AIR NAVIGATION

A PRIMER OF AIR NAVIGATION

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PREFACE

PRIOR to the year 1919 there was little occasion for long-distance air navigation: the subject has, however, become one which requires study, and the remarkable trans-atlantic flights of 1919 have shown that the art of sea navigation can with suitable modifications be applied to the air. The purpose of this little book is to show in simple language how this can best be done and how when sea-going methods fail others can be devised to take their place. The net result is that although the long experience and tradition of sea navigators forms the convenient basis on which air pilotage and navigation is founded, the latter have individuality enough to substantiate a claim for separate treatment.

As at sea, so in the air, the navigator's duty is to keep his dead reckoning position and, as opportunity offers, to check this position by such other methods as are available. This D.R. work is equally necessary in Air Pilotage and Air Navigation. The checks in the case of the former are references to maps and the recognition of the country flown over; in the latter they are the methods of directional wireless and astronomical observation.

The author is principally indebted to Flight Lieutenant R. S. Capon, R.A.F., for assistance with this book, especially in the collection, and working out, of the Examples; he is indebted also to Mr. L. C. Bygrave, late Captain R.A.F., and to Mr. C. Hall. Notes on "Compass Deviation," furnished by Mr. G. T. Bennett, F.R.S., of Emmanuel

College, Cambridge, were of great use, though the author naturally takes responsibility for the actual methods suggested in the text.

The author thanks *The Times* for permission to include their maps of the Atlantic and Australia flights: the Hydrographer of the Navy for permission to include the variation chart of Chapter II: and his thanks are due in innumerable cases to the kind assistance of officers of the Royal Air Force and the Air Ministry.

GORING-ON-THAMES,
28 *February*, 1920

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A PRIMER OF
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CHAPTER I

INTRODUCTORY

Navigation—Atlantic flights—Australia flights—Limiting range—Position lines—The sextant—Artificial horizons.

1. Navigation.—Navigation is the art of determining the course and position of any craft whether on the surface of the sea or in the wider ocean of the air. Centuries of experience have taught the sea navigator the quickest and most accurate means of determining the position of his ship; some of these methods can be grafted into air navigation with little change, but others are unsuited to this end, whilst certain methods found inapplicable at sea are of material use in the air.

As has been well expressed by Kelvin, to find a ship's place at sea is a practical application of pure geometry and astronomy. This stands also for aircraft, but addition must now be made of the methods of direction finding wireless telegraphy.

The fundamental and most important method of preserving a record of the craft's position is by means of what is called "dead reckoning." This requires a knowledge of both the direction and the speed of flight. The former may be derived from the compass reading, the latter—so far as still air is concerned—from the air-speed indicator. Hence, in the absence of wind, the navigator's task is simple. In the general case, however, the effect of atmospheric currents, which we recognize as wind, will take the craft far away from the position it would have occupied had there been no wind. Hence the navigator needs to keep a current estimate, to the best of his ability, of the velocity and direction of the wind at the height at which he is travelling and compute his position accordingly. This position he calls his "D.R." (Dead Reckoning) position, and he checks it by means of astronomical observations and by wireless telegraphy.

Pilotage sometimes requires no more of the airman than recognizing the towns, rivers and railway junctions near which he flies. Of this simple form of the art nothing more need be said. Sometimes, however, the land will be obscured by fog or cloud; sometimes the course will lie across the ocean; sometimes across deserts. If the safety of the craft and its crew is to be reasonably assured in each of these cases, provision must be made for navigation by the best methods available.

Fig. 1 is a map prepared by the American journal *Aeronautics*, to show the distances by alternative routes for crossing the Atlantic. It affords an illustration of the need for accurate navigation. Suppose an aircraft to start from Newfoundland on the 1200 miles flight to Flores, in the Azores; an error of no more than 3 degrees in the course made good would lead the craft 60 miles astray by the time the neighbourhood of the island was reached: as will be seen from the map, such an error might lead to consequences very serious to the craft.

This diagram also assists the visualization of the important meteorological fact that an aircraft able to select its route and altitude of flight in such a way as to have a 30-mile-an-hour wind behind it would shorten its journey by 720 miles in each 24 hours of flight. On a transatlantic crossing this would immensely increase the ease and safety of the passage. It appears from a study which has been made of the prevailing Atlantic winds that of the routes shown on the map, the upper route is on the average the more favourable for an eastward crossing, whilst for a favourable westward one a route further south than the

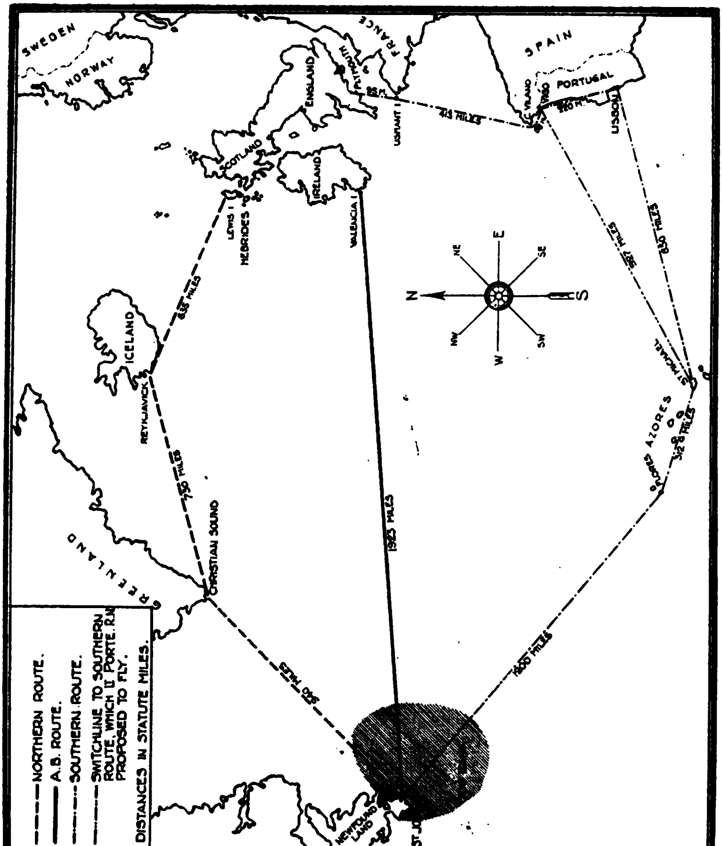
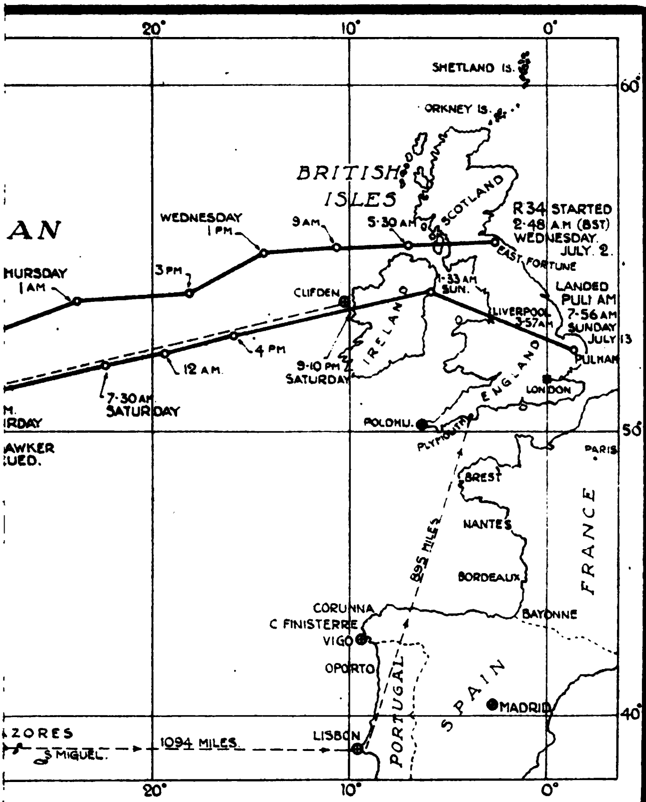


FIG. 1.—Atlantic Air Routes.

most southerly one shown, and therefore much longer, would be necessary. The middle one shown involves a "jump" of nearly 2000 miles, which, without the assistance of a favourable wind, is, as the notable transatlantic flights of 1919 have proved, near the limit of what is feasible for an aeroplane of the present day. It is well within the range, however, of a rigid airship, such as the famous R 34, and it is possible that it would be feasible to build a commercial airship equal to this range even when carrying 12 to 15 tons of cargo. A more speculative suggestion has been made elsewhere of an airship—not of 2,000,000 but 10,000,000 cubic feet capacity—capable of a range of 20,000 miles and having a freight capacity of as much as 200 tons.

2. Atlantic flights.—The summer of 1919 saw four attempts on a flight across the Atlantic. The first craft to make the passage by air was the NC 4, one of a group of four American flying boats; the passage chosen was via the Azores, and several days were occupied by the flight. The total distance was 4513 miles and the actual flying t me 53 hours 31 minutes. Great credit is due to the American Government and its officers for this successful pioneer flight. Almost simultaneously Hawker and Grieve's Sopwith two-seater aeroplane made its plucky attempt, but owing to engine trouble only succeeded in covering half the distance. The first aircraft to make the crossing in a single "non-stop" flight was the Vickers-Vimy manned by Alcock and Brown. The distance covered in its passage from Newfoundland to Ireland was 1890 miles and the time taken 15 hours 57 minutes, giving an average speed of 117 m.p.h. On landing there was still enough petrol in its tanks for a further 800 miles. Shortly afterwards—July, 1919—the large rigid Admiralty airship, R 34, made the direct flight from England to the United States, and after a short stay there made its return passage equally successfully. A map showing the routes taken appears in Fig. 2. The journey was a very remarkable one. The outward journey, delayed by head winds,


20°
10°
0°
60°
50°
40°
SHETLAND IS.
ORKNEY IS.
BRITISH ISLES
SCOTLAND
AN
R 34 STARTED
2·48 A.M (BST)
WEDNESDAY
JULY 2.
EAST FORTUNE
WEDNESDAY
1 PM
9 AM
5·30 AM
HURSDAY
1 AM
3 PM
CLIFDEN
1·33 AM
SUN.
LANDED
PULHAM
7·56 AM
SUNDAY
JULY 13
IRELAND
LIVERPOOL
3·57 AM
4 PM
12 AM
9·10 PM
SATURDAY
PULHAM
ENGLAND
LONDON
7·30 AM
SATURDAY
M.
RDAY
POLDHU.
PLYMOUTH
AWKER
UED.
PARIS
BREST
NANTES
895 MILES
BORDEAUX
FRANCE
CORUNNA
C FINISTERRE
VIGO
BAYONNE
OPORTO
PORTUGAL
SPAIN
MADRID
AZORES
S MIGUEL
1094 MILES
LISBON

ATLANTIC.

few seconds through a hole in the fog and cloudbank. I obtained the drift of the machine by noting the breaking waves through the drift indicator, and we were then at 4000 feet and climbing. The drift was 10 degrees to the right of our course, which I had already allowed for on starting, owing to the north-east wind then blowing from St. John's."

