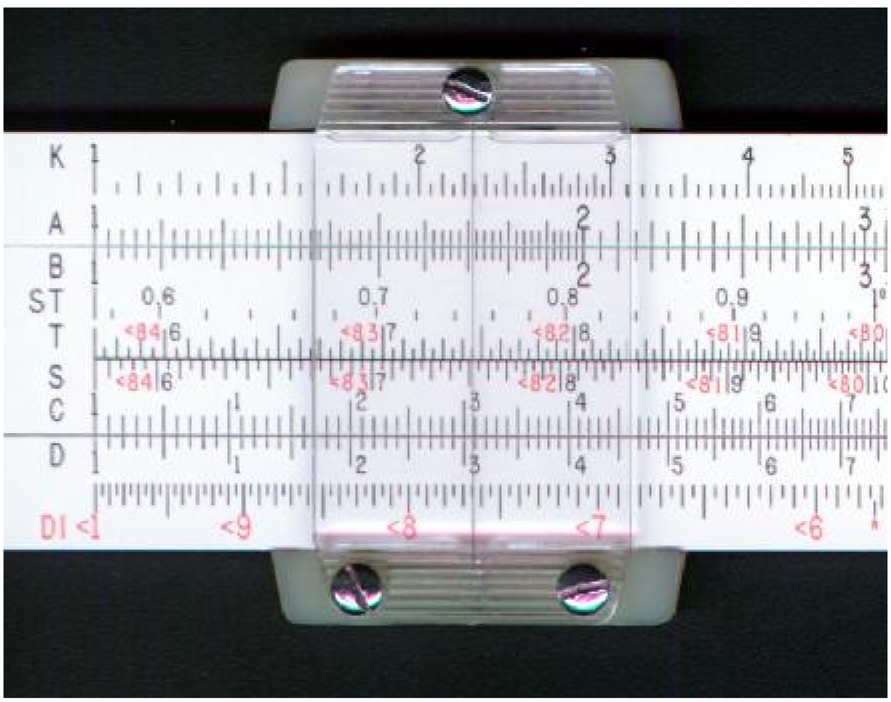


Fig 14-2

Note:

(a) In the above example we could take the same setting on the ST scale as 7.5°, 750°, 0.75°, etc., in which case the equivalent value in radians would have been 0.131, 13.1, 0.0131, etc. respectively.

(b) To convert from radians to degrees, we find the radians value on the D scale and its degrees equivalent is read off the ST scale.

Exercise 14(b)

Covert to radians:

(i)	4.5° =	(iv)	0.36° =
(ii)	36° =	(v)	91.36° =
(iii)	360° =	(vi)	12' =

Convert to degrees:

(vii)	0.65 radians =	(x)	5 radians =
(viii)	1.2 radians =	(xi)	0.625 radians =
(ix)	0.12 radians =	(xii)	0.0314 radians =

14.4 Arc Length and Area of a sector

Recall the formulae:

Arc Length, $L = r\theta$ (for r= radians and θ angle in radians)

Area of a Sector, $A = \frac{1}{2}r^2\theta$

Thus, if the angle θ was given in degrees (instead of radians), we could still evaluate the above quite readily.

Example: $L = r\theta$ for $r = 14$ and $\theta = 60°$ (Fig. 14-3)

1. Set the hair line over 60° (marked as 0.6) on the ST scale.
2. Place the left index of the C scale under the hair line.
3. Reset the hair line over 14 on the C scale.
4. Under the hair line read off 14.65 on the D scale as the answer.
   $\therefore$ L = 14.65

Note: The area of a sector could be similarly found but would necessitate a second movement of the slide to multiply by the extra factor of r and the ½. A better method of obtaining the area of a sector is found in Unit 20.

Exercise 14(c)

Find the arc length given:

(i)	$r = 2.5$ cm, $\theta = 45°$.	(iii)	$r = 15$ cm, $\theta = 2°30'$.
(ii)	$r = 4$ cm, $\theta = 120°$.	(iv)	$r = 6.8$ cm, $\theta = 315°$.

Find the area of the sector given:

(v)	$r = 2.5$ cm, $\theta = 45°$.	(vi)	$r = 4$ cm, $\theta = 120°$.

Chapter 15 – Solutions of Triangles

In general we will be given at least two of the three sides and an angles of a triangle, and asked to find one or more of the remaining sides of the angles.
We label the vertices of the triangle A, B and C and the sides opposite each angle are then labeled a, b, and c respectively. When we have a right angle triangle (as in section 15.1, 15.2 and 15.3) we will assume the right angle is at C and thus c is the hypotenuse.
The following methods are for the trigonometrical scales on the slide, interchanging the C scale for D, and CI scale for DI, and vise versa, will produce a suitable method.

15.1 Right Angle Triangle (given 2 sides)
We know angle C = 90° and sides a and b. Angle A and B, and the hypotenuse c are unknown. Thus we use

$$\tan A = \frac{a}{b}$$

B = 90° - A

And $c = \dfrac{a}{\sin A}$ (from $\sin A = \dfrac{a}{c}$)

Example: Given a = 44.5 cm and b = 59 cm
To find angles A, B and the hypotenuse c.

1. Set the hair line over 44.5 on the D scale.
2. Place the left index of the CI scale under the hair line. (In some examples it will be the right index).
3. Reset the hair line read off 37° on the T scale as the value for A.
4. Reset the hair line over 37° on the S scale.
5. Under the hair line read off 74 on the CI scale as the value for c.
   Then B = 90° - A = 90° - 37° = 53°
   Thus A = 37°, B = 53° and c = 74cm

Exercise 15(a)
(Note C = 90° and side c is the hypotenuse.)
Find angles A and B, and side c, given:

(i) a = 14.1 cm and b = 9.6 cm
(ii) a = 150 cm and b = 83 cm
(iii) a = 3.6 cm and b = 5.9 cm
(iv) a = 13 cm and b = 30.4 cm

15.2 Right Angle Triangle (given the hypotenuse and an angle).
We know angles C = 90° and A, and side c. Then sides a and b and the angle B are unknown.

Thus we use a = c sin A (from $\sin A = \dfrac{a}{c}$)

b = c cos A (from $\cos A = \dfrac{b}{c}$)

and B = 90° - A

Example; Given c = 9.5 cm and B = 41°
To find angle A and sides a and b.

1. Place 9.5 of the C scale over the right index of the D scale.
2. Set the hair line over 41° on the S scale.
3. Under the hair line read off 6.23 on the C scale as the value for a.
4. Reset the hair line over 41° (red) on the S scale.

5. Under the hair line read off 7.16 on the C scale as the value for b.
   B = 90° - A = 90° - 41° = 49°
   Thus B = 49°, a = 6.23 cm and b – 71.6 cm

Exercise 15(b)
(Note C = 90° and side c is the hypotenuse.)
Find angles B and sides a and b, given:

(i) A = 34°18’ and c = 18.5cm
(ii) A = 48.4° and c = 42.5cm
(iii) A = 26° and c = 7.3cm
(iv) A = 56.2° and c = 315cm

15.3 Right Angle Triangle (given the hypotenuse and a side)
We know angle C = 90° and sides c and a. The angles A and B and side b are unknown.

Thus we use

$$\sin A = \frac{a}{c}$$

$$B = 90 - A$$

and $$b = c \sin B \quad \text{(from } \sin B = \frac{b}{c}\text{)}$$

Example: Given a = 31.5 cm and c = 79
To find angles A and B and side b.

1. Place the 79 of the C scale over the right index of the D scale.
2. Set the hair line over 31.5 on the C scale.
3. Under the hair line read off 23.5° on the S scale as the value for A.
   Then B = 90° - A = 90° - 23.5° = 66.5°
4. Reset the hair line over 66.5° on the S scale.
5. Under the hair line read off 72.4 on the C scale as the value for b.
   Thus A = 23.5°, B = 66.5° and b = 74.2 cm

Exercise 15(c)
(Note Angle C = 90° and side c is the hypotenuse.)
Find angles A and B and side b given:

(i) a = 43 cm and c = 74 cm
(ii) a = 17.2 cm and c = 29.6 cm
(iii) a = 35 cm and c = 52 cm
(iv) a = 6.3 cm and c = 15.8 cm

15.4 Area of a Triangle
Knowing two sides and the included angle of any triangle, its area is given by:

Area = ½ a b sin C,
Or = ½ b c sin A,
Or = ½ a c sin B.

Example: Given a =15 cm., b = 17 cm and C = 36°.
$\therefore$ area = ½ x 15 x 17 x sin 36°

1. Set the hair line over 36° on the S scale.
2. Place the 15 of the CI scale under the hair line.
3. Reset the hair line over 17 on the C scale.
4. Place the 2 of the C scale under the hair line.
5. Below the right index of the C scale read off 75 on the D scale as the answer.
   i.e. area = 75 cm^2

Exercise 15(d)
Find the area of the following triangles given that:

(i) a = 6 cm, b = 9 cm and C = 38°.
(ii) b = 4.1 cm, c = 2.9 cm and A = 67°.
(iii) a = 15.4 cm, c = 13.8 cm and B = 43°24'.
(iv) a = 29.1 cm, b = 35.3 cm and C 105°.

15.5 Sine Rule (Scalene Triangles)
The sine rule is usually expressed as

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

In this unit we will deal with the cases where two angles and a side, or two sides and an angle opposite one of the sides are given. See the Appendix for cases where two sides and an angle not opposite one of the sides, or three sides are given.

Example 1: Given a 48 = cm, A = 55° and B = 76°
To find an angle C and sides b and c.

C = 180° - (55° + 76°)
∴ C = 49°

Thus we have: $\frac{\sin 55°}{48} = \frac{\sin 76°}{b} = \frac{\sin 49°}{c}$

1. Set the hair line over 55° on the scale.
2. Place the 48 of the C scale under the hair line. (This sets up the known ratio.)
3. Reset the hair line over 76° on the S scale.
4. Under the hair line read off 56.8 on the C scale as the value for b.
5. Reset the hair line over 49° on the S scale.
6. Under the hair line read off 44.2 on the C scale as the value for c.