3. Route to Australia.—On December 10th, 1919, a Vickers-Vimy machine which had started from London on November 12th arrived at Port Darwin, Australia. For this splendid pioneer effort great credit is due to the captain of the aircraft—Captain Sir Ross Smith. The total distance of 11,300 miles was covered in two days within the specified period of one month. The route taken and the several stopping-places are shown in Fig. 3. This route will probably prove to be one of the most travelled air-ways of the future.

4. Limiting range of flight.—The limiting range of flight is mainly governed by the fuel supply which can be carried. Obviously this will be increased by adding to the overall efficiency of the engine and propeller. Taking these factors at the best values of the present time, it appears that for heavier-than-air craft, the product of total laden weight, in tons of 2240 lb., by air miles flown per gallon of fuel may be as much as 15. And if the total amount of petrol carried be 40 per cent of the total laden weight, the fuel available will be 900 lb., or 125 gallons, per ton. So if one ton can fly 15 miles on one gallon, on 125 gallons it will fly about 1850 miles; or allowing for the loss of weight as the fuel is used, about 2300 miles, and this is therefore the limit of air mileage. For seaplanes it would be appreciably less.

It is interesting to compare this figure of 15-ton-miles per gallon of fuel with that for road motor vehicles which may be put at 50 for a speed of 20 m.p.h.—if, however, the car ran at 100 m.p.h. the tractive resistance would rise from 70 lb. per ton to about ten times this figure, and the ton mileage coefficient would sink from 50 to 5. Hence an


MONGOLIA
PEKING
VLADIVOSTOK
TOKIO
CHINA
PACIFIC
OCEAN
NOV 27
NOV 28
CALCUTTA
NOV 28
NOV 29
AKYAB
NOV 29
NOV 30
RANGOON
HONG KONG
PHILIPPINE IS
BANGKOK
8038 M
DEC 1
DEC 2
SINGORA
DEC 2
DEC 4
PENANG
SUMATRA
SINGAPORE
BORNEO
BANKA
DEC 4
NEW GUINEA
DEC 6
KALIDJATTI
SAMARANG
SURABAYA
BALI
LOMBOK
BIMA
REO
LARANTUKA
ATAMBOEA
DEC 8
PORT DARWIN
11.294 M.
CAPT ROSS-SMITH
ARRIVED AT
PORT DARWIN · DEC 10.
AUSTRALIA

IGHT

aeroplane is three times as efficient as a " distance coverer " for high speed work. The two would be equally efficient at about 60 m.p.h., the aeroplane being ahead at all higher speeds.

FIG. 3A.

5. Sea craftsmanship.—In the early days of sailing ships a knowledge of the surface winds in all parts of the navigable oceans was of first importance. Such knowledge was entirely empirical, and those who possessed it thought little indeed about such effects as surface friction and still less of any influence of the weather conditions tens of thousands of feet above them. Coupled with an empirical knowledge of the average behaviour of surface wind was an empirical knowledge of the ocean tides.

With the advent of the higher speeds of steam-driven ships the tides became of less importance as a factor deflecting the ship from its course, although the greater momentum due to swifter motion made the penalty of misjudgment very much heavier.

Aircraft are concerned with another ocean—that of the air itself. Its tides are far less easy to predict than the tides in the sea and have far, far higher velocity. Its tides are the winds themselves, and their intensity may range to 100 miles an hour or even more. Here there are no permanent tide charts to guide the would-be navigator, however pressing his need. Long experience of surface winds had enabled his marine predecessor to form some rough judgments of the ways of such winds, thus Conrad tells :—

"The narrow seas around these isles, where British admirals keep watch and ward upon the marches of the Atlantic Ocean, are subject to the turbulent sway of the west wind. Call it north-west or south-west, it is all one—a different phase of the same character, a changed expression on the same face. In the orientation of the winds that rule the seas, the north and the south directions are of no importance. The north and the south winds are but small princes in the dynasties that make peace and war upon the sea. They never assert themselves upon a vast stage. In the polity of winds, as amongst the tribes of the earth, the real struggle lies between east and west. The end of the day is the time to gaze at the kingly face of the westerly weather, who is the arbiter of ships' destinies. Benignant and splendid, or splendid and sinister, the western sky reflects the hidden purposes of the royal mind. Clothed in a mantle of dazzling gold or draped in rags of black clouds like a beggar, the might of the westerly wind sits enthroned upon the western horizon with the whole North Atlantic as a footstool for his feet and the first twinkling stars making a diadem for his brow."

Not less impressive to the airman are the majestic turbulences of the atmospheric ocean in which he moves; and still more powerful are they in determining the motions of his craft.

6. Position lines.—A navigator's effort is constantly directed to locating on a chart the position of his craft. If he can draw a straight line such that he is certain his position is somewhere along it he has made a first step towards its determination. If he can draw two such lines, then his position is completely determined by their point of intersection. These are called "position lines." There are two ways of drawing them on the chart, the older method—Sumner's—is by means of measurements of the altitude of heavenly bodies, the newer is by means of wireless telegraphy. Both are equally available and equally important for air navigation.

The sphere of the earth is shown in profile in Fig. 4; at point A some heavenly body is seen to be at a certain distance—say 40 degrees—from the zenith. Now another point B on the same meridian can be found at which this same heavenly body would also be 40 degrees from the zenith, but on the other side of it; the locus of all such points as A or B on the surface of the sphere is a circle as shown in perspective in Fig. 5. If the zenith distance of another star be measured at the same time, we get the second circle shown by dotted lines. It follows that the observer's position must therefore have been at one of the two points at which these circles intersect,

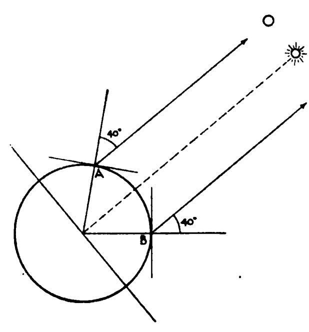


FIG. 4.

It is not usual, of course, to include the whole earth on any one chart, and only the part of immediate interest is so shown, hence the whole circumference of these circles would not appear, but a portion only; and such sectors can be approximately represented by short straight lines, which strictly are the tangents to the circles. Thus in

FIG. 5.

Fig. 6 the two position lines are A B and C D, and the observer's position is at E. These two lines could equally have been obtained by astronomical measurements or by wireless telegraphy—or the first line by one method and the second by the other.

To obtain them by astronomical means requires the use of an instrument called a sextant.

7. The Sextant.—This important instrument, invented by Sir Isaac Newton, was first made by Hadley about the year 1730.

Let there be a frame on which are mounted two small mirrors, B and C, as in Fig. 7. An eye placed at E and looking in the direction of the arrow would see a distant object S. The angle of elevation of S above the line B E is evidently the angle S F B.

Now this construction has the remarkable property that the eye at E will still see S in the same apparent direction even if the wooden base is turned clock-wise, or counter clock-wise, in the plane of the paper. This property is of the greatest value, since it means that once

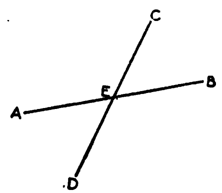


FIG. 6.

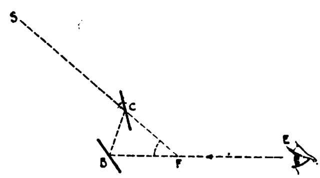


FIG. 7.

it is incorporated in an instrument the rolling or pitching oscillations of the craft will not require the direction in which the eye is looking to alter. If this is important at sea it is still more important in the air.

For use at sea the mirror B is only half silvered, and the sea horizon is seen through the unsilvered half; the mirror C is then turned on its mounting until the heavenly body appears to coincide with the sea horizon, when the angle S F B at once gives the angular elevation above the horizon, and this angle is known as the "altitude" of the heavenly body. A small correction has to be made for the dip of the horizon due to the deck of the ship being above the sea level.

Experiments with this system of mirrors will show that an extra 10 degrees of elevation corresponds to moving the mirror C by 5 degrees, and the scale of angles attached to the frame of the instrument is graduated accordingly.

8. Artificial horizons.—Sometimes the horizon cannot be seen distinctly or at all. Then to ensure that the line E B is horizontal a pendulum or spirit level can be incorporated, but this brings in the acceleration errors discussed later. Another way is to view a second image of the same star through the mirror B by looking down at the still surface of some still mercury or other level reflecting surface. This gives the effect shown in Fig. 8, where M is the reflecting surface. It is easy to see that the angle of elevation S E H, which it is wished to ascertain, is exactly half the angle S E M which the sextant will read. Hence in this case we halve the sextant reading.

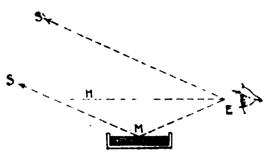


FIG. 8.—Artificial horizon.

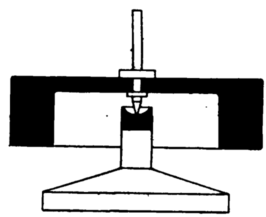


FIG. 9.—Gyrostatic horizon.

In an aircraft there is too much vibration for a mercury surface to remain free from ripples, so instead it is better to make use of another sea navigator's device—the flat-surfaced spinning-top, known as the "Selsen" or "Fleuriais" horizon. See Fig. 9. In its simplest construction it is spun by the fingers, but in more complete designs it is spun by air blast, by springs, or by clockwork. It is necessary that the centre of gravity should be close to the pivot or else the effect of acceleration will be to cause it to make a slow wobble, known as "precession," and so upset the observations.

9. Adaptation to air use.—For air use it is possible to employ an ordinary sextant and make use of the true horizon, if it is visible, or in the alternative of a cloud or haze horizon. The latter alternative is usually forced on the navigator unless flying at a low height, but has the disadvantage that the height of the cloud or haze horizon at a distance has to be guessed, and may be guessed wrong. A method has been suggested by which the opposite horizons are simultaneously observed by suitable optical means and the position half-way between taken to give the true vertical, but this amounts to assuming that the two haze horizons are at the same height (which may or may not be true) and that the navigator's position in the aircraft is such as to give him a clear view in two specific but opposite directions (which is not always the case).

Probably the best way of all of making sextant observations in the air is to use an artificial horizon method, either by incorporating some pendulum or bubble device in the sextant or by using a gyrostatic top of the Selsen, Fleuriais, or other type.

The method by which the altitudes so measured are used to draw position lines on the chart will be described later.

CHAPTER II

GENERAL PRINCIPLES

Shape of earth—Latitude and longitude—Definitions—Maps and charts —Contrast between sea and air conditions—Effect of acceleration on a pendulum—Resonance in vibration—Magnetic Compass—Variation—Deviation—Swinging—Types of compass—Gyrostats.

10. Shape of Earth.—The earth is very nearly an exact sphere of 8000 miles diameter. A model to scale of 8 inches in diameter, i.e. 1000 miles to the inch, would appear completely spherical and completely smooth, the highest mountains would only project 0·005 inch, an amount about equal to the thickness of the paper on which this book is printed. The oceans would be represented by a water film no thicker than this same amount, which on an 8-inch globe might be noticed as a slight dampness. The chief difference between the shape of the earth and an exact sphere is the bulge at the Equator due to the action of centrifugal force in distant ages when the earth was still plastic.

For the purposes of air navigation any small departures from the exact spherical form may be neglected.

11. Latitude and longitude.—Points on a sphere are most easily defined as so many degrees north or south of the Equator (or if it be preferred, so many degrees north of the South Pole), and so many degrees east or west of some definite place, usually Greenwich. Thus in Fig. 10 the horizontal bands are lines of latitude and the lines reaching from pole to pole are lines of longitude. It is useful to remember that the metre was first defined as a distance such that 10,000,000 of them would stretch from the Equator to the Pole, so that the earth's circumference would be 40,000 kilometres or about 25,000 miles.

Latitude is measured in *degrees* north or south of the Equator, and longitude either in degrees or in hours (24 to the complete circle) east or west of Greenwich.

Celestial Sphere. The position of heavenly bodies is

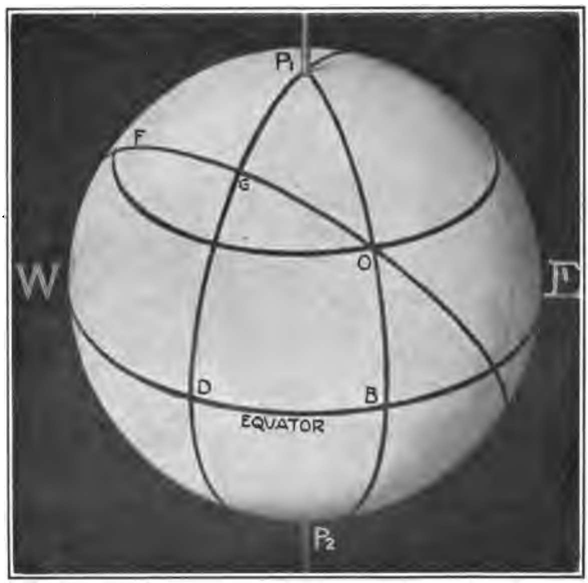


FIG. 10.—Globe of Earth.

P_1, P_2, poles; P_1 G D Greenwich meridian; D B longitude of place O; O B latitude of place O; O G F great circle running through place and Greenwich.

defined in a very similar way, and the following parallelism is easily remembered :—

Terrestrial Sphere.	Celestial Sphere.	Observer's Sphere.
Latitude N. or S.	= Declination N. or S.	= Altitude.
Longitude	= Right Ascension	= Azimuth.
Greenwich, the zero for longitude	= First point of Aries, the zero for Right Ascension	= Zero azimuth.

12. Meridian of a place.—The meridian of a place is the vertical plane which passes through the north and south points.

13. Prime vertical.—The prime vertical is the vertical plane which runs through the east and west points.

14. Solar day.—A solar day is the mean interval between two successive passages of the sun across the meridian. The day is divided into 24 hours.

15. Sidereal day.—A sidereal day is the interval between two successive passages of the same star across the meridian. This period of time is divided into 24 sidereal hours and is shorter than the solar day by four minutes, since 365 solar days equals 366 sidereal days.

16. Hour angle.—The hour angle of a heavenly body is measured by the number of hours since it crossed (or before it will cross) the meridian. It is measured from 0 to 180 degrees E. or W.

17. Azimuth.—The azimuth is the angle between the vertical plane containing the observed object and the meridian. It is usually reckoned as so many degrees east or west of south.

18. Names of bearings.—The names of bearings are as shown in Fig. 11. Each is separated by 45 degrees from its neighbour.

19. Mean Sun.—Partly owing to the earth's orbit being an ellipse, and partly owing to the plane in which it moves—the ecliptic—being inclined to the equatorial plane, the sun appears to move around the earth at different rates at different times of the year. This affects the length of the day.

It would be inconvenient to have a day of variable length: so astronomers, for time measuring purposes, have replaced the true sun by a fictitious body, the mean sun, which moves around the earth at a constant rate, viz. the average for the year of the rate of the true sun.

Hence, the true sun does not always cross the meridian exactly at midday. The difference between the time as recorded by the true sun, and the time as recorded by the mean sun, is called the **Equation of Time.**

Hence,

Mean time = Apparent time + equation of time.

20. Greenwich Mean Time (G.M.T.).—G.M.T. is the number of hours since the mean sun crossed the meridian at Greenwich; in other words, the mean sun's hour angle

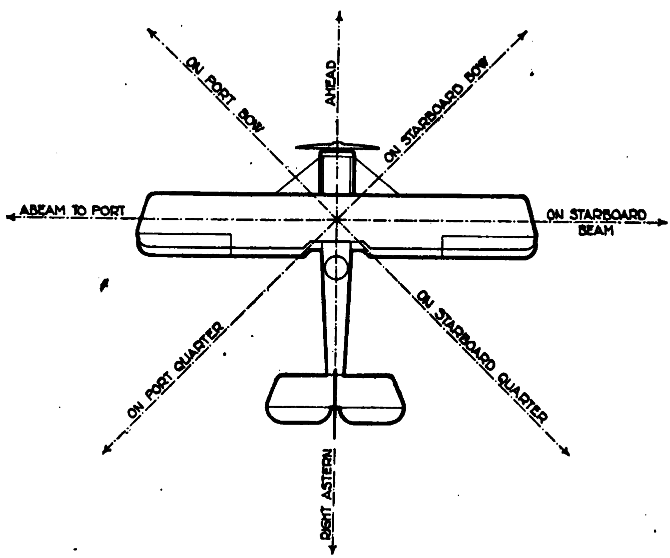


FIG. 11.—Air bearings.

at Greenwich. All long-distance aircraft carry a clock or watch keeping G.M.T.

Local Time. Local time is the number of hours since the mean sun crossed the local meridian. If, for example, local time is found to be 4 hours 30 minutes, and a chronometer keeping G.M.T. shows 9 hours 40 minutes, we know that the mean sun must be higher at the place of the local observation than it is at Greenwich and that the locality must be west of Greenwich, and in longitude 5 hours

10 minutes west, being the difference between the two times.

21. Sidereal Time.—Sidereal time is the number of sidereal hours since the first point of Aries crossed the meridian of the place: a little consideration will show that it is always equal to the Right Ascension (R.A.) of any heavenly body *then* on the meridian of the place. A clock or watch keeping sidereal time is a part of the equipment of all observatories, and of all long-distance aircraft.

It is useful to remember that :—

Hour angle + Sidereal time = Right Ascension.

22. Nautical mile.—A nautical, or geographical mile, is the distance on the earth's surface corresponding to a change of one degree of latitude. It is 6080 feet.

23. Knot.—A knot is a velocity of one nautical mile per hour. This is equal to 1·15 m.p.h., or 1·7 ft. per sec.

24. Astronomical day.—The astronomical day consists of 24 solar hours. At present it is reckoned from midday: so that 9 a.m. on 23 October, 1917, in civil reckoning would be 21 hours on 22 October, 1917, in astronomical reckoning. This, however, will change in 1925, and the astronomical reckoning will be the same as the civil.

25. Difference of latitude.—The difference of latitude (d. lat.) between two places is the angular distance between their respective parallels of latitude.

26. Difference of longitude.—The difference of longitude (d. long.) between two places is the angular distance between the two meridians of the two places.

27. Meridional parts.—Tables are included in many reference books of "meridional parts." These figures show the distance from the Equator to any parallel of latitude on the Mercator chart in terms of minutes of arc at the Equator. Thus the Mercator parts for 1 degree latitude=60; for 10 degrees latitude=603; for 30 degrees latitude=1888; for 45 degrees latitude=3030; for 60 degrees latitude=4527, and so on.

28. Compass Rose.—"Compass Rose" is the name given to the card carried by the sea-going type of compass. It shows the division of the circle into the traditional 32 "points of the compass" (an eight-point turn corresponds to a turn of 90 degrees); it is sometimes graduated in degrees also. For use in the air this division of the circle into "points" is obsolete and the graduations are always in degrees E. of N. up to 360 degrees.

29. Track.—The track is the line on the map or chart along which the aeroplane or airship is travelling.

30. The track angle is the angle made by this line with the direction of north. It is measured from north through east from 0 degrees to 360 degrees (in the same manner as the compass bearings). It may be either "true" or "magnetic."

31. Drift angle is the angle between the fore and aft axis of the aeroplane, or airship, and the track. It is measured in degrees to port or starboard.

32. Bearing.—The bearing of a place B relative to a place A in the angle made by the "rhumb" (see par. 34) line A B with the meridian of longitude. It may be either "true" or "magnetic."

33. Maps and charts.—Maps and charts are diagrams of the earth's surface. They can never be really true to scale, since it is impossible truly to represent a sphere by a plane any more than it is possible to flatten out the cover of a tennis ball without distorting the shape of the outline, or of any lines drawn upon it. A map or chart is therefore some geometrical, mathematical, or conventional way of placing the terrestrial meridians and parallels of latitude on paper with as little distortion as may be. Charts are usually much more distorted than maps, since in the former the great convenience of having meridians and parallels shown as straight lines instead of curves is worth some sacrifice of accuracy in other directions—the fact that on a Mercator chart all *angles* are shown without distortion compensates for Greenland being shown much larger than India, which it is not really.

Maps are often called "projections" as though they were geometrically projected, which they sometimes are but not always. Only those maps which are formed by some geometrical—as distinct from mathematical or conventional—treatment of the sphere are really entitled to be called projections. Thus a photograph of the moon taken from the earth gives a "map" which is a true projection—being what an engineer would term a plan view—but it makes a very bad map, since only the parts near the middle would be true to scale whilst those near the margin would be shown very inaccurately in the radial direction though true to scale circumferentially, so that all circular craters near the moon's limb would be shown elliptical.

The most important projections are :—

(1) Mercator's.
(2) Gnomonic.
(3) Stereographic.
(4) Conical.
(5) Rectangular (Cassini's).

34. Mercator's.—In Mercator's method—which is the basis of almost all charts—all the parallels of latitude are stretched out until they are of the same length as the Equator. This means that the parallel of latitude 60 degrees north or south, is shown twice as long as it should be, and all east and west dimensions in that latitude are consequently shown twice as big as they really are : to balance this east and west distortion Mercator stretched the north and south dimensions in just the same proportions, so that the shape of any small island in latitude 60 degrees would be shown twice as big in both directions, and therefore true to shape.