∴ C = 49°, b = 56.8 cm and c = 44.2 cm

Note:

(a) In the above procedure we have set up the ratio of $\frac{\sin 55°}{48}$ on the C and S scales. Thus for a given angle on the S scale the side opposite it is read directly off the C scale, and for a given side on the C scale the angle opposite it is found on the S scale.
(b) Instead of using the C scales, we could have used the CF and S scales. (See example 2 below).
(c) For an S' scale on the slide (or a Slide Rule with the S scale on the slide), it can be used with the D scale on the body of the Slide Rule.)

Example 2: Given a = 8.3 cm, A = 48.8° and B = 60°
To find angle C and sides b and c.

C = 180° - (48.4° + 60°)
∴ C = 71.6°

Thus we have: $\frac{\sin 48.4°}{8.3} = \frac{\sin 60°}{b} = \frac{\sin 71.6°}{c}$

1. Set the hair line over 48.4° on the scale.
2. Place the 8.3 of the CF scale under the hair line.
3. Reset the hair line over 60° on the S scale.
4. Under the hair line read off 9.6 on the CF scale as the value for b.
5. Reset the hair line over 71.6° on the S scale.

6. Under the hair line read off 10.51 on the CF scale as the value for c.
   $\therefore$ C = 71.6°, b = 9.6 cm and c = 10.51 cm

Note: Had we set up Example 2 on the C and S scales, we would have found that for the hair line over 71.6° on the S scale, it would have been off the end of the C scale. In such circumstances a Slide Rule such as the Faber-Castle 2/83N or 2/82N with extended scale graduations can be a great help.

CASE II – Two Sides and an Angle Opposite One of Them (S.S.A)

A. One Triangle Possible, as the side opposite the given angle is longer than the other side given.

Example 3: Given A = 59°, a = 7.8 cm and c = 6.2 cm
To find angles B and C and side b.

Thus we have: $\dfrac{\sin 59°}{7.8} = \dfrac{\sin \mathbf{B}}{b} = \dfrac{\sin C}{6.2}$

a) Set the hair line over 59° on the S scale.
b) Place the 7.8 of the C scale under the hair line.
c) Reset the hair line over 6.2 on the C scale.
d) Under the hair line read off 43° on the S scale as the value for angle C.
   Thus B = 180° **-** (59° + 43°)
   $\therefore$ B = 78°
e) Reset the hair line over 78° on the S scale.
f) Under the hair line read off 8.9 on the C scale as the value for b.

B. Two Triangles Possible, as the side opposite the given angle is shorter than the other side give.

Example 4: Given a = 17.2 cm, b = 19.6 cm and A = 51°.
To find angles B and C, and side c.

Thus we have: $\dfrac{\sin 51°}{17.2} = \dfrac{\sin \mathbf{B}}{19.6} = \dfrac{\sin C}{c}$

1. Set the hair line over 51° on the S scale.
2. Place the 17.2 of the CF scale under the hair line.
3. Reset the hair line over 19.6 on the CF scale.
4. Under the hair line read off 62.3° on the S scale as one of the possible values for angle B.
   $\therefore$ B_1 = 62.3° Then B_2 = 180 – 62.3°
   = 117.7°
   Thus C_1 = 180° **-** (51° + 62.3°)
   = 66.7°
   and C_2= 180° **-** (51° + 117.7°)
   = 11.3°

For C_1 = 66.7°

5. Reset the hair line over 66.7° on the S scale.
6. Under the hair line read off 21.5 on the CF scale as the value for c_1

For C_1 = 117.7°

7. Reset the hair line over 11.3° on the S scale.
8. Under the hair line read off 4.34 on the CF scale as the value for c_2

Thus the two possible answers are:
B_1 = 62.3°, C_1 = 66.7° and c_1 = 21.5 cm
B_2 = 117.7°, C_2 = 11.3° and c_2 = 4.34 cm
(Note: we used the CF scale with the S scale as it would otherwise have run off the end of the C scale.)

Exercise 15(e)

Use the Sine Rule to find the remaining sides and angles, given:

(i) A = 66°, C = 48° and a = 42cm

(ii) B = 75°, C = 62° and a = 10.4cm
(iii) A = 126°, B = 15° and c = 17cm
(iv) a = 73cm, c = 28cm and A = 42°
(v) a = 9.1cm, b = 7.7cm and A = 51.2°
(vi) b = 25.4cm, c = 3.3cm and B = 100°
(vii) b = 83cm, c = 96.8cm and B = 48.4°
(viii) a = 62.5cm, b = 98cm and A = 38.3°

15.6 Cosine Rule (Scalene Triangles)
The cosine rule is usually expressed as:

$$a^2 = b^2 + c^2 - 2bc\cos A\ ,$$

or $$b^2 = a^2 + c^2 - 2ac\cos B\ ,$$

or $$c^2 = a^2 + b^2 - 2ab\cos C\ .$$

These also can be expressed in the form – $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$, etc.

We use the slide rule to square, multiply, divide and obtain square roots in these expressions.

Exercise 15(f)
Use the cosine and sine rules to solve the triangles, given:
(i) a = 45cm, b = 20cm and c = 60cm
(ii) a = 2.2cm, b = 3.4cm and c = 2.4cm
(iii) a = 25cm, b = 35cm and C = 38°
(iv) b = 14.9cm, c = 7.24cm and A = 104°36’

Chapter 16 – Complex Numbers

16.1 Basic Relationships

The Cartesian form of a complex number is $a+jb$, where $j=\sqrt{-1}$. The complex number $a+jb$ is represented on the complex plane by the vector OP, for P with coordinates (a, b).

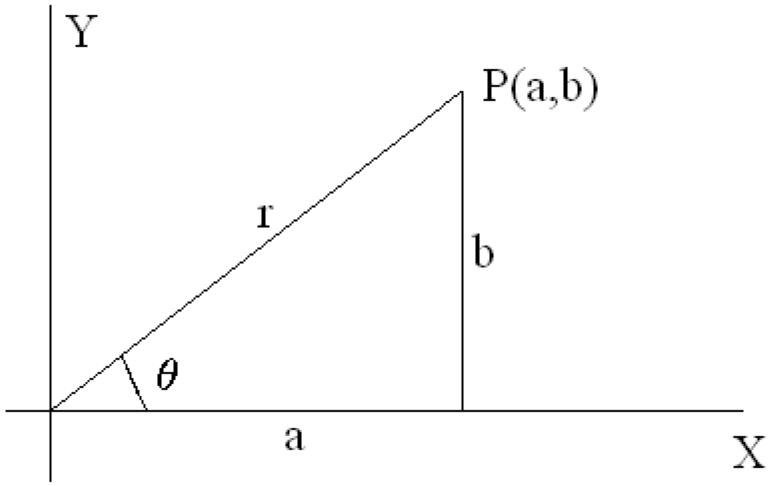


Fig 16.1

On occasions it is necessary to convert to Cartesian form to the polar form (i.e. $r(\cos\theta + j\sin\theta)$ and vise versa. The polar form can also be expressed as $re^{j\theta}$ by Euler's Equation. Often we express $r(\cos\theta + j\sin\theta)$ as $r\angle\theta$ for brevity.
From Fig. 16.1 it is clear that the following relationships connect the polar and Cartesian forms –

$$a = r\cos\theta$$
$$b = r\sin\theta$$
$$r = \sqrt{a^2+b^2}$$
$$\theta = \tan^{-1}\frac{b}{a}$$

(the last coming from $\tan\theta = \frac{b}{a}$).

16.2 Converting Cartesian Form to Polar

To convert $a+jb$ to $r\angle\theta$ we use $\tan\theta = \frac{b}{a}$ (i.e. $\theta = \tan^{-1}\frac{b}{a}$) to find θ, and $r = \frac{b}{\sin\theta}$ (from $\sin\theta = \frac{b}{r}$) to find r.

Note: when 'a' and/or 'b' are negative, this means the complex number lies in the 2nd, 3rd, or 4th quadrant.
The angle θ is thus affected, but not the amplitude, r. Hence, for placing a and b on the Slide Rule, we take their absolute values (i.e. $|a|$ and $|b|$).
If φ is the angle obtained in any of the methods above (using the absolute values of a and b) then for the various quadrants θ is obtained by –

a) Second Quadrant (a<0, b>0)
   θ = 180° - φ
b) Third Quadrant (a<0, b<0)
   θ = 180° + φ

c) Forth Quadrant (a>0, b<0)
$\theta = 360° - \varphi$

A. For S, T_1 and T_2 scales on the body of the Slide Rule.

Example 1: Convert 4 + 3j to polar form:
1. Set the hair line over 3 on the D scale.
2. Place the left index of the CI scale under the hair line. (in some cases the right index)
3. Reset the hair line over 4 on the CI scale.
4. Under the hair line read off 36.85° on the T_1 scale as the value for θ. (Use T_1 scale if $\frac{b}{a} < 1$ and T_2 scale if $\frac{b}{a} > 1$.)
5. Reset the hair line over 36.85° on the S scale.
6. Under the hair line read off 5 on the CI scale as the value for r.

$$\therefore 4 + 3j = 5\angle 36.85°$$

B. For S and T scales on the slide and a DI scale there are two cases.

For $\theta < 45°$ (i.e. $\frac{b}{a} < 1$)

Example 2: Convert 4 + 3j to polar form.
1. Set the hair line over the left index of the DI scale.
2. Place the 3 of the C scale under the hair line.
3. Reset the hair line over the 4 on the DI scale.
4. Under the hair line read off 36.85° on the T scale as the value for θ.
5. Reset the hair line over 36.85° on the S scale as the value for θ.
6. Under the hair line read off 5 on the DI scale as the value for r.

$$\therefore 4 + 3j = 5\angle 36.85°$$

For $\theta > 45°$ (i.e. $\frac{b}{a} > 1$)

Example 3: Convert 3+4j to polar form.
1. Set the hair line over the left index of the DI scale.
2. Place the 3 of the C scale under the hair line. (Note, we have '3' here as the value of 'a', in contrast to the '3' in step 2 of Example 2 which was then the value for 'b').
3. Reset the hair line over the 4 on the DI scale.
4. Under the hair line read off 36.85° on the T scale, so that $\theta = 90° - 36.85° = 53.15°$.
5. Reset the hair line over 36.85° on the S scale as the value for θ.
6. Under the hair line read off 5 on the DI scale as the value for r.

$$\therefore 3 + 4j = 5\angle 53.15°$$

Note: These two cases can be brought into one general method by using first the C scale, whichever of a and b is the smaller. Then if $\frac{b}{a} < 1$, the angle is taken as read off the T scale, otherwise for $\frac{b}{a} > 1$, we take the complement of the angle found on the T scale.

C. For S and T scales on the slide and no DI scale, there are two cases:

For $\theta < 45°$ (i.e. $\frac{b}{a} < 1$)

Example 4: Convert 4 + 3j to polar form.

1. Set the hair line over 4 on the D scale.
2. Place the right index of the C scale under the hair line.
3. Reset the hair line over 3 on the D scale.
4. Under the hair line read off 36.85° on the T scale as the value for θ.
5. Place the 36.85° on the S scale under the hair line.
6. Below the right index of the C scale read off 5 on the D scale as the value for r.

$$\therefore 4+3j = 5\angle 36.85°$$

For θ > 45° (i.e. $\frac{b}{a} > 1$)

Example 5: 3 + 4j to polar form.

1. Set the hair line over 4 on the D scale.
2. Place the right index of the C scale under the hair line.
3. Reset the hair line over 3 on the D scale.
4. Under the hair line read off 36.85° on the T scale so that θ = 90° - 36.85° = 53.15°
5. Reset the hair line over 36.85° on the S.
6. Under the hair line read off 5 on the D scale as the value for r.

$$\therefore 3+4j = 5\angle 53.15°$$

Exercise 16(a)

Convert the following to polar form:

(i) 8 + 6j =
(ii) 5 + 12j =
(iii) 12 + 5j =
(iv) -3 + 4j =
(v) -41 – 11j =
(vi) 34 – 7.2j =
(vii) -7 + 25j =
(viii) 2 – 3j

16.3 Converting Polar Form to Cartesian

To convert $r\angle\theta$ to a + jb we use b = r sin θ to find b and a = $\frac{b}{\tan\theta}$ (from $\tan\theta = \frac{b}{a}$) to find a.

Note: for angles, θ, greater than 90° (that is complex numbers in the 2nd, 3rd, or 4th quadrant) we express the angle as φ (for φ<90°) by –

d) Second Quadrant (90°a<θ<180°)
   φ = 180° - θ
e) Third Quadrant (180°a<θ<270°)
   φ = θ – 180°
f) Forth Quadrant (270°a<θ<360°)
   φ = 360° - θ

A. For S, T_1 and T_2 scales on the body of the Slide Rule.

Example 1: Convert $13\angle 59°$ to Cartesian form:

1. Set the hair line over 59° on the S scale.
2. Place the 14 of the CI scale under the hair line.
3. Reset the hair line over the index of the CI scale.
4. Under the hair line read off 11.15 on the D scale as the value for b.
5. Reset the hair line over 59° on the T_2 scale. (Use the T_1 scale if θ < 45°)
6. Under the hair line read off 6.7 on the CI scale as the value for a.

$$\therefore 13\angle 59° = 6.7 + 11.15j$$

B. For S and T scales on the slide and a DI scale there are two cases.

For θ < 45°

Example 2: Convert $13\angle 31°$ to Cartesian form.

1. Set the hair line over 13 on the DI scale.
2. Place the 31° of the S scale under the hair line.
3. Reset the hair line over the right index of the DI scale.
4. Under the hair line read off 11.15 on the C scale as the value for a.
5. Reset the hair line over 31° on the T scale.
6. Under the hair line read off 6.7 on the DI scale as the value for b.

$$\therefore 13\angle 31^\circ = 11.15 + 6.7\,j$$

Example 1: Convert $13\angle 59^\circ$ to Cartesian form:
For the complement of 59° = 90° - 59° = 31°

1. Set the hair line over 13° on the DI scale.
2. Place the 31° of the S scale under the hair line.
3. Reset the hair line over the index of the DI scale.
4. Under the hair line read off 11.15 on the C scale as the value for b.
5. Reset the hair line over 31° on the T scale.
6. Under the hair line read off 6.7 on the DI scale as the value for a.

$$\therefore 13\angle 59^\circ = 6.7 + 11.15\,j$$

Note: These two cases can be brought into one general method by using the angle as given, if it is less than 45°, otherwise we use its complement. If the angle is less than 45°, we read ‘a’ off the C scale and ‘b’ off the DI scale. If the angle given is greater than 45°, we read ‘b’ off the C scale and ‘a’ off the DI scale.

C. For S and T scales on the slide and no DI scale.

To convert $13\angle 31^\circ$ to Cartesian form, we evaluate –

a = 13 cos 31° = 11.5
and b = 13 sin 31° = 6.7
to obtain 11.15 + 6.7j

Exercise 16(b)

Convert the following to polar form:

(i) $4\angle 60^\circ$
(ii) $4\angle 30^\circ$
(iii) $6.5\angle 42^\circ$
(iv) $6.5\angle 132^\circ$
(v) $10.6\angle 216^\circ$
(vi) $34\angle 304^\circ$
(vii) $105\angle 110^\circ$
(viii) $15\angle 143.15^\circ$

16.4 Miscellaneous Problems

Recall:

$$r_1\angle\theta_1 \times r_2\angle\theta_2 = r \ \times r_2\angle\theta_1 + \theta_2$$
$$r_1\angle\theta_1 \div r_2\angle\theta_2 = r \ \div r_2\angle\theta_1 - \theta_2$$

Exercise 16(c)

Express the answer to the following in polar form:

(i) $35\angle 21^\circ \times 19\angle 53^\circ$
(ii) $4.7\angle 34^\circ \div 8.6\angle 41^\circ$
(iii) $(4+3j)\times(2+3j)$
(iv) $(6+9j)\div(5+3j)$

Express the answer to the following in Cartesian form:

(v) $2.5\angle 133^\circ \times 6.88\angle 68^\circ$
(vi) $42\angle 110^\circ \div 72\angle 140^\circ$
(vii) $(3+37)\times(-2+5j)$
(viii) $(-2-2j)\div(3-4j)$

Chapter 17 – Logarithms to Base 10 (L Scale)

Recall the Laws of Logarithms:

(i) $\log ab = \log a + \log b$

(ii) $\log \frac{a}{b} = \log a - \log b$

(iii) $\log a^N = N \log a$

And the equivalent Logarithmic and Exponential form:

$\log_b N = L$ and $N = b^L$

17.1 Logarithms and Antilogarithms Using L and D scales.

(i.e. usual L scale on body of the slide rule)

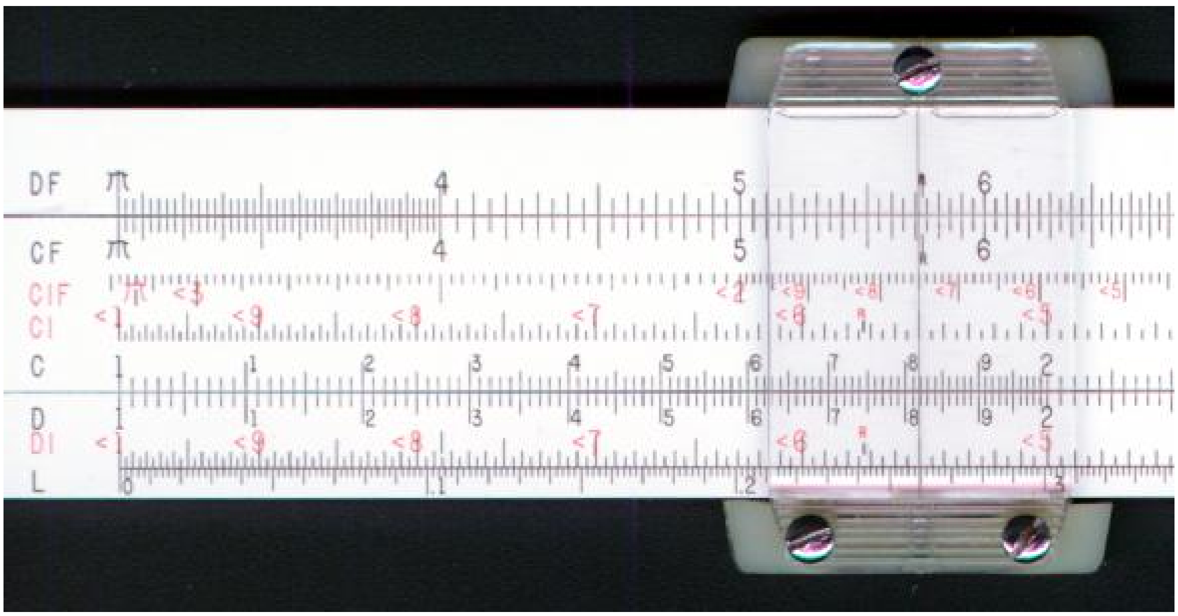


Fig 17-1

Example 1: $\log_{10} 1.82 = 0.26$ (Fig. 17-1)

(Or this could be stated $1.82 = 10^{0.26}$)

1. Set the hair line over 1.82 on the D scale.
2. Under the hair line read off 0.26 on the L scale as the answer.

Note:

(a) For numbers between 1 and 10 on the D scale, their logarithms are read directly off the L scale as the values between 0 and 1.

(b) For the logarithms of numbers outside the range 1 to 10, we have to decide their characteristic ourselves. e.g. $\log_{10} 182 = 2.26$ or $\log_{10} 0.182 = -1.26$

The Slide Rule gives us only the mantissa, as do logarithm tables.

(c) If we have the logarithm of a number and wish to find the number, we work the opposite way, (i.e. from the L scale to the D scale.)

Example 2: antilogarithm of 3.26 = 1,820

(or this could be stated $10^{3.26} = 1{,}820$)

1. Set the hair line over 0.26 (i.e. mantissa only) on the L scale.
2. Under the hair line read off '182' on the D scale as the answer.

   $\therefore answer = 1{,}820$ (as the characteristic is 3)

   (See exercise 17(a) at the end of 17.2 for problems)

Note:
As sines, cosines and tangents are found on the D scale, the value of log sin, log cos and log tan can be obtained by reading from the angle on the appropriate trigonometrical scale directly onto the L scale.