In a Mercator chart if one inch represents one degree in *longitude*, one degree in latitude has the following lengths :—

In latitude	0 degrees	.. .. ..	1·00 inch.
,,	20 ,,	.. .. ..	1·06 ,,
,,	40 ,,	.. .. ..	1·31 ,,
,,	60 ,,	.. .. ..	2·00 ,,

This preservation of shape is a great advantage, and provided that a scale suitable to the latitude is always used little inconvenience results. Moreover, because the extensions east and west are balanced by an equal degree of extension north and south all bearings are shown correctly. Thus if some small island bore on the chart at 193 degrees from Plymouth, a ship sailing steadily on a 193 degrees bearing would reach the island. The path so taken is called a *rhumb line;* its property is that it cuts all meridians at the same angle, but it is not necessarily the shortest path that could be chosen. The shortest path would be a great circle and this would be shown as a slightly curved line on a Mercator chart. See Fig. 36 in Chapter IV.

Since the length of a latitude circle at latitude l is cos l times the length of the Equator, and since all latitude circles become the same length as the Equator circle on the Mercator map, it follows that at latitude l lengths on the Mercator map are sec. l times too long.

35. Gnomonic.—A gnomonic chart is the shadow picture which would be got on any tangent plane to the earth, if there were imagined to be a bright light at the earth's centre which could cast a shadow of the land surfaces on to this tangent plane. It is useful for charts of harbours and small areas generally.

Since each meridian lies in a plane passing through the earth's centre, the "shadow" of any meridian must lie along the intersection of this plane with the tangent plane on which the map is being made. And since the intersection of any two planes must always be a straight line, therefore all great circles of the earth will be shown as straight lines—and this is the chief virtue of the gnomonic chart. The shortest distance between any two points is always shown as a straight line, and not a curved line as in the case of Mercator charts. The gnomonic is a true projection.

36. Stereographic.—This resembles the gnomonic, but the "light" instead of being at the centre of the sphere is at the other end of the diameter which passes through the tangency point. It is chiefly remarkable because by this method circles on the globe are always represented by circles on the map. It also is a true projection. It is not much used, however.

37. Conical.—These maps are developments of the earth on a conical surface, but are not really projections though at first sight they may seem to be. They are formed by imagining a conical cap fitted on the earth—its axis coinciding with the earth's axis. The angle of the cone is set so that the line of contact of the two surfaces shall be any desired parallel of latitude, choice naturally being made of the latitude which comes about the middle of the desired map. Now unroll the cone until it is flat. The meridians become straight lines running to the apex of the cone and the parallels are circular arcs placed at such distance apart as gives the least distortion of the areas represented. This projection is often used by map makers.

38. Rectangular (Cassini's).—This is a method of plotting on squared paper an area of the globe. The procedure is as follows: Draw the meridian through the centre C of the area on the globe it is desired to reproduce, and through C the other great circle, which cuts the central meridian at right angles. These are represented on the squared paper by two lines at right angles. Call them O y and O x respectively. Then to place any point A on the globe on to the squared paper, draw through A the great circle which cuts the central meridian at right angles at M. Then measure off along O x a length equal to A M, and along O y a length equal to C M, and draw through the points on O x and O y thus obtained lines parallel to O x and O y. The intersection of these lines is the point representing A on the globe.

39. The contrast between sea and air conditions.—Sea navigation differs from air navigation in two chief ways. It is more difficult to make measurements with instruments on aircraft than it is on sea-going vessels, mainly because of the absence of a horizon and of the disturbing effect of acceleration on any pendulum or level carried on an aircraft; but the working out of the position from the observations made is simpler. The greater ease of reducing the observations is due to the less degree of accuracy which air navigation calls for; at sea it has long been customary to aim at an accuracy of position fixing of one mile either way. In the air, however, there are neither rocks nor shoals to encounter. Further, the range of vision is usually greater and from a more convenient angle, hence fixing positions within ten miles amply suffices—and on many occasions even 20 or 30. From the deck of a ship 50 feet above the sea surface the horizon is about eight miles away and the area of a circle of this radius is about 200 square miles; from an aircraft at 5000 feet the horizon is 80 miles away and the area of the circle is 20,000 square miles—just one hundred times as much.

40. Effect of acceleration on a pendulum.—The effect of increase in velocity, i.e. acceleration, is to cause the pendulum to lag behind the direction in which its point of support is moving. It will lag behind by an angle proportional to the amount of this acceleration—indeed, this is one way of measuring acceleration. The effect on the pendulum is just as though gravity ceased to act vertically but acted at an angle inclined to the vertical. This is commonly expressed by saying that the pendulum takes up the line of the "apparent vertical." The direction of this line is that given by the "parallelogram of velocities" applied to acceleration. Thus in Fig. 12, if A B be the gravitational acceleration (32 feet per sec. per sec.) and a steady acceleration C A act on a pendulum, then the pendulum will take up the position A D inclined to the vertical by the angle D A B. Note, moreover, that the *pull* in the string of the pendulum will also have grown from the length A B to the length A D (this explains why the occupant of an aircraft finds his "weight" much greater in a rapid turn). When an aircraft moves in a circle,

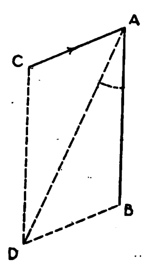


FIG. 12.

an acceleration at once acts radially and is measured by the square of the velocity divided by the radius of the turn. This is the most frequent cause of the acceleration which is so often found to be affecting the readings of instruments carried on aircraft—of course it will only affect those instruments which contain *unbalanced* masses able to act as little pendulums. When an aircraft turns, the turn is always accompanied by a proportional amount of radial acceleration; this combines with the vertical gravitational acceleration and gives rise to an "apparent vertical" inclined outwards from the circular path; unless, therefore, the aeroplane banks until the wings are perpendicular to the "apparent vertical" (as is shown by the cross-level being central), the machine will side-slip. The same effect occurs when a railway train passes around a curve; the outer rail on railway curves is always "super-elevated," as it is called, so that at the average speed of running the floor of the coach will still be perpendicular to the apparent vertical.

In aircraft there are also accelerative forces due to vertical "bumps," and these also affect instrumental readings, though they are perhaps less insidious in their effects than the radial or longitudinal accelerations.

Sextants containing pendulums, or bubbles, are obviously liable to be affected by acceleration. For this reason such instruments have been little used on board ship—experience has shown that it is impossible to attain the requisite accuracy on a surface craft rolling, pitching and yawing in the waves. In the air, however, although the acceleration errors are just as bad—or even worse—nevertheless the accuracy of reading required is so much less that this type of instrument has every chance of being a successful means of astronomical navigation in the air.

41. Resonance in vibration as affecting the motion of pendulums.—If a weight, held up by a spring as shown in Fig. 13, is pulled down and let go, it will oscillate up and down. Each of these oscillations will take the same time and the vibration will be observed to keep time with a simple pendulum (e.g. a small mass swinging at the end of a thread) whose length is equal to the length by which the spring was originally extended when the weight was put on. The oscillations will gradually die away, due to air friction and any other forms of resistance which may happen to be present.

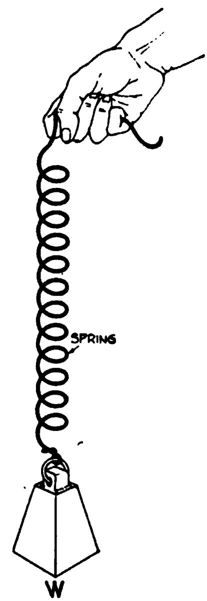


FIG. 13.

Now, while the weight is still vibrating up and down (the time of a complete vibration being *t* secs.), let the hand supporting the upper end of the spring also oscillate up and down. It will be seen that when the hand oscillates at the same rate as the weight the excursions of the latter grow and grow until the spring becomes in danger of breaking—this picking up of an increasing vibration is called **resonance.** It is to avoid such effects, *inter alia*, that troops crossing a light bridge are ordered to break step.

Next, repeat the experiment after bringing everything to rest. Move the hand up and down *very* slowly, say through one inch, the weight will be observed to oscillate at the same rate as the hand, also through one inch. Now oscillate the hand a little faster—the weight will still keep in step, but will move through a little more than an inch. In fact, it will move through a distance equal to

$$\frac{t_1^2}{t_1^2-t_0^2}$$

where t_1 is the time of an oscillation of the hand and t_0 has its previous value (i.e. the time of a free oscillation by the weight when the hand is steady). This expression shows that when t_1 is twice t_0 the oscillation will be 1·33 inches. When t_1 is 1·5 times t_0 the oscillation of the weight will be 1·8 inches. When t_1 is 1·1 times t_0 the oscillation will be nearly 6 inches. Until when $t_1=t_0$ the oscillation becomes infinite and the spring will break unless there is friction present to reduce its oscillations.

Pursue the matter further : suppose the vibration of the hand becomes faster than that of the free oscillation of the weight—say $t_0=2t_1$—then the oscillation becomes 0·33 inches, but is of negative sign. This means that the oscillation of the weight is less than that of the hand, and that when one is up the other is down (i.e. the two are no longer in step but are separated by a half period or 180 degrees). When t_0 is ten times t_1 the distance moved through by the weight is hardly one-hundredth of an inch—so the weight remains nearly at rest, which is the condition aimed at for example in designing the springs of an automobile where the rapid vibrations due to a rough road fail to " get through " to the slowly sprung car-body.

Note here that it is the motion of the weight relative to the rest of the room which has been considered. The motion *relative* to the hand, that is the motion of the weight as seen by a fly sitting on the hand, would appear quite differently. With slow oscillations of the hand the fly would not think the weight was moving at all. As the hand began to move faster, the fly would begin to detect a small motion of the weight. As resonance was approached it would detect a growing motion of the weight, and at resonance a very large one. Once this point was passed it would notice a less oscillation, until in the end, with the growing rapidity of the hand motion, the oscillation became exactly one inch.

Now the fly on the hand is the observer in an aeroplane watching the motion of a pendulum, or bubble. The period of oscillation of an aeroplane is nearly constant whether the oscillation be large or small. Hence a very slow-moving pendulum—one having a much longer period than the aeroplane—would remain undisturbed in its vertical position, and therefore could be used to give the vertical for sextant and other observations. The trouble is to make such a long period pendulum without incurring prohibitive weight or undue complexity. A pendulum of the same period as the machine would oscillate wildly. The usual short-period pendulum scarcely shows any motion, relative to the aeroplane, at all, unless affected by vibrations from the engine, or wings, which it may pick up by resonance. To prevent this latter phenomenon from complicating the use of such apparatus it is usual to introduce "damping," which means the introduction of some dash-pot, oil-bath, or other frictional means of deadening the effect of such vibrations. But in any case, with or without damping, a short period pendulum (and all ordinary pendulums and bubbles in aircraft are short period) cannot show the true vertical.