17.2 Logarithms and Antilogarithms Using L and W (Root) scales.
This is the system applicable to the Faber-Castell Slide Rules 2/83N, 62/83 etc. The L scale is on the slide and it is used in conjunction with the W scales. It is best to use the W'$_1$ and W'$_2$ scales instead of the W_1 and W_2 scales to avoid any error, should the slide be slightly displaced.

(i) For a number on the W'$_1$ scale, that is a number less than about 3.2 (3.5 for the 2/83 N), we read its logarithm off the L scale according to the numbers to the left of the graduations.

Example 1: $\log_{10} 1.385 = 0.1415$

1. Set the hair line over 1.385 on the W' scale.
2. Under the hair line read off 0.143 on the L scale (according to numbers to left of graduations) as the answer.

(ii) For a number on the W'$_2$ scale, that is a number greater than about 3 (2.8 for the 2/83N), we read its logarithm off the L scale according to the numbers to the right of the graduations.

Example 2: $\log_{10} 82.4 = 1.916$

1. Set the hair line over 82.4 the W'$_2$ scale.
2. Under the hair line read off 0.916 on the L scale (according to numbers to right of graduations) as the mantissa of the answer.

   $\therefore$ answer = 1.916 (as 82.4 is between 10 and 100).

Exercise 17(a)

(i) $\log_{10} 2.3 =$

(ii) $\log_{10} 46 =$

(iii) $\log_{10} 192 =$

(iv) $\log_{10} 0.67 =$

(v) $10^{0.477} =$

(vi) $10^{1.699} =$

(vii) $10^{2.45} =$

(viii) $10^{5.855}$

Find X in the Following:

(ix) $\log_{10} X = 0.908$

(x) $\log_{10} X = -2.805$

(xi) $\log_{10} X = 2.015$

(xii) $\log_{10} X = 4.262$

(xiii) $\log_{10} X = 8.5$

(xiv) $\log_{10} X = 32$

(xv) $\log_{10} X = 0.065$

(xvi) $\log_{10} X = 2{,}500$

17.3 Raising Numbers to Powers and Solving Exponential Equations:
A. Raising a Number to Power. (A better method using LL scales is given in unit 19.)

Example 1: $33.4^{4.95} = 35{,}450{,}000$

Express as $\log_{10} 33.4^{4.95} = 4.95 \log_{10} 33.4$ (by Law III)

$= 4.95 \times 1.524$ (Using Slide Rule to Find logarithm)

$= 7.55$ (Multiply using Slide Rule)

(Use the Slide Rule again to obtain the antilog of 0.55 and position the decimal point according to the characteristic, 7.)

$\therefore$ answer = 35,450,000

B. Solving an Exponential Equation.

Example 2: solve $3^x = 5$ for x

If two quantities are equal, then their logarithms will be equal.

i.e. $\log_{10} 3^x = \log_{10} 5$

$\therefore\ x \log_{10} 3 = \log_{10} 5$ (evaluate each using Slide Rule)

$$\therefore x = \frac{\log_{10} 5}{\log_{10} 3} \text{ (divide using Slide Rule)}$$

$$= \frac{0.699}{0.477} \text{ (evaluate each using Slide Rule)}$$

$$\therefore x = 1.466$$

Exercise 17(b)

(i) $1.5^{7.8} =$

(ii) $16.5^{2.5} =$

(iii) $157^{0.68} =$

(iv) $0.98^{3.6} =$

Find x in the following:

(v) $2.3^x = 7.6$

(vi) $56^x = 29.5$

(vii) $0.8^x = 0.2$

(viii) $x^{1.5} = 12.5$

(Hint, write as $x = 12.5^{\frac{1}{1.5}}$)

Chapter 18 – Logarithms to Base e and Power of e (LL scale)

18.1 The form of the LL scale.

The LL scales are found on the body of the Slide Rule and are used in conjunction with the D (or C scale). The LL_3, LL_2, and LL_1 (some also have an LL_0) scales are in black and give the positve powers of e. A number of slide rules have the inverted LL_{03}, LL_{02}, and LL_{03} (some also have LL_{00}) scales in red (reading from right to left) to give the negative powers of e. Note that the decimal point is given with all numbers marked on these scales. In the following, we have dealt with Slide Rules with 3, 6, 8 LL scales.

Example: $e^{1.52} = 4.75$

1. Set the hair line over 1.52 on the D (or C scale).
2. Under the hair line read off 4.57 on the LL_3 (i.e. e^x) scale as the answer.

Note:

(a) we read $e^{0.152} = 1.164$ on the LL_2 scale (i.e. $e^{0.1x}$) scale.
$e^{0.0152} = 1.0153$ on the LL_1 scale (i.e. $e^{0.01x}$) scale.
and $e^{0.00152} = 1.00152$ on the LL_0 scale (i.e. $e^{0.001x}$) scale.

For a on the D scale between	***e^a read off***
1 and 10	LL_3 scale
0.1 and 1	LL_2 scale
0.01 and 0.1	LL_1 scale
0.001 and 0.01	LL_0 scale

(b) As $e^N = x$ is equivalent to $\log_e x = N$ we can either say (for example) evaluate $e^{2.5}$, or solve $\log_e x = 2.5$ for x.

Exercise 18(a)

(i) $e^{4.3} =$
(ii) $e^7 =$
(iii) $e^{0.95} =$
(iv) $e^{0.035} =$
(v) $e^{0.062} =$
(vi) $e^{0.48} =$
(vii) $e^{0.003} =$
(viii) $e^{0.0051} =$

Solve the following for x:

(ix) $\text{Log}_e x = 3.1$
(x) $\text{Log}_e x = 0.62$
(xi) $\text{Log}_e x = 0.007$
(xii) $\text{Log}_e x = 0.049$

18.3 Negative Powers of e

Example 1: $e^{-7.9} = 0.00037$

1. Set the hair line over 7.9 on the D (or C) scale.
2. Under the hair line read off 0.00037 on the LL_{03} (i.e. e^{-x}) scale.

Note:

(a) we read $e^{-0.79} = 0.454$ on the LL_{02} scale (i.e. $e^{-0.1x}$) scale.
$e^{-0.079} = 0.924$ on the LL_{01} scale (i.e. $e^{-0.01x}$) scale.
and $e^{-0.0079} = 0.99213$ on the LL_{00} scale (i.e. $e^{-0.001x}$) scale.

For a on the D scale between	***e^a read off***
1 and 10	LL_{03} scale
0.1 and 1	LL_{02} scale
0.01 and 0.1	LL_{01} scale
0.001 and 0.01	LL_{00} scale

(b) For Slide Rules without the e^{-x} scales we can still obtain negative powers of e by using the fact $e^{-x} = \dfrac{1}{e^x}$.

Example 2: $e^{-0.79} = .454$

1. Set the hair line over 0.79 on the D (or C) scale).
2. Under the hair line read off 2.205 on the LL_2 scale as the answer of $e^{0.79}$.
3. Reset the hair line over 2.205 on the C scale.
4. Under the hair line read off 0.454 on the CI scale as the value for $\frac{1}{e^{0.79}}$ (i.e. $e^{-0.79}$)

Exercise 18(b)

(i) $e^{-2.1} =$
(ii) $e^{-9} =$
(iii) $e^{-0.4} =$
(iv) $e^{-0.19} =$
(v) $e^{-0.062} =$
(vi) $e^{-0.024} =$
(vii) $e^{-0.0019} =$
(viii) $e^{-0.0083}$

Solve the following for x:

(ix) $Log_e x = -2$
(x) $Log_e x = -0.4$
(xi) $Log_e x = -0.068$
(xii) $Log_e x = -0.0032$

18.4 Miscellaneous Powers of e

A. Very Small Powers of e can be approximated as follows:

for any x $e^x \approx 1 + x$

This can be easily verified and is useful when a Slide Rule does not have LL_0 and LL_{00} scales.

e.g. $e^{0.0027} \approx 1 + 0.0027 = 1.0027$,
$e^{-0.0018} \approx 1 - 0.0018 = 0.9982$, etc.

B. Very Large Powers of e (i.e. e^x for x > 10).

Example 1: $e^{15} = 3,320,000$

Express $e^{15} = (e^5)^3$
$= (149)^3$ (obtain $e^5 = 149$ using D and LL_3 scales)
$= 3.320,000$ (cube using D and K scales)

OR

Express $e^{15} = e^{10+5} = e^{10}e^5$
$= 22,000 \times 149$
$= 3,380,000$

(Note, discrepancies may occur in the third or forth significant figures from the different methods due to limitations of accuracy obtainable with various scales.)

Example 2: $e^{-15} = 3.01 \times 10^{-7}$

Express $e^{-15} = \left(e^{-5}\right)^3$
$= (0.0067)^3$
$= (6.7 \times 10^{-3})^3$
$= 301 \times 10^{-9} = 3.01 \times 10^{-7}$

C. Some other powers of e which may be worthy of note are covered in the following table.

Set the H.L. over		Under H.L. on LL scales read	Under H.L on LL_0 scales read
a on	A scale	$e^{\sqrt{a}}$	$e^{-\sqrt{a}}$
a	K	$e^{\sqrt[3]{a}}$	$e^{-\sqrt[3]{a}}$
θ	S	$e^{\sin\theta}$	$e^{-\sin\theta}$
θ (red)	S	$e^{\cos\theta}$	$e^{-\cos\theta}$
θ	T_1 or T_2	$e^{\tan\theta}$	$e^{-\tan\theta}$

a	DI(CI)	$\sqrt[a]{e}$	$\frac{1}{\sqrt[a]{e}}$
a	BI	$e^{\frac{1}{\sqrt{a}}}$	$e^{-\frac{1}{\sqrt{a}}}$
a	DF	$e^{\frac{a}{\pi}}$	$e^{-\frac{a}{\pi}}$
a	P	$e^{\sqrt{1-a^2}}$	$e^{-\sqrt{1-a^2}}$
a	W	e^{a^2}	e^{-a^2}

18.5 Logarithms to Base e

To find $\log_e N$ (or ln N as we often write it), the hair line is set over N on the LL scale and under the hair line on the D (or C) scale ln N is found.

The sign and position of the decimal point is dependent upon which LL scale N is located on.

Example 1: ln 4.57 = 1.52

(i.e. the steps shown in Fig. 18-1 in reverse order)

1. Set the hair line over 4.57 on the LL_3 Scale.
2. Under the hair line read off 1.52 on the D scale as the answer.

Example 2: ln 1.02 = 0.0198

1. Set the hair line over 1.02 on the LL_1 scale.
2. Under the hair line read off 0.0147 on the D scale as the answer. (It reads as 0.0147 on the D scale, as 1.02 is found on the LL_1 scale (i.e. $e^{0.01x}$)).

Example 3: ln 0.6 = -0.511

1. Set the hair line over 0.6 on the LL_{02} scale.
2. Under the hair line read off –0.511 on the D scale as the answer. (It reads as –0.511 on the D scale, as 0.6 is found on the LL_{02} scale (i.e. $e^{-0.1x}$)).

Note:

(a)

Found on the –	ln is read off the D scale as a value between –		
LL_3	1.0	and	10.0
LL_2	0.1	and	1.0
LL_1	0.01	and	0.1
LL_0	0.001	and	0.01
LL_{00}	-0.001	and	-0.01
LL_{01}	-0.01	and	-0.1
LL_{02}	-0.1	and	1.0
LL_{03}	-1.0	and	-10.0

(We do not attempt to write the logarithm of numbers less than 1 with a negative characteristic and positive mantissa, but leave it as a negative number.)

(b) For Slide Rules without e^{-x} scales we can still obtain natural logarithms of numbers less than 1, by using the fact $\ln x = -\ln \frac{1}{x}$.

Example: ln 0.6 = -0.511

(i.e. ln 0.6 = -ln $\frac{1}{0.6}$)

1. Set the hair line over 0.6 on the C scale.
2. Under the hair line read off 1.677 on the CI scale as the value for $\frac{1}{0.6}$.
3. Reset the hair line over 1.667 on the LL_2 scale.
4. Under the hair line read off –0.511 on the D scale as the value for ln 0.6.

(c) As ex = N is equivalent to ln N = x, we can either say, (for example) solve ex = 2.5 for x or evaluate ln 2.5.

(d) To solve $\sqrt[x]{e} = N$ for x, we note it can be written as $e^{\frac{1}{x.}} = N.$

i.e. $\ln N = \frac{1}{x}$

Thus to find what root of e is equal to a number N, we set the hair line over N on the LL scales and read off x on the DI scale.

(e) Logarithms of numbers near 1 are very small positive or negative values, depending upon whether the number is greater or less than 1. (ln 1 = 0). The logarithms of such numbers can be approximated, as follows:
ln 1.0023 $\approx$ 1.0023 – 1 = 0.0023
ln 0.9984 $\approx$ 0.9984 – 1 = -0.0016 (etc.)
This is useful for Slide Rules with out LL_2 and LL_{03} scales.

Exercise 18(d)

(i)	ln 9.5	(vii)	ln 1.005	(xiii)	ln 0.63
(ii)	ln 170	(viii)	ln 1.0019	(xiv)	ln 0.422
(iii)	ln 2.4	(ix)	ln 0.9985	(xv)	ln 0.035
(iv)	ln 1.14	(x)	ln 0.992	(xvi)	ln 0.0002
(v)	ln 1.07	(xi)	ln 0.95		
(vi)	ln 1.01	(xii)	ln 0.98		

Solve the following for x:

(xvii) $e^x = 4{,}000$
(xviii) $e^x = 1.06$
(xix) $e^x = 0.1$
(xx) $e^x = 0.9$

(xxi) $\sqrt{e}$ = 1.65
(xxii) $\sqrt{e}$ = 1.0202

Chapter 19 – Further Applications of the LL scales

19.1 Reciprocals

The LL (e^x) scales and the corresponding LL_0 (e^{-x} or $\frac{1}{e^x}$)Scales are the reciprocal of each other.

Example 1: $\frac{1}{4.1}$ = 0.244

1. Set the hair line over 4.1 on the LL_3 scale.
2. Under the hair line read off 0.244 on the LL_{03} scale.

Example 2: $\frac{1}{0.9335}$ = 1.071

1. Set the hair line over 0.9344 on the LL_{01} scale.
2. Under the hair line read off 1.071 on the LL_1 scale as the answer.

Note

(a) The reciprocal of any number from 0.000045 to 22,000 (10^{-5} to 10^5 for Slide Rules with extended scales) can be obtained by locating the number on the particular e^x or e^{-x} scale and reading off on the corresponding e^{-x} or e^x scale as in examples above.

(b) Even for a Slide Rule with LL_0 and LL_{00} scales, there is a gap between 0.9991 and 1.0009, but this range would not often be encountered. For number between 0.1 and 6, the LL scales give more accuracy the CI and D scales, while outside this range (i.e. greater than 6 and less then 0.1) the CI and D scales are more accurate. One of the greatest advantages of the LL scales for reciprocals, is that the decimal point is read directly off the scales.

e.g. 270 on the LL_3 scale gives $\frac{1}{270} = 0.0037$ on the LL_{03} scale.

Exercise 19(a)

(i) $\frac{1}{23}$ =

(ii) $\frac{1}{3.9}$ =

(iii) $\frac{1}{1.46}$ =

(iv) $\frac{1}{1.051}$ =

(v) $\frac{1}{0.98}$ =

(vi) $\frac{1}{0.9}$ =

(vii) $\frac{1}{0.48}$ =

(viii) $\frac{1}{0.0032}$ =

19.2 Tenth and Hundredth Powers and Roots

Example: Find 2.5^{10}, $\sqrt[10]{2.5}$, $\frac{1}{2.5^{10}}$, and $\frac{1}{\sqrt[10]{2.5}}$. (Fig 19.2)

1. Set the hair line over 2.5 on the LL_2 scale.

Under the hair line red off –

2. 9,500 on the LL_3 scale as the value of 2.5^{10}
3. 1.096 on the LL_1 scale as the value for $\sqrt[10]{2.5}$.
4. 0.000105 on the LL_3 scale as the value for $\frac{1}{2.5^{10}}$.

5. 0.9124 on the LL_{01} scale as the value for $\dfrac{1}{\sqrt[10]{2.5}}$.

Further Examples:
For 'a' on the LL scale as shown, the other LL scales give –

LL_3	LL_2	LL_1	LL_0	LL_{00}	LL_{01}	LL_{02}	LL_{03}
a	$\sqrt[10]{a}$	$\sqrt[100]{a}$	$\sqrt[1000]{a}$	$\dfrac{1}{\sqrt[1000]{a}}$	$\dfrac{1}{\sqrt[100]{a}}$	$\dfrac{1}{\sqrt[10]{a}}$	$\dfrac{1}{a}$
a^{10}	a	$\sqrt[10]{a}$	$\sqrt[100]{a}$	$\dfrac{1}{\sqrt[100]{a}}$	$\dfrac{1}{\sqrt[10]{a}}$	$\dfrac{1}{a}$	$\dfrac{1}{a^{10}}$
a^{100}	a^{10}	a	$\sqrt[10]{a}$	$\dfrac{1}{\sqrt[10]{a}}$	$\dfrac{1}{a}$	$\dfrac{1}{a^{10}}$	$\dfrac{1}{a^{100}}$
a^{1000}	a^{100}	a^{10}	a	$\dfrac{1}{a}$	$\dfrac{1}{a^{10}}$	$\dfrac{1}{a^{100}}$	$\dfrac{1}{a^{1000}}$
$\dfrac{1}{a^{1000}}$	$\dfrac{1}{a^{100}}$	$\dfrac{1}{a^{10}}$	$\dfrac{1}{a}$	a	a^{10}	a^{100}	a^{1000}
$\dfrac{1}{a^{100}}$	$\dfrac{1}{a^{10}}$	$\dfrac{1}{a}$	$\dfrac{1}{\sqrt[10]{a}}$	$\sqrt[10]{a}$	a	a^{10}	a^{100}
$\dfrac{1}{a^{10}}$	$\dfrac{1}{a}$	$\dfrac{1}{\sqrt[10]{a}}$	$\dfrac{1}{\sqrt[100]{a}}$	$\sqrt[100]{a}$	$\sqrt[10]{a}$	a	a^{10}
$\dfrac{1}{a}$	$\dfrac{1}{\sqrt[10]{a}}$	$\dfrac{1}{\sqrt[100]{a}}$	$\dfrac{1}{\sqrt[1000]{a}}$	$\sqrt[1000]{a}$	$\sqrt[100]{a}$	$\sqrt[10]{a}$	a

Exercise 19(b)

(i) $1.08^{10} =$

(ii) $1.08^{100} =$

(iii) $\dfrac{1}{1.08^{10}} =$

(iv) $\dfrac{1}{1.08^{100}} =$

(v) $\sqrt[10]{54} =$

(vi) $\sqrt[100]{54} =$

(vii) $\dfrac{1}{\sqrt[10]{54}} =$

(viii) $\dfrac{1}{\sqrt[100]{54}} =$

(ix) $\sqrt{0.65} =$

(x) $0.65^{10} =$

(xi) $\dfrac{1}{\sqrt[10]{.65}} =$

(xii) $\dfrac{1}{0.65^{10}} =$

(xiii) $\sqrt[10]{9.5} =$

(xiv) $\sqrt[100]{.03} =$

(xv) $1.7^{10} =$

(xvi) $0.97^{100} =$

Note:

(a) There is no difficulty with locating decimal points, as all values are read off the particular LL scale.

(b) Recall $\sqrt[10]{a} = a^{\frac{1}{10}} = a^{0.1}$

$$\sqrt[100]{a} = a^{\frac{1}{100}} = a^{0.01}$$

$$\sqrt[1000]{a} = a^{\frac{1}{1000}} = a^{0.001}$$

19.3 Positive Numbers to Any Powers

The LL scales can be used to obtain a^N for a > 0 and N, any power (positive, negative or fractional). The procedure is as follows: For 'a' on the LL scale, the D scale give 'ln a' (see unit 18). If we multiply this value of 'ln a' on the

D scale by 'N' (using the C and D scale) we then obtain 'N ln a' (i.e. ln a^N) on the D scale. If the multiplication was done using the left index of the C scale (see example 1 below), the value of 'a^N' is read off –

(i) The original LL scale for $1 < N < 10$.