Damping is of two kinds, one—the usual variety—the damping of all motions *relative* to the machine, and the other damping relative to space. The former would be achieved by letting the pendulum bob move to and fro in an oil-bath carried on the machine. The latter by filling a hollow pendulum bob with mercury, the internal friction of which would gradually bring the pendulum to rest. Note that an infinitely effective machine damping would cause the pendulum to follow all the motions of the machine, whereas an equally effective space damping would cause it to ignore all such motions.

The subject is a large and important one, but cannot be further pursued here. What has been said, however, has a vital consequence in compass design.

42. The Magnetic Compass.—If a short piece of steel is suspended exactly at its centre of gravity it will lie in any direction indifferently. If now it be magnetized and again suspended from the same point it will be observed that the north end will point downwards, as shown in Fig. 14. (This is for all countries north of the Equator—the south end dips in countries south of the Equator.) The magnet (as it now is) dips down until it points towards the north magnetic pole of the earth; since the position of north magnetic pole on the earth is known, it is clear that the magnetic needle can be of use in direction finding. The line in which the vertical plane containing the needle cuts the horizontal plane, gives the direction of magnetic north, and this is what is shown by the ordinary compass card.

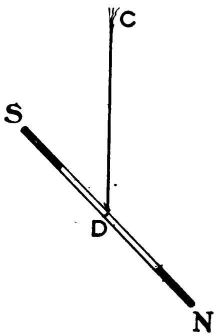


FIG. 14.

Observe, however, that this always requires the vertical plane to be known, and if acceleration forces are acting on the aircraft, due to turning or some other cause, the instrument will treat the apparent vertical as if it were the true vertical, and an incorrect reading of the compass card will result—this is the well-known "turning error" in compasses. The great effect of such accelerations upon compass performance was not at first realized, but that something appeared to go wrong with the compass whenever an aeroplane flew into a cloud was evident enough. The following extract (dated 1915) from a pilot's record of his experience shows this:—

"A huge bank of black clouds loomed ahead. Our orders were to land if clouds were too bad, but as two machines pushed on ahead of me, I pushed on too. It started with a thin mist and then gradually got thicker. I continued so for about ten minutes and then found that according to my compass I had turned completely round and was heading out to sea. The clouds got thicker and the compass became useless, swinging round and round. I was about 7000 feet up and absolutely lost. The next thing I realized was that my speed indicator had rushed up and the wind was fairly whistling through the wires. I pulled her up, but had quite lost control. I nose-dived, side-slipped, stalled, etc., time after time, my speed varying wildly. I did not get out of the clouds until I was only 1500 feet up. I came out diving headlong for the earth. As soon as I saw the ground, I of course adjusted my sense of balance and flattened out. I was, however, hopelessly lost—the sea was nowhere in sight. I steered by my compass (which had recovered, being out of the clouds), and after a short time picked up the coast."

As a measure of practical convenience in constructing compasses a small mass is added to the S end of the needle, so that the compass card to which it is attached will be horizontal; this does not affect the existence of the abovementioned turning error, however.

It can be got over by omitting any additional weight at the S end and by having a double pivot suspension, so that the needle is compelled to move in a plane perpendicular to the line of these pivots—but this is no real solution unless the line of the pivots can be kept vertical by some additional apparatus, usually of a bulky and inconvenient character.

43. Variation of the Compass.—Magnetic north is not the same direction as geographical north, because the magnetic pole and the geographical pole do not quite coincide. The magnetic North Pole is the extreme north of North America, and it follows that the deflection of the magnetic north line from the geographical or true north will vary in different parts of the world: this angle of deflection is called "variation." It is at present about 15 degrees west in London and 6 degrees west in Washington. This is shown by the map reproduced in Fig. 15.

44. Deviation of the Compass.—The craft to which any compass is fitted is sure to have some parts made of iron or steel, and hence may pick up some magnetism of its own. This affects the compass of course, but can on the average be neutralized by putting little balancing magnets under the compass. Any remaining angular error produced by the magnetism of the craft is called "deviation"—and it is customary for this error to be neutralized as far as may be and a diagram of remaining deviation errors to be supplied for each compass.

Owing to certain parts of the aircraft being made of steel there is always the likelihood—indeed, probability—that such parts will become magnetic. An unmagnetized piece of steel which is hammered briskly whilst lying in a north and south line will become slightly magnetic; this means that almost all engineering structures will in course of their erection become somewhat magnetized and aeroplanes are no exception.

Now whatever complicated magnetic disturbances are caused by the steel parts of an aeroplane being more or less magnetic, it is possible to compensate for the effects so produced at the compass position. At that position the net magnetic effect can be divided up into three components, one fore and aft, one athwartships, and one vertical. The vertical one, since it acts at right angles to the compass card, will introduce little difficulty; but the other two components will pull the compass needle one way or the other in a horizontal plane. It is very easy to prevent this. Let the aeroplane be headed—on the ground and in flying attitude—due magnetic north; then the fore and aft component of the aeroplane's own magnetism cannot deflect the needle from the fore and aft line. The athwartship's component,* however, is in a maximum effect position, and will cause the needle to swing some degrees to port or starboard. Place correcting magnets in the holes provided, choosing those holes which are **perpendicular to the magnetic meridian**; when enough magnets have been put in the needle will read zero. Next head the machine south and note whether the compass reads 180 degrees; if it does, well and good. If not, change the athwartship correcting magnets so as to bring the needle *half* way in to the 180 degrees mark: this splits the residual error north and south. Now turn the aeroplane till it points east. The athwartship's component of the aeroplane's magnetism has been neutralized, but the fore and aft component† is at its maximum effect. Put sufficient magnets in that hole which, in the new position of the aeroplane, is **perpendicular to the magnetic meridian,** so that the needle reads exactly 90 degrees from north. Turn the machine west and see if the compass reads 270 degrees; if so, well and good; if not, split the residual error exactly as before.

All this assumes that the lubber line is travelling parallel

* Sometimes known as the Q component.

† Sometimes known as the P component

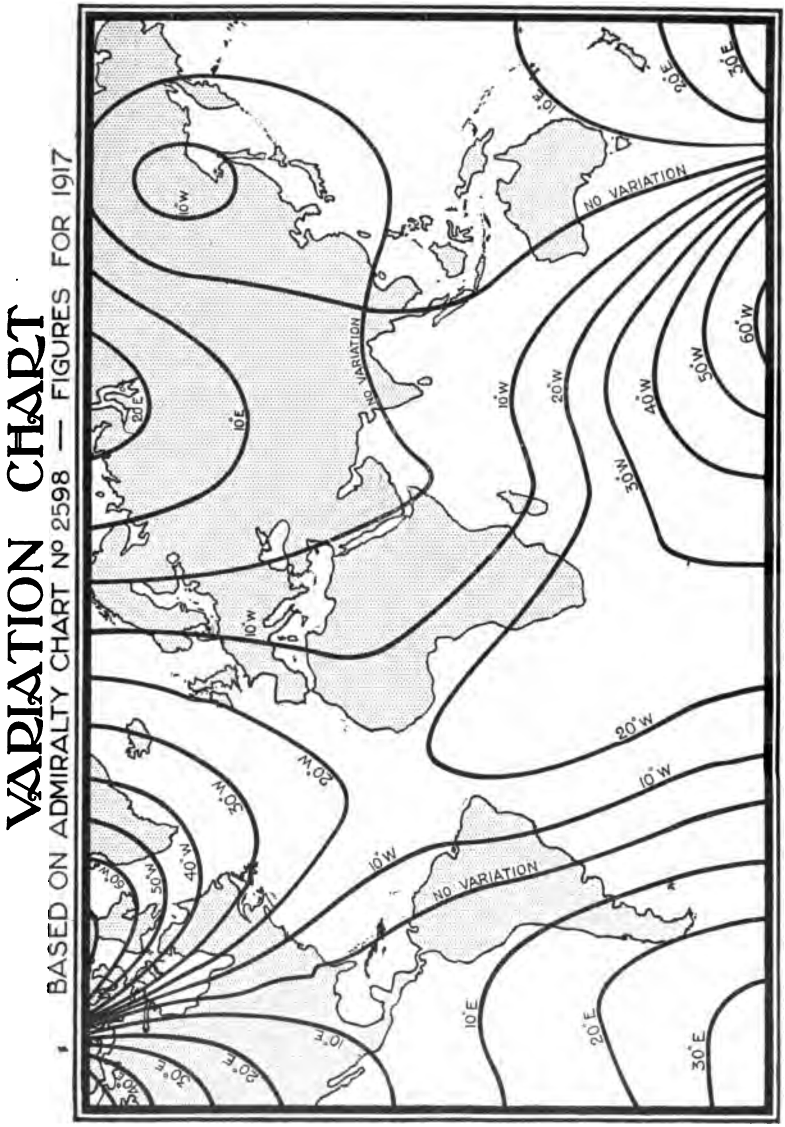


Fig. 15.—Lines of equal magnetic variation.

to the fore-and-aft line of the machine. This can be checked by adding the deviations algebraically when pointing north, south, east and west, and dividing by four. The result is the angular error in the lubber line; this error can then be corrected.

The compass is now protected against the magnetic effects of the aeroplane's permanent magnetism. There may be residual effects, due to temporary induction, but these cannot conveniently be neutralized in an aeroplane (in a surface ship they are neutralized by adding masses of soft iron). All that remains to do is to put the aeroplane on each inter-cardinal point (i.e. every 45 degrees from 0 to 360) and note how much the compass is out. Then make a list of deviations—" a deviation table "—and carry it on the aircraft near the compass. A sample one is given here :—

Heading by Compass.	True Magnetic Bearing.
0	3
45	47
90	87
135	133
180	183
225	227
270	267
315	313
360	3

Summarizing, the procedure for compass corrections should be as follows :—

1. Take deviations on north, east, south, west.
2. Head west. Put in fore-and-aft magnets to make the deviation equal to the mean of the deviations got on east and west.
3. Head north. Put in athwartship magnets to make the deviation equal to the mean of the deviations got on north and south.
4. Swing on eight points and note deviations.

5. Shift lubber mark to the mean of these deviations.
6. Make table of residual deviations by subtracting this mean from the eight observed deviations.

This process is called "swinging the compass," and is usually thought to be a much more complicated matter than it is.

FIG. 16.—Type A.C.O. 5/17 compass (for small machines only).

45. Northerly turning error.—If an aircraft on a banked turn is heading magnetic north the compass card will not be horizontal but will be banked also. This will allow the vertical component of the earth's magnetism to turn the north pole of the needle downwards, hence the card tends to turn the same way as the machine. In some cases it will actually turn faster than the machine does, and hence will appear to indicate a turn in the opposite direction.

This is called the *northerly turning error*. It occurs only in a dangerous form on courses in the neighbourhood of magnetic north, but in the days before it was understood it caused much perturbation. The compass was most needed when flying in clouds or mist, since this was the occasion when the existence of an involuntary turn was least easy to recognize, due to there being no external objects

FIG. 17.—Type A.C.O. 253 compass.

to judge by, and the compass was looked to to afford the necessary guidance. But if the pilot happened to be on a northerly course the compass might indicate a turn to port when one to starboard had really been made. The pilot, to compensate for the apparent turn to port, put on starboard helm, with the consequence of increasing his real turn to starboard still more. The needle still indicated a turn to port and the pilot put on more rudder still, speedily leading to a spin—and the compass appearing to rotate like a top. This phenomenon led to the compass falling into discredit. After a while the cause was known, but it was not easy to prevent it from happening even then—for if the magnetic needle was made to move less rapidly (say by reducing the strength of the magnetization of the needle), the compass tended to be sluggish for ordinary

FIG. 18.—A.C.O. aperiodic compass—6/18 (Bennett).

work. It unfortunately happens that the periodic time of roll or pitch of most aeroplanes is almost the same as the periodic time of the average war pattern compass, so leading to resonance and an unsteady compass needle. In future designs the compass will probably be made non-periodic (by increasing the damping), to avoid resonance effects, whilst on the other hand sharp turns on the part of the pilot will be indicated to him by the gyrostatic turn indicator described below, the compass not being referred to at all until the turn indicator shows that the craft is again flying straight.

46. Typical Compasses.—Typical compasses are shown in Figs. 16, 17 and 18. The first is the A.C.O. 5/17 type fitted to small machines, the second the A.C.O. 253 type for large machines, and the third is the A.C.O. 6/18, an aperiodic type introduced for the reasons above given. The 6/18 aperiodic compass is also fitted with a rotatable bearing plate which can be set by the fixed lubber point to any desired magnetic course, and the machine be flown so that the magnet needle is parallel to the 0/180 line—marked with parallel wires—of the bearing plate.

47. Gyrostats.—Under certain conditions of mounting, a gyrostat, or spinning top, tends to set its axis parallel to the earth's axis. This useful property is made use of in the gyro compass used at sea. It has the great merit of pointing to the true north instead of magnetic north. Gyro compasses have not yet been produced in a form suitable for use in the air : it is difficult to make them light and compact enough ; moreover, the high ground speed sometimes attained requires a rather large and difficult correction to be applied.

For use in aircraft the gyro's most successful use is as a turn indicator. In Fig. 19 G is a gyro spinning about an axis C D, which is carried by a frame mounted on a bear-

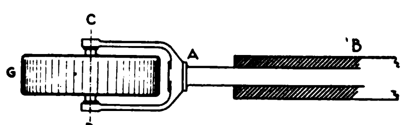


FIG. 19.—Gyro turn indicator.

ing A B. If this be carried in an aircraft and a turn be made to port or starboard in the plane of the paper the gyro will precess, as it is called, that is turn itself and its frame about the axis A B. If this precession be limited by a spring and the angle turned through be indicated by a geared-up pointer, it is clear that by flying the machine so that the pointer stands at its zero position the machine cannot be turning but must be flying straight. Immediately the craft is put on a turn the turn indicator will indicate that this has been done and will show the direction of the turn. Its use for cloud flying and for facilitating the taking of navigational observations, by compass, sextant, or other instrument, is undeniably great.

If a gyrostat could be balanced so that its centre of gravity was exactly central and mounted on central gimbals entirely free from friction, the axis could be used to indicate latitude or longitude. So far the instrumental difficulties have made it impossible to reach this ideal, but it remains a theoretical possibility.

EXAMPLES

Find the distances and track angles (true) for the following courses :

(i) Brighton to Scarborough.
(ii) Harwich to Gloucester.
(iii) Southampton to Edinburgh.
(iv) Gravesend to Ipswich.
(v) Worthing to Hastings.

CHAPTER III

DEAD RECKONING NAVIGATION

Instruments necessary—Air speed indicator—Altimeter—Compass—The drift indicator—Dead reckoning in the past—Allowance for wind—Prediction of wind—Aero bearing plate—Determination of wind velocity during flight—Wind gauge bearing plate—Course setting sight—Measurement of wind velocity from ground—Course and distance calculator—Bigsworth protractor.

48. Instruments necessary. The instruments necessary for keeping a record of the D.R. position are :—

1. An air-speed indicator and altimeter.
2. A compass.
3. A drift indicator.

49. Air-speed Indicator (Ogilvie).—An air-speed indicator is a pressure gauge for measuring the difference in the air pressure in two tubes, of which one has a hole facing in the direction of motion of the aircraft, and the other at right angles thereto. This instrument needs to be very sensitive, since the difference is very small, being at ground level only about one-fifth of a pound per square inch at an air speed of 100 miles an hour. At any height above the ground it is less still, since the air density will be less : the pressure difference (in pounds per square foot) is given by the expression $\frac{1}{2}\rho v^2$, where ρ is the weight in pounds of a cubic foot of air at the altitude of observation and v is the velocity in feet per second of the aircraft. Columns 1 and 2 in the following table show how the mean density (taking the density at 800 feet as unity) varies with altitude ;—

Height in feet.	Mean Density. "Standard Density."	Corrective factor for air-speed indicator.
0	1·024	0·988
1,000	0·994	1·003
2,000	·964	1·019
3,000	·934	1·035
4,000	·905	1·052
5,000	·876	1·069
6,000	·847	1·087
7,000	·820	1·104
8,000	·794	1·122
9,000	·769	1·140
10,000	·744	1·159
11,000	·720	1·179
12,000	·696	1·199
13,000	·673	1·219
14,000	·651	1·239
15,000	·630	1·259
16,000	·610	1·280
17,000	·590	1·302
18,000	·570	1·325
19,000	·551	1·347
20,000	·533	1·370
21,000	·515	1·393
22,000	·498	1·417
23,000	·482	1·441
24,000	·465	1·466
25,000	·449	1·492
26,000	·433	1·519
27,000	·418	1·547
28,000	·403	1·576
29,000	·388	1·606
30,000	·373	1·637

"Standard density" (for which air-speed indicators are graduated) = 1221 grams per cubic metre.

Now at 20,000 feet the air density is only 0·533, and an air-speed indicator which read 100 when the aircraft was travelling near ground level at a 100 m.p.h. would indicate 73 m.p.h. when travelling at the same speed at 20,000 feet. This lower figure is called the **indicated air speed,** to distinguish it from the **true air speed**. To convert indicated air speed into the true air speed, which it is necessary to know for navigational purposes, an Appleyard Ring is often fitted around the dial of the instrument. Column 3 in the above table shows the multiplier by which the indicated air speed needs to be multiplied at each altitude to give the true air speed.

50. The Altimeter.—The altimeter is a form of aneroid barometer for reading the pressure of the atmosphere at any altitude. The pressure at the earth's surface is between 14 and 15 lb. per square inch, and is due to the weight of the atmosphere. The pressure at the bottom of an ocean one mile deep is about one ton per square inch, and gets less and less as the point of observation rises from the ocean bed. In just the same way the pressure in the air ocean gets less as the observer rises to a higher and higher altitude above the earth's surface. Owing to the fact that air is easily compressible and water is not, the rate at which the pressure falls is different in the two cases. In air the logarithm of the pressure ratio is proportional to the altitude. This law is taken account of in the construction of the altimeter.

Before the days of flying all aneroid barometers were graduated for a uniform temperature of 10 degrees Cent. Such a rule is out of place for an aviation instrument, nevertheless in the early days of flying it was necessary to employ such instruments as were available, and in those times the heights reached were so small that the "temperature error" thus incurred was not very detrimental. Unfortunately, however, the development of the aeroplane took place just at a time—in the Great War—when it was very difficult to "swap horses," and this curious convention, that the temperature is everywhere 10 degrees Cent., continued to be the basis of altimeter calibration. Hence an altimeter will commonly read

21,000 feet when the height is really 20,000 feet. It is, however, possible to correct these readings by a suitable table. Aneroids are usually corrected for the effect of temperature change on their *internal* parts (due to the expansion and contraction of the various levers, etc.); it is the effect of temperature change on the weight of the air which is neglected. Fortunately at the moderate heights of navigation flights the error is not troublesome.

51. The Compass.—The compass must be easily read to one degree, and it must be as little affected by the oscillations of the machine as possible. Much has been done to reach the maximum of efficiency in the details of air compass design, but much still remains.

52. The Drift Indicator.—Three methods of observing drift are available :—

1. By observing ground markings ahead and noting at what angle to the fore and aft line they must be in order subsequently to pass right under the aircraft.

2. By observing points almost directly under the machine and noting at what angle they move past relative to the fore and aft line of the aircraft.

3. By choosing some prominent object under the aircraft and noting at what angle it bears after some minutes of flight: these latter are called tail observations, as the measurements are made towards the tail of the aircraft.

For all these methods to be successful the aircraft must be flown steadily and straight. The rolling and pitching of the machine tends to cause errors in them all, but much the least when method 3 is followed.

The drift indicator used will depend on the machine, the choice lying between :—

1. The "**aero bearing plate**" (for vertical observations).

2. The "**wind gauge bearing plate**" (for tail observations).

3. The "**course setting sight**" (for forward observations).

Of these three, the second is the most accurate method of observation. Each of these instruments also provides for the ground speed to be measured by a stop-watch when required.

Details of these devices are given later in this chapter. Before coming to them it is necessary to consider the effect of wind on the motion of aircraft, and the measurement of its velocity and direction.

53. Dead Reckoning in the past.—If air speed be known and wind be known, and a good compass be available, the navigator is in the position of being able to predict his position from hour to hour with complete precision. He is in the same position as would be the navigator of a ship who knew the record of his log and the precise effect of the tides. The maritime navigator makes use of his knowledge—or of the best approach he can make to it—by plotting on his chart the position his ship would be in were there no tides at all, and this he calls the "dead reckoning," or D.R. position. He then corrects this position for the effect of the assumed tides and so arrives at his "estimated position." This needs always to be recorded in the log, no matter whatever other navigational methods may be employed as a check. The importance of keeping note of the D.R. and estimated positions has long been known; thus in Hakluyt's voyages we find the following graphic "Instructions and notes very necessary and needful to be observed," and dated 1580:—

"When you come to Orfordnesse,* if the winde doe serve you to goe a seabord the sands, doe you set off from thence, and note the time diligently of your being against the saide Nesse, turning then your glasse, whereby you intende to keepe your continuall watch, and apoint such course as you shal thinke good, according as the winde serveth you. And in keeping your dead reckoning, it is necessary that you doe note at the ende of every foure glasses what way the shippe hath made (by your best proofes, to be used) and howe her way hath beene through the water, considering withall for the sagge of the sea, to leewards, accordingly as you shall finde it growe. Doe you diligently

* It is interesting to find that the importance of Orfordness does not date from 1915.

observe the latitude as often, and in as many places as you may possible ; and also the variation of the compasse (especially when you may bee at shoare upon any land) noting the same observations truely, and the place and places where, and the time and times when you do the same. Afterwards when you have sailed 1, 2, 3 or 4 glasses (at the most) noting diligently what way your barke hath made, and upon what point of the compasse, do you againe set that first land seene, or the parts thereof, that you first observed, if you can well perceive or discerne them, and likewise such other notable points or signes upon the land that you may then see, and could not perceive at the first time, distinguishing it also by letter from the other, and drawing in your booke the shape of the same land, as it appeareth unto you, and so the third time, etc. Thus, I beseech God Almightie to bless you, and prosper your voyage with good and happie successe, and send you safely to returne home againe, to the great joy and rejoycing of the adventurers with you, and all your friends, and our whole countrey. Amen."