(ii) The LL scale above the original for $10 < N < 100$ (e.g. 'a' on the LL_2 then 'a^N' on the LL_3 or 'a' on LL_{02} then 'a^N' on LL_{03}).

(iii) The LL scale below the orginal for $0.1 < N < 1$. (e.g. 'a' on the LL_2 then 'a^N' on the LL_1 or 'a' on LL_{02} then 'a^N' on LL_{01}).

In multiplying 'N' by 'ln a', on the C and D scales, if the right index of the C scale is used, the answers are found on the LL scale above the LL scales listed in the foregoing method. (see example 2).
This method can be extended to larger or smaller number values of N by moving two or more LL scales instead of one.

Example 1: Find $1.4^{2.5}$,1.4^{25}, and $1.4^{0.025}$

1. Set the hair line over 1.4 on the LL_2 scale.
2. Place the left index of the C scale under the hair line.
3. Reset the hair line over 2.5 on the C scale.
   Under the hair line read off –
4. 2.32 on the LL_2 scale as the value for $1.4^{2.5}$.
5. 4,500 on the LL_3 scale as the value for 1.4^{25}.
6. 1.0878 on the LL_1 scale as the value for $1.4^{0.25}$.
7. 1.0084 on the LL_0 scale as the value for $1.4^{0.025}$.

Example 2: Find $0.6^{6.15}$, $0.6^{0.615}$, $0.6^{0.00615}$, $0.6^{-6.15}$, $0.6^{-0.0615}$

1. Set the hair line over 0.6 on the LL_2 scale.
2. Place the right index of the C scale under the hair line.
3. Reset the hair line over 0.615 on the C scale.
   Under the hair line read off –
4. 0.043 on the LL_{03} scale as the value for $0.6^{6.15}$.
5. 0.73 on the LL_{02} scale as the value for $0.6^{0.615}$.
6. 0.99686 on the LL_{00} scale as the value for $0.6^{0.00615}$.
7. 23.3 on the LL_3 scale as the value for $0.6^{-6.15}$.
8. 1.032 on the LL_1 scale as the value for $0.6^{-0.0615}$.

Note:

(a) For negative power we simply transfer to the reciprocal LL scale (see 19.1)

(b) Instead of using the C scale in step 2 and 3 of the above Examples, we could have used the CF scale. Step 2 would then be –Place the index of the CF scale under the hair line. Note, we must use either the C or CF scale in a problem, and not mix them, otherwise it means the power is either multiplied or divided by π.

(c) For powers of numbers which take us outside the range of the LL scales, one of the following procedures can be used.

(i) $5.9^9 = 5.9^{4.5} \text{ x } 5.9^{4.5} = (5.9^{4.5})^2$. Use the LL scales to evaluate $5.9^{4.5}$ and then square the results in the usual way.

(ii) $36.4^5 = (3.64 \text{ x } 10)^5 = 3.64^5 \text{ x } 10^5$.

(iii) $1.52^{29} = 1.52^{19} \text{ x } 1.52^{10}$. Evaluate each separately and multiply together in the usual way or $1.52^{29} = 1.52^{14.5} \text{ x } 1.52^{14.5} = (1.52^{14.5})^2$

(iv) $163^{13.8} = (1.63 \text{ x } 10^2)^{13.8} = 1.63^{13.8} \text{ x } 10^{27.6} = 1.63^{13.8} \text{ x } 10^{0.6} \text{ x } 10^{27}$. Evaluate the first term using the LL scales. To find the second term, use the L scale as $10^{0.6} = x$ is equivalent to $\log_{10}x = 0.6$ (i.e. find 0.6 on the LL scale and read off 3.98 on the D scale (or W'_2 scale for the 2/83N) as $10^{0.6}$)

(v) $0.0021^{5.7} = (.21 \text{ x } 10^{-2})^{5.7} = 0.21^{5.7} \text{ x } 10^{-11.4} = 0.21^{5.7} \text{ x } 10^{0.6} \text{ x } 10^{-12}$
Again, we evaluate the first two terms as in (iv) above.

Exercise 19(c)

(i)	$3.7^{6.1} =$	(iii)	$3.7^{-0.61} =$	(v)	$0.179^{4.3} =$
(ii)	$3.7^{0.61} =$	(iv)	$0.625^3 =$	(vi)	$0.02^{1.7} =$

(vii) $79^2 =$
(viii) $70^{0.14} =$
(ix) $79^{-0.002} =$
(x) $4{,}200^{0.003} =$
(xi) $0.9154.2 =$
(xii) $22^{-1.3} =$
(xiii) $44^{-0.75} =$
(xiv) $1.01^{70} =$
(xv) $0.98^{34} =$
(xvi) $14^{22} =$
(xvii) $e^{14} =$
(xviii) $1{,}100^{-4.8} =$
(xix) $0.0057^{8.3} =$
(xx) $0.042^{-19} =$

19.4 Miscellaneous Powers and Roots of Positive Numbers

Expressing $\sqrt[N]{a} = a^{\frac{1}{N}}$ we can the LL scale to obtain the root of any positive number. As previously with "powers" in 19.3, for 'a' on an LL scale the D scale gives 'ln a'. If we divide this value of 'ln a' by 'N', we then obtain '$\frac{1}{N}\ln a$' (i.e. $a^{\frac{1}{N}} = \ln\sqrt[N]{a}$) on the D scale. We must note the location of the decimal point for '$\frac{1}{N}$' so as to decide on which LL scale the value of '$\sqrt[N]{a}$' will be found. This is done according to the same rules used in 19.3.

Example 1 Find $\sqrt[3]{16}$ and $\sqrt[30]{16}$

1. Set the hair line over 16 on the LL_3 scale.
2. Place the 3 of the C scale under the hair line.
3. Reset the hair line over the right index of the C scale.
   (Note: $\frac{1}{3} \approx 0.33$ and $\frac{1}{30} \approx 0.033$, thus we will find $\sqrt[3]{16}$ on the LL_2 scale and $\sqrt[30]{16}$ on the LL_1 scale.)
   Under the hair line read off –
4. 2.52 on the LL_2 scale as the value for $\sqrt[3]{16}$.
5. 1.0968 on the LL_1 scale as the value for $\sqrt[30]{16}$.

Example 2: Find $\sqrt[5]{420}$ and $\frac{1}{\sqrt[5]{420}}$

1. Set the hair line over 420 on the LL_3 scale.
2. Place the 5 of the C scale under the hair line.
3. Reset the hair line over the left index of the C scale.
   Under the hair line read off –
4. 3.35 on the LL_3 scale as the value for $\sqrt[5]{420}$.
5. 0.298 on the LL_{03} scale as the value for $\frac{1}{\sqrt[5]{420}}$.

Note: If we use the right index of the C scale when we divide by 'N' for 1 < N < 10 the value for '$\sqrt[N]{a}$' will be found on the LL scale below the original LL scale on which 'a' was located. While, if 10 < N < 100 the value for '$\sqrt[N]{a}$' will be found on the second LL scale below the original one.

If we use the left index of the C scale when we divide by 'N', for 1 < N < 10 the value for '$\sqrt[N]{a}$' will be located on the same LL scale on which 'a' was located. While, if 10 < N < 100 the value for '$\sqrt[N]{a}$' will be found on the LL scale below the original one.
Some special powers are given in the following table.

On appropriate LL scale Set the H.L over -	Under the H.L place	Reset H.L. over	On appropriate LL scale under H.L answer

a	Index of CI scale	N on the CI scale	$a^{\frac{1}{n}}$
a	Index B	N B	$a^{\sqrt{N}}$
a	Index BI	N BI	$a^{\frac{1}{\sqrt{N}}}$
a	Index K'	N K'	$a^{\sqrt[3]{N}}$
a	Index W	N W	a^{N^2}
a	Index C	N CF	$a^{\frac{N}{\pi}}$
a	Index CF	N C	$a^{n\pi}$
a	N CI	M C	a^{NM}
a	M C	N C	$a^{\frac{N}{M}}$
a	N C	M CI	$a^{\frac{1}{NM}}$
a	Index C	θ S'	$a^{\sin\theta}$

Exercise 19(d)

(i) $\sqrt[4]{280} =$

(ii) $\sqrt[7]{1.85} =$

(iii) $\sqrt[9]{0.64} =$

(iv) $\sqrt[3]{7.8} =$

(v) $\sqrt[14]{0.78} =$

(vi) $\frac{1}{\sqrt[5]{33}} =$

(vii) $3.2^{\frac{1}{5.3}} =$

(viii) $1.26^{\sqrt{13}} =$

(ix) $52^{\sin 53} =$

(x) $0.75^{\frac{1}{\sqrt{0.9}}} =$

(xi) $4^{\sqrt[3]{30}} =$

(xii) $0.96^{4.1^2} =$

(xiii) $1.05^{3\pi} =$

(xiv) $121^{\frac{1}{6\times 2.4}} =$

(xv) $2{,}100^{\frac{0.1}{13}} =$

(xvi) $0.7^{3.6\times 4.1} =$

19.5 Logarithms To Any Base and Solving Exponential Equaltions

A. Logarithm

The LL scales can be used to obtain logarithms to any base, by placing the left or right index of the C scale over the base as found on the LL scale. Then for any number on an LL scale its logarithm to the chosen base is read off the C scale.

Example 1: $\text{Log}_6 23.5=1.765$

1. Set the hair line over 6 on the LL_3 scale.
2. Place the left index of the C scale under the hair line.
3. Reset the hair line over 23.5 on the LL_3 scale.
4. Under the hair line read off 1.765 on the C scale as the answer.

Note:

(a) We could have used the CF scale above, by placing the index of the CF scale under the hair line in step 2, and thus reading 1.765 off the CF scale in step 4.

(b) For logarithms of other numbers to base 6, leave the slide as positioned in step 2 and reset the hair line over the number on an LL scale. (e.g. $\log_6 2 = 0.3875$ as 2 on the LL_2 scale give 0.3875 on the C scale.)

Example 2: Find $\log_2 1.4$, $\text{Log}_2 23$ and $\log_2 650$.

1. Set the hair line over 2 on the LL_2 scale.
2. Place the right index of the C scale under the hair line.
3. Reset the hair line over 1.4 on the LL_2 scale and read off 0.485 on the C scale as the value for $\log_2 1.4$.