Our *air* navigator needs to keep his D.R. position just as carefully, but he is in the more fortunate position in that he can almost always determine the velocity and direction of the wind (his aerial "tides") from time to time by observation on the visible earth beneath. The ship navigator could only do this if the sea were transparent and he could see the bottom of the ocean and so observe and measure his angle of drift. The air navigator can almost always count on obtaining a glimpse, from time to time, of the earth, and so check his current estimate of the wind.

54. Allowance for wind.—It is easiest to allow for the effect of the wind by considering the aircraft to move straight ahead through still air, and then to consider separately the simultaneous shift—which we call wind—of this air ocean relative to the earth. If, for instance, an airplane starting from Hendon heads due north for an hour at an air speed of 100 miles an hour, it would, if there were no wind, then find itself 100 miles north of Hendon; but if in the meantime there had existed a uniform wind such that the air in which the machine mounted at Hendon had since arrived at Reading, the airplane would at the end of the hour find itself 100 miles north, not of Hendon but of Reading. Thus, in Fig. 20 the machine flies from H to A whilst the air moves from H to R and from A to B; so that at the end of the hour the machine finds itself not at A but at B. This is called a vector diagram, and the velocity vector H A is said to be added to the wind velocity vector A B, giving H B as the answer.

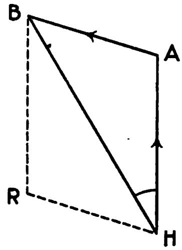


FIG. 20.

Evidently this means also that a machine wishing to fly from H to B must set its head along the direction H A, and not in the direction H B; the angle A H B, the angle between the course steered and the course-made-good, is a very important one and known as the angle of drift. H A is the course steered and H B the course-made-good.

Note that on an "out and home" journey in the same wind and at the same air speed the drift angles out and home will be equal. Thus, to fly A to C, see Fig. 21, when the wind is equal to B D in velocity and direction means setting the head of the machine along A B on the outward flight and along C B on the home flight. Then as A B and B C are equal, the angles of drift (B A D and B C D) must also be equal. Thus, if a navigator knows the right angle of drift to allow for on his outward journey, he also knows it for his home journey if the same conditions as to wind and air speed remain.

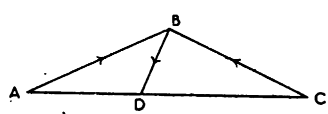


FIG. 21.

The gradual development of the science of meteorology has at last enabled it to include in its wide world survey the study of the winds of the upper air. Such study is absolutely essential to the future of air navigation, and that this is beginning to be realised, witness the daily publication since 21 January, 1919, in *The Times* of tables of the "Wind Direction and Velocity in the Upper Air."

AVIATION WEATHER

MONDAY, JANUARY 20TH, 1919.—MORNING OBSERVATIONS.
WIND DIRECTION AND VELOCITY IN THE UPPER AIR.

	Height above ground.			
	2000 ft.	5000 ft.	10,000 ft.	15,000 ft.
District.	m.p.h.	m.p.h.	m.p.h.	m.p.h.
Scotland, E.	S. 22	S. 28	—	—
England, N.	S.S.E. 37	—	—	—
England, N.W.	S.S.W. 14	S.W. 13	—	—
England, E.	—	S.S.E. 19	N.W. 43	W. 23
Ireland, S.W.	—	—	—	—

Velocities are given in miles per hour. Note here the importance of choosing the right level in which to fly. On this day in Eastern counties the wind at 5000 feet was but 19 m.p.h., whereas at 10,000 feet it was more than twice as much, whilst at 15,000 feet the velocity had almost sunk to that at the 5000 feet level. Here the wind is seen first to increase with height and then to decrease, whilst the direction also changes considerably. But the more variable are the weather conditions, the more necessary is it to have the right sort of meteorological information before setting out on an air voyage.

55. The Prediction of Wind.—Air navigation requires two things of the meteorologists, first that they shall provide accurate information as to the average weather conditions along any specified air route, and secondly that they shall provide a forecast of the probable winds in the upper air during the ensuing days. The former is necessary when planning out standard air routes with their chains of landing grounds, and the latter to enable the navigator to select the best altitude for flight and the probable time-table for his journey. Forecasting surface winds is, however, a difficult matter, and it may be that the winds of the upper air will prove as little tractable.

The ability to predict the upper wind depends upon a knowledge of the pressure and temperature changes in the atmosphere. A few words may perhaps be appropriately inserted here to indicate the method by which this information is utilised. A knowledge of the barometric pressure throughout a given area enables a series of equal pressure lines, or isobars, to be drawn. The pressure gradient across these lines enables the resulting wind to be calculated ; this is known as the gradient wind. The gradient wind is made up of two parts, the geostrophic component and the cyclostrophic component, the former, which is the more important, is due to the earth's rotation, and the latter to any curvature of the isobars. The origin and nature of the geostrophic wind cannot be better described than in the following extract from the official "Barometer Manual" :—

"Let us consider the case of an arctic bird, or an aeroplane, that starts from some point on the parallel of 84 degrees north, and makes a bee-line for the pole (supposed in sight for the purpose of keeping a straight course), and keeps 'straight on' beyond it, flying at 60 nautical miles an hour. It will reach the pole by direct line after six hours' flight, and at the end of twelve hours will have done a journey of 720 miles, and have got to latitude 84 degrees again, and meanwhile the place from which it started will have come round with the earth and have made the journey of about 1130 miles, and the pilot who has made a straight course will find himself at home again. If he had marked his trail by dropping bombs at intervals, or in some other effective manner, he would have provided conclusive evidence that he never 'set out' for the pole at all, but after once getting up speed he made off at a great pace to some point about west-north-west, gradually slackened his speed and got drifted towards the pole, and when he arrived there, turned slowly round and came back to where he started from, and arrived at the starting point again from the east-north-east. Whenever anything flies or floats in the air, as airships or winds do, the rotation of the earth has to be reckoned with, and its effect is to turn the course of a body that is left to its own momentum at the rate of

15 degrees × sin λ per hour, where λ is the latitude. On an earth that does not rotate, a body that is left to its own momentum keeps in a vertical plane and moves along a great circle. If the earth rotates, the moving body left to its own momentum is diverted from the great circle at the rate of 15 degrees sin λ per hour, to the right in the northern hemisphere and to the left in the southern. If, on the other hand, it is to be kept in the great circle it has to be pushed from the side on which it would be left behind by the rotation of the earth underneath it. The push must always be at right angles to the direction of the motion otherwise it would do more than alter the course, it would accelerate or retard the velocity, and that is not wanted. With a current of air in the free atmosphere we get, so far as we are able to tell, exactly the conditions required, the pressure difference on the two sides of a moving stream of air is always at right angles to its motion, and just provides the push necessary to steer the air with no appreciable effect upon the speed. We get a proper balance, and the air moves under its own momentum without being diverted from its path along a great circle if the push represented by the pressure gradient γ is balanced by a speed of motion v such that

$$\gamma = 2\omega v \rho \sin \lambda$$

when λ is the latitude, ω the angular velocity of the earth's rotation, ρ the density of the moving air."

This formula allows the geostrophic wind to be calculated.

Careful researches along these lines have brought to light a surprising and most fortunate closeness of connection between the geostrophic wind and the actual wind. It is found, in fact, that the two may without grave error be regarded as substantially identical. Sir Napier Shaw* remarks on this :—

" To assume that this balance of wind and pressure in the upper air is an operative principle of atmospheric structure may be thought a hazardous mode of procedure,

* *Manual of Meteorology.*

and it requires the most scrupulous examination; but the proper course seems to be to accept it, at least until the proved exceptions are numerous enough to show that, under the prescribed conditions of motion approximately in a great circle, finite differences of pressure do exist in the air without the compensating velocity in the air currents. It need not be supposed that the balance is always strictly perfect, but only that in ordinary circumstances the accelerating forces operating in the air are so small in relation to the pressures that we measure that they are beyond our powers of observation."

Shaw also points out that in ordinary circumstances there is a deviation of some 20 to 30 degrees between the direction of the surface wind and that of the geostrophic wind, due to surface friction and in the direction which that friction would indicate. Dobson has shown that although the geostrophic velocity may be arrived at within 1000 feet of the earth, the calculated direction will not usually be obtained till 2500 feet; it follows, therefore, that if the surface friction effects, always local and uncertain, are to be avoided, as accuracy in air navigation requires that they should, flight at a lower height than 2000 or 3000 feet over land is undesirable. The effect is much less over the sea and the equivalent heights would be lower. As a general rule, the velocity of the surface wind over the sea is one-third less than it would be were there no surface friction; and over the land two-thirds less than it would otherwise be. In the absence of surface friction these velocities would each be that of the gradient wind. This loss of velocity of 33 per cent. over the sea involves the loss of about half the wind's kinetic energy; the missing half is found in the energy of the ocean billows.

56. The Aero Bearing Plate.—An instrument to measure the drift angle by means of vertical observations is the aero bearing plate, which in early patterns was merely a dummy compass card in metal, fitted with vanes for taking bearings. Later models have an open centre fitted with a number of horizontal parallel wires, and a vertical hinged height scale marked in hundreds of feet. It is shown in Fig. 22. To use it as a drift indicator, set the azimuth circle to zero, look vertically downwards through

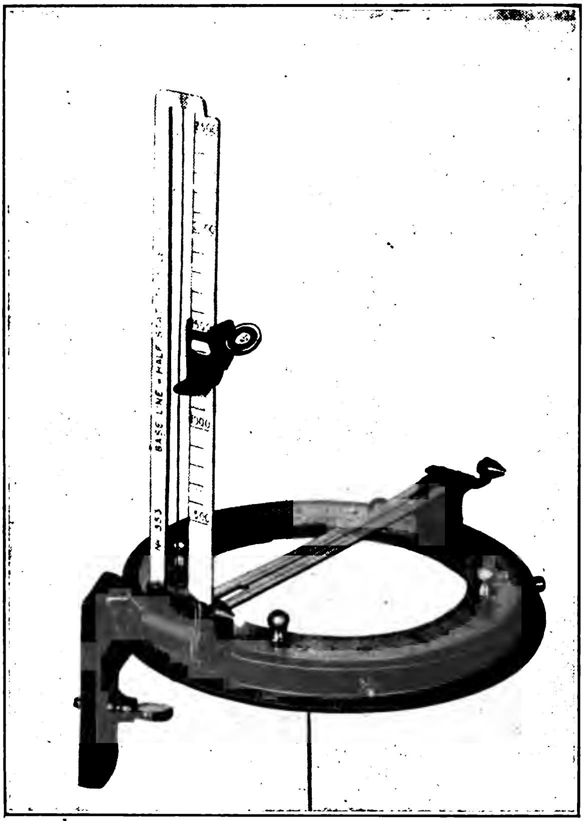


FIG. 22.—Aero Bearing Plate.

the open centre and turn the parallel wires until objects on the ground or sea surface appear to pass along them from end to end. Read off the difference in angle, to starboard or port, between the zero of the instrument and the line of the wires. This gives the drift in degrees, and if this angle be applied to the course steered, it gives the course made good, i.e. the actual track angle.