4. Reset the hair line over 23 on the LL_3 scale and read off 4.52 on the C scale as the value for $\log_2 23$.
5. Reset the hair line over 650 on the LL_3 scale and read off 9.34 on the C scale as the value for $\log_2 650$.

B. Solving exponential equations

Because $\text{Log}_b N = x$ is equivalent to $N = b^x$ the above problems are the same as solving an exponential equation for an unknown power. Example 1 could have been stated – Find x for 6x = 23.5. Thus to solve for an unknown power or exponent we could either express the equation in logarithmic form or leaving it as an exponential equation proceed as follows for $N = b^x$.

1. Set the hair line over the base 'b' on the appropriate LL scale.
2. Place the left or right index of the C scale under the hair line.
3. Reset the hair line over the number 'N' on its appropriate LL scale.
4. Under the hair line read off the value for x on the C scale and locate the decimal point according to the LL scales used.

Note:

(a) To solve the equation $b^{\frac{1}{x}} = N$ for x, we follow the same method as outlined above, except in step 4 the value of x is read off the CI scale.

(b) To solve the equation $b^{kx} = N$ for x, the following method can be used –

Example: $e^{1.4x} = 9$

1. Set the hair line over e on the LL_3 scale.
2. Place the 1.4 of the CIF scale under the hair line. (We can also use CI scale here if suitable).
3. Reset the hair line over 9 on the LL_3 scale.
4. Under the hair line read off 1.57 on the CF scale as the value for x.
   (When the CI scale is used in 2, the answer in 4 will be on the C scale.)

Exercise 19(e)

(i) $\log_5 1.9 =$
(ii) $\log_8 24 =$
(iii) $\log_2 57 =$
(iv) $\log_3 0.53 =$

(v) $\log_{1.5} 2 =$
(vi) $\log_{15} 1.3 =$
(vii) $\log_5 260 =$
(viii) $\log_{2.5} 17 =$

Find x in the Following:

(ix) $5^x = 30$
(x) $12^x = 76$
(xi) $15^x = 3.5$
(xii) $456^x = 21$
(xiii) $0.3^x = 0.95$
(xiv) $0.16^x = 0.045$

(xv) $2.5^{\frac{1}{x}} = 1.02$
(xvi) $16e^x = 48$
(xvii) $e^{\pi x} = 64$
(xviii) $e^{-3x} = 0.45$

Chapter 20 – Root (W) Scales

20.1 The Form of the W Scales.

The Faber-Castell Slide Rules 2/83N and 62/83, incorporate these scales. In the case of 2/83N, the W scales are actually 50cm. "C and D scales" cut in hat at $\sqrt{10}$ (i.e. 3.162) and fitted on a 25 cm (10 inch) Slide Rule. For the 62/83 they are 25 cm. "C and D scales" cut in half and fitted on a 12.5 cm (5 inch) slide rule. For the 2/83N, the W scale gives approximately four significant figures of accuracy, wile the 62/83 can be read to about three figures, thus making calculations with the W scales on the 12.5 cm (5 inch) Slide Rule as accurate as the normal C and D scales on a 25 cm (10 inch) Slide Rule.

20.2 Multiplication

The procedure differs somewhat from that used with the C and D scales. Depending on the numbers involved we have to use either the left (1) or right (10) black index marks, or one of the red index marks located on the right hand end of the W'_1 and W_1 scales, or the left hand end of the W_2 and W'_2 scales. The following examples show each of the four possible variations.

Example 1: 1.2 x 21 = 25.2

1. Place the left (black) index of the W'_1 scale over the 1.2 on the W_1 scale.
2. Set the hair line over 21 on the W'_1 scale.
3. Under the hair line read off 25.2 on the W_1 scale as the answer.
   (Note, under the hair line 6.64 (at 4) on the W'_2 scale gives 1.2 x 6.64 = 7.97 (at 5) on the W_2 scale.)

Example 2: 8.86 x 19.3 = 171

1. Place the right (black) index of the W'_2 scale over 8.86 on the W_2 scale.
2. Set the hair line over 19.3 on the W'_1 scale.
3. Under the hair line read off 171 on the W_1 scale as the answer.
   (Note, under the hair line 6.1 (at 4) on the W'_2 scale gives 8.86 x 6.1 = 54.1 (at 5) on the W_2 scale).

Example 3: 2.37 x 6.85 = 16.25

1. Place the right (red) index of the W'_1 scale over 2.37 on the W_1 scale.
2. Set the hair line over 6.85 on the W'_2 scale.
3. Under the hair line read off 16.25 on the W_1 scale as the answer.
   (Note, under the hair line 21.65 (at 4) on the W'_1 scale gives 2.37 x 21.65 =51.4 (at 5) on the W_2 scale.)

Example 4: 39.9 x 50.6 = 2020

1. Place the left (red) index of the W'_2 scale over 39.9 on the W_2 scale.
2. Set the hair line over 50.6 on the W'_2 scale.
3. Under the hair line read off 2020 on the W_1 scale as the answer.
   (Note, under the hair line 1.6 (at 4) on the W'_1 scale gives 39.9 x 1.6 = 63.9) (at 5) on the W_2 scale.)

The following table summarizes these procedures for easy reference.

20.3 Rules for Division

(i) When the numbers involved in the division are located on adjacent scales, the answer is read off the W_1 or W_2 scales under either of the black index marks.

(ii) When the numbers involved in the division are located on scales on opposite sides of the rule, the answer is read off the W_1 or W_2 scales under either of the red index marks.

Exercise 20(b)

(i) 360 ÷ 18 =

(ii) 4,800 ÷ 0.6 =

(iii) 12.25 ÷35 =

(iv) 43.75 ÷ 0.0304 =

(v) 3,025 ÷ 55 =

(vi) 1,925 ÷ 17.5 =

(vii) π ÷ 6 =

(viii) 93 ÷ 9,600 =

(ix) $\dfrac{219}{17 \times 28}$

(x) $\dfrac{35}{0.12 \times 0.47}$

(xi) $\dfrac{805}{(104 \times 0.043)}$

(xii) $1{,}406 \div 52^2 =$

20.4 Squares and Square Roots

A. Squares

For numbers on the W'$_1$ and W'$_2$ scales, their squares are found on the C scale. We use the W'$_1$ and W'$_2$ scales in preference to the W_1 and W_2 scales, as the former are on the slide with the C scale.

Example: $2.46^2 = 6.05$

1. Set the hair line over 2.46 on the W'$_1$ scale.
2. Under the hair line read off 6.05 on the C scale as the answer.

Note: In Fig 20.7, under the hair line the 7.78 9at 3) on the W'$_2$ scale would be read off the scale (a 1) as $7.78^2 = 60.5$.

Exercise 20(c)

(i) $7.65^2 =$

(ii) $0.9^2 =$

(iii) $65^2 =$

(iv) $207^2 =$

(v) $0.084^2 =$

(vi) $0.00022^2 =$

(vii) $30.25^2 =$

(viii) $(5.4 \text{ x } 10^3)^2 =$

B. Square Roots

For a number on the C scale, its square root is read off the W'$_1$ scale if the number is between 1 and 10, or off the W'$_2$ scale if the number is between 10 and 100. For numbers larger than 100 or less than 1, we use the procedure as outlined in Unit 5.

Example 1: $\sqrt{13} = 3.605$

1. Set the hair line over 13 on the C scale. (as the number is between 10 and 100 we find its square root on the W'$_2$ scale.)
2. Under the hair line read off 3.605 on the W'$_2$ scale as the answer.

Example 2: $\sqrt{130} = 11.4$

1. Set the hair line over 130 on the C scale. (Express $\sqrt{130} = \sqrt{1.3 \times 100} = \sqrt{1.3} \times 10$, thus we find $\sqrt{1.3}$).
2. Under the hair line read off 1.14 on the W'$_1$ scale as the value for $\sqrt{1.3}$.

$\therefore$ answer = 1.14 x 10

= 11.4

Example 3: $\sqrt{0.098} = 0.313$

1. Set the hair line over 9.8 on the C scale. (Express $\sqrt{0.098} = \sqrt{\dfrac{9.8}{100}} = \dfrac{\sqrt{9.8}}{10}$, thus we find $\sqrt{9.8}$).
2. Under the hair line read off 3.13 on the W'$_1$ Scale as the value for $\sqrt{9.8}$.

$\therefore$ answer = $\dfrac{3.13}{10}$

= 0.313

Exercise 20(d)

(i)	$\sqrt{6.25}$	(vi)	$\sqrt{2{,}450}$
(ii)	$\sqrt{93.5}$	(vii)	$\sqrt{0.06}$
(iii)	$\sqrt{1.125}$	(viii)	$\sqrt{0.143}$
(iv)	$\sqrt{324}$	(ix)	$\sqrt{0.4}$
(v)	$\sqrt{960}$	(x)	$\sqrt{0.0025}$

20.5 Miscellaneous Calculations

The following tables list a number of calculations which make use of the W scales, and form a supplement to the table given in 10.3. For squares and square roots the appropriate W scale must be used according to the numbers involved.

Example	Set the H.L over	Under the H.L Place	Reset the H.L over	Under the H.L. answer
a^2b	a on W scale	Index of W' scale	B on the C scale	on D scale.
ab^2	a D	Index C	b W'	D
ab^4	b W	a CI	b W'	D
a^2b^2	a W	Index W'	a W'	D
a^2b^2c	a W	c CI	b W'	D
$\frac{a}{b^2}$	a D	b W'	Index W'	D
$\frac{a^2}{b^3}$	a W	b W'	b CI	D
$\frac{a^2}{b}$	a W	b C	Index C	D
$\frac{a^3}{b}$	a W	b C	a C	D
$\frac{1}{ab^2}$	a DI	b W'	Index W'	D
$\frac{1}{a^2b^2}$	a W	Index W'	b W'	DI
$\frac{1}{a^4}$	Index W	a W'	Index W'	A
$\frac{a^2b}{c}$	a W	c C	b C	D
$\frac{a^2b}{c^2}$	a W	c W'	b C	D
$\frac{ab}{c^2}$	a D	c W'	b C	D
$a\sqrt{b}$	a W	Index W'	b C	W
$\sqrt{ab}$	a D	Index C	b C	W
$\sqrt{abc}$	a D	b CI	b C	W

$a^2\sqrt{b}$	a W	Index W’	b B	D
$\sqrt{\dfrac{a}{b}}$	a D	b C	Index C	W
$\dfrac{\sqrt{a}}{b}$	a D	b W’	Index W’	W
$\dfrac{a}{\sqrt{b}}$	a W	b C	Index W’	W
$\sqrt{\dfrac{ab}{c}}$	a D	c C	b C	W
$\sqrt{\dfrac{a}{bc}}$	a D	b C	c CI	W
$\dfrac{1}{\sqrt{ab}}$	a DI	Index CI	b CI	W
$\dfrac{1}{a\sqrt{b}}$	Index W	a W’	b CI	W
$a\sqrt{bc}$	b D	c CI	a W’	W
$a\sqrt{\dfrac{b}{c}}$	b D	c C	a W’	W
$\pi r^2 h$	r W	h CI	Index C	DF
$\dfrac{1}{2}r^2\theta$	θ (degrees) ST θ (degrees) D	2 C 2 C	r W’ r W’	D D

Chapter 21 – Appendix

21.1 Hyperbolic Functions
The hyperbolic functions are defined (for x in radians) as follows –

$$\sinh x = \frac{1}{2}(e^x - e^{-x})$$

$$\cosh x = \frac{1}{2}(e^x + e^{-x})$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$$

(Note, for x > 3, sinh x $\approx$ cosh x $\approx \frac{e^x}{2}$)

These can be obtained by calculating the required e^x, e^{-x}, etc. with the LL scales and substituting into the above formulae. If x is given in degrees, it must first be coverted to radians.

Exercise 21(a)
- (i) sinh 1.7 =
- (ii) cosh 54° =
- (iii) tanh 30° =
- (iv) tanh 1.35 =

21.2 Cursor Lines
These vary for different Slide Rules. Most Slide Rules have marks for converting kW to HP. This is done by setting the kW line over the given value and reading the corresponding HP under its line, or vise versa. The decimal point is lovated by remembering 0.746 kW = 1HP.

Many Slide Rules have a line labelled ‘d’ over the C and D scales and a secong labelled ‘S’ over the A and B scales. (or ‘d’ could be over W scales and ‘S’ over C and D scales). This allows us to set the ‘d’ line over the value for the diameter of a circle and under the ‘S’ line we read off directly the area of the cirlce.
For any other marks and lines, see the instruction leaflet with your Slide Rule.

21.3 Reciprocals
The C and D scales or the W scales can be used by themselves to obtain reciprocals.
Example 1: ¼ = 0.25
1. Place the left index of the C sclae over 4 on the D scale.
2. Above the right index of the D sclae read off 0.25 on the C scale as the answer.

Example 2: ½ = 0.5
1. Place the left (black) index of the W'_1 scale over 2 on the W_1 scale.
2. Below the right (black) index of the W_2 scale read off 0.5 on the W'_2 scale.

Note:
- (a) The latter example using the W scales affords the most accurate method for obtaining reciprocals. Combing this with the LL scales method to obtain the decimal point (as outlined in 19.1), we have the ideal way of finding a reciprocal with the Slide Rule.
- (b) Sometimes in dividing quantities (e.g. a÷b) it is more convient to do the division upside down (e.g. b÷a), the latter being read off the D scale, usually under the index of the C scale. Thus to obtain the reciprocal, which is the desired value we read it off the C scale above the index of the D scale. A typical example would be if we

required $\dfrac{a}{\sin\theta}$ or $\dfrac{a}{\tan\theta}$ when solving a right triangle. It would be much eiser to find $\dfrac{\sin\theta}{a}$ or $\dfrac{\tan\theta}{a}$ if the trignometrical scales are on the body of the Slide Rule.

Example: $\dfrac{8.75}{\tan 35°}$ = 12.5

1. Set the hair line over 35° on the T^1 scale.
2. Place the 8.75 of the C scale under the hair line.
3. Above the left index of the D scale read off 12.5 on the C scale as the value for $\dfrac{8.75}{\tan 35°}$

Exercise 21(b)

(i) $\dfrac{3.6}{\sin 21°}$

(ii) $\dfrac{19.4}{\tan 58.5°}$

(iii) $\dfrac{26.3}{\cos 72°}$

(iv) $\dfrac{23.6}{\sqrt[3]{335}}$

21.4 Addition and Subtraction

A. a $\pm$ b.

This can be calculated by noting the relationship $a \pm b = b\left(\dfrac{a}{b} \pm 1\right)$ and using the following procedures.

1. Set the hair line over ‘a’ on the D scale.
2. Place the ‘b’ on the C scale under the hair line.
3. Below the index of the C scale we have the value for $\dfrac{'a'}{b}$ on the D scale.
4. Reset the hair line over $\left(\dfrac{a}{b} \pm 1\right)$ on the D scale.
5. Place the ‘b’ fo the CI scale under the hair line.
6. Below the index of C scale read off $a \pm b = b\left(\dfrac{a}{b} \pm 1\right)$ on the D scale as the answer.

B. $a^2 \pm b^2$

This can be calculated by noting the relationship $a^2 \pm b^2 = b^2\left(\dfrac{a^2}{b^2} \pm 1\right)$ and using the following procedure.

1. Set the hair line over ‘a’ on the D scale.
2. Place the ‘b’ of the C scale under the hair line.
3. Above the index of the B sclae we have the value for $\dfrac{a^2}{b^2}$ on the A scale.
4. Reset the hair line over $\left(\dfrac{a^2}{b^2} \pm 1\right)$ on the A scale.
5. Place the ‘b’ of the CI scale under the hair line.

6. Above the index of the B scale read off $a^2 \pm b^2 = b^2\left(\dfrac{a^2}{b^2} \pm 1\right)$ on the A scale as the answer.

(Note, this method can be used to solve a right angle triangle using the Theorem of Pathagoras.)

This can be calculated by noting the realationship $\sqrt{a} \pm \sqrt{b} = \sqrt{b}\left(\dfrac{\sqrt{a}}{\sqrt{b}} \pm 1\right)$ and using the following procedure.

1. Set the hair line over 'a' on the A scale.
2. Place the 'b' of the B scale under the hair line.
3. Below the index of C scale we have the value $\dfrac{\sqrt{a}}{\sqrt{b}}$ on the D scale.
4. Reset the hair line over $\left(\dfrac{\sqrt{a}}{\sqrt{b}} \pm 1\right)$ on the D scale.
5. Place the index of the B scale under the hair line.
6. Reset the hair line over 'b' on the B scale.
7. Under the hair line read off $\sqrt{a} \pm \sqrt{b} = \sqrt{b}\left(\dfrac{\sqrt{a}}{\sqrt{b}} \pm 1\right)$ and the D scale as the answer.

Exercise 21(c)

(i) $8.1^2 - 6.3^2$ =

(ii) $\sqrt{29.1} - \sqrt{19.4}$ =

(iii) $45^2 - 31^2$ =

(iv) $\sqrt{65} - \sqrt{49}$ =

21.5 Soltuion of Quadradic Equations

For quadradic equation of the form $ax^2 + bx + c = 0$ we recall that if the roots (or solutions) are p and q then:

$$p + q = -\frac{b}{a}$$

and $$pq = \frac{c}{a}$$

Thus to solve a quadradic equation, it amounts to finding two values whose sum and products are known. We do this by successive trials using a rather novel apporach.

(Note, that if we place the right index of the C scale over, for example 9, on the D scale, the for any setting of the hair line the product of the numbers read off under it on the D and CI scale will always equal 9.)

Example: Solve $x^2 - 6x + 8 = 0$

(Note the product of the roots is 8.)

1. Set the right index of the CI scale over 8 on the D scale.
2. (By trial and error, set the hair line in variouis positions, checking each time whether the sum of the values read off the CI and D scale is 6). The correct setting is shown in Fig 20.3 with the solutions x = 2 and 4.

Note:

(a) It is advisable to tabulate the successive trials.

(b) If the quadradic equation has roots of opposite sings (i.e. $c < 0$) then the roots we are looking for will have a difference of $-\frac{b}{a}$.

Exercise 21(d)
Solving the following for x:

(i) $x^2 + x - 6 = 0$
(ii) $2x^2 + x - 6 = 0$
(iii) $20x^2 – 29x + 6 = 0$
(iv) $2x^2 + 9x - 5 = 0$

21.6 Sine Rule
A. Two Sides and an Angle not Opposite of one of the sides.
Example: Given a = 5.45 cm, b = 4.88 cm and C = 48°.
To find side C and angels A and B.
We have

$$\frac{\sin A}{5.45} = \frac{\sin B}{4.88} = \frac{\sin 48}{c}$$

1. Set the hair line over 48° on the S scale.
2. By trial and error, set an estimate of the value for 'c' on the C scale under the hair line. (If we try c = 4.5 on the C scale, the values of A and B on the S scale are read off oppisite the values a = 5.45 and b = 4.88 respectively on the C scale – i.e. A = 64° and B = 53.7°. We then check A + B + C = 64° + 53.7° + 48° = 165.7°. This is short of 180, so we try another value (say c = 4.3) and repeat, checking the sum of the angles again).
   The correct answer will be found to be, side c = 4.24 and Angle A = 73° and B = 59°.

B. Three Sides.
Example: Given a = 48cm, b = 56.6cm and c = 44.2cm.
To find angles A, B, and C.

$$\frac{\sin A}{48} = \frac{\sin B}{56.6} = \frac{\sin C}{44.2}$$

1. We estimate the value for A (say 60°) and set the hair line over this value on the S scale.
2. Place 48 (the value for 'a') under the hair line, and then find the angles B and C corresponding to b = 56.6 and c = 44.2. Checking A + B + C and if it is not 180° try another value for A and repeat steps 1 and 2 again. The correct answers will be found to be A = 55°, B = 76° and C = 49°.

Exercise 21(e)
Using the Sine Rule to find the remaining sides and angles given:

(i) a = 65 cm, b = 51 cm and c = 45 cm.
(ii) a = 60.3 cm, b = 64 cm and c = 58 cm.
(iii) a = 4.15 cm, b = 4.81 cm and C = 71.6°.
(iv) a = 62.5 cm, b = 98 cm and C = 65.2°.