To obtain the ground speed, set the sight bar on the height scale to the height of the aircraft above the ground which is being passed over. Align the eye-hole with the foresight on any object which lies in the actual track, and when alignment occurs start a stop watch. When the same object is vertically below the observer, as indicated by the rear sight coming into alignment between the eye-hole sight and the object, stop the watch. The aircraft has then passed over a certain distance in the time indicated by the stop watch, and from this the ground speed can be calculated. The distance referred to varies with the instrument, and may be half a statute mile or half a nautical mile.

This instrument suffers from the disadvantage that *vertical* observations for drift are only accurate at low altitudes.

57. Determination of wind velocity whilst in flight.—A curious and convenient relationship here comes to the air navigator's aid. If he uses a horizontal bearing plate having a transparent centre and lays across it a rod pivoted at one end, as shown in the diagram in Fig. 23, so that its length A B is parallel to the direction of the drift of the ground—A C, the line of symmetry being parallel to the fore and aft lines of the machine, and if he then draws a pencil line A B on the bearing plate (previously oriented correctly) and repeats this operation on a number of courses, he will obtain a wind star, and the intersection

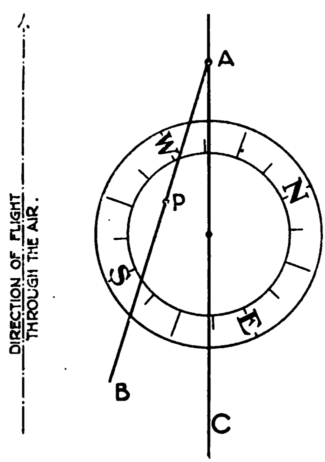


FIG. 23.

point, known as the wind point, will give him the velocity and direction of the wind. Thus, in Fig. 24 three such courses are shown, and all, it will be observed, intersect at the wind point; P C is then the velocity of the wind to scale and the direction of P C relative to the bearing plate scale gives the direction from which the wind is blowing. The point P can of course be got by a cut of two lines only, but for accuracy such change of course should not be less

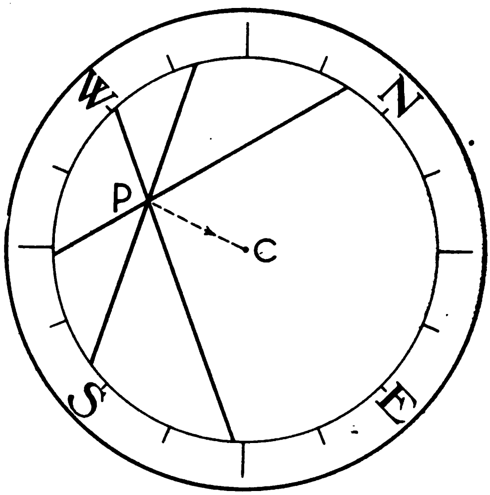


FIG. 24.—"Wind Star."

than say 50 degrees; this can conveniently be done by turning first 25 degrees one way and then the other and for equal times, since this does not interfere appreciably with the course desired to be made good. Of course aircraft on closed patrols, for war purposes, change course several times on each patrol, and a wind determination or check can be made at each "corner." On a long straight flight the position of the wind point along the line A B in Fig. 23 can, if desired, be obtained by timing over some object on the ground and so determining the length of P A and hence the position of the wind point. Equally, once the wind point is obtained, the length P A will give the ground speed and the bar A B can be graduated directly in m.p.h.

The procedure for setting A B to the drift angle depends on the height. At low heights it suffices to view the ground vertically below, but at considerable heights it is much more accurate to observe the angle which an object passed over some minutes previously bears to the fore and aft line of the machine (a tail bearing). When over the sea a flare may be dropped to afford the necessary fixed point when nothing else is available.

So much for the actual use of the method; the reason why the drift lines in Fig. 24 all pass through the wind point in this convenient manner may seem to need explanation to those who are not prepared to take such construction for granted.

Thus, in Fig. 25, if an aeroplane head due north along C A and the observed drift line be A B we know the closing side of the vector diagram will represent the wind, and that it must start *somewhere* along the line A B and must end at C. In short, A B is what is called a "*position line*" for the wind point. Now head the craft on a new bearing, say 75 degrees east of north, along C D, and let D E be the observed drift line. Then we get a new vector diagram, and a new position line D E for the wind point; these two intersect at P, and this must therefore be the wind point, and P C the velocity and direction of the wind to scale (the scale being the same one as that by which C A and C D represent the air speed).

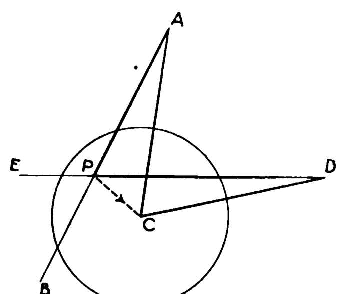


FIG. 25.

This is for two courses only, but obviously the drift line for any third, fourth, or additional course must equally pass through the point P. Probably the obtaining of a wind star in this way by flying on several courses in succession affords as accurate a way as can be devised for the ascertaining of the wind elements at any desired altitude and time. Moreover, in the nature of the case, meteorological records must always be received some little time—not infrequently in practice some hours—after the moment at which they were measured, whereas by this method, provided the ground is visible, the instantaneous values can be obtained and can be checked as often as desired.

It should also be noted that A P and D P, in Fig. 25, will measure the ground speeds along each of these courses. Hence, once the wind point is found and the instrument set accordingly, figures can be given at once for the ground speed along any contemplated course and the probable duration of flight to cover any desired mileage over the ground.

58. Wind Gauge Bearing Plate (Wimperis).—The wind gauge bearing plate is an instrument to enable the drift angle to be measured by tail bearings, and it also enables the velocity and deviation of the wind to be obtained whilst in flight by the method just described. The general arrangement of the instrument is shown in the illustration in Fig. 26. It is capable of being fixed on the port or starboard side of the aircraft. A glass panel occupies the centre of the bearing plate and is marked with concentric circles corresponding to wind speeds by intervals of 10 m.p.h. Beneath the glass is the red wind point (A) mounted on the wind arrow (B), and capable of sliding so that the red spot may be set to any wind speed by means of the circles. The wind arrow (B) is carried by the wind ring (R) and may be turned by knobs (N) beneath, so that it points in any desired direction relative to the bearing plate.

A drift bar (D) lies athwart the bearing plate; it comprises a central bar with speed and time scales, and a bevelled edge for drawing drift lines on the bearing plate glass; also drift wires at the sides, carrying timing-beads. The drift bar is pivoted at one end to a nut; the position of this nut in its guides is adjusted by turning the handle (S) until the index points to the *true air speed* on the scale (X). The nut carries also a quadrant drift scale (Q) over which the tail of the drift bar moves, the pointer indicating the angle of drift. The height bar (H) is hinged at its base and when not in use it is folded down. The slider (C) is movable

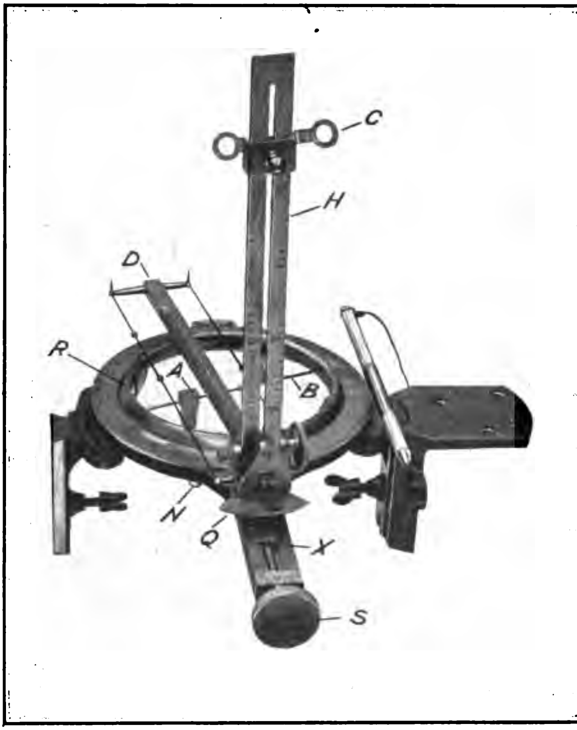


Fig. 26.—Wind Gauge Bearing Plate.

on the height bar its ring backsights are used for observing the drift against the vee foresight or along the drift wires of any object over which the aircraft has passed some minutes previously. A dropped flare may be used, if desired, when over the sea.

Method of use. Fly on a steady course. Turn bearing plate until lubber mark points to the course being steered. Turn drift bar (D) until it is parallel to the earth drift, best noted by sighting on the new position of some object over which the machine was observed to have passed some minutes before (or on a dropped flare). Draw a line on the glass along the bevelled edge of the drift bar, using the glass marking pencil. Fly steadily on some new course preferably not less than 45 degrees from the line of the first. Turn the bearing plate until the lubber mark reads the new course steered. Set the drift bar to the drift and draw a second line along the bevelled edge. These two lines will intersect *at the correct position* for the wind point.

Or if it be preferred to determine the wind point without altering course, then whilst on the original course time the passage of any stationary object on the earth from one bead to the next one, viewing them through the back-sight (C) set to the correct altitude. This will give a timing run of half a mile. When timing on the red beads set the backsight to the red scale or on the black beads set it to the black scale on the height bar. Read on the stop watch the number of seconds, then make a mark on the glass opposite this red figure on the drift bar. This will give the correct position for the wind point.

The wind point on the instrument is thus set to its correct position by one or other of these methods. Once this has been done the reading on the black scale of the drift bar opposite the wind point will give the ground speed in m.p.h., whilst the red scale will show the number of minutes to cover thirty statute miles.

At any time the quadrant (Q) near the pivot of the drift bar will give the drift angle in degrees which added to, or subtracted from, the course steered will give the course made good.

59. Course Setting Sight (Wimperis).—In flying boats it is usually most convenient to make drift observations by sighting ahead, and for this purpose the course setting sight is suitable. This instrument—which was exhibited at the

British Scientific Products Exhibition in 1919—can be used to determine the velocity and direction of the wind, and in consequence the course to steer. The course setting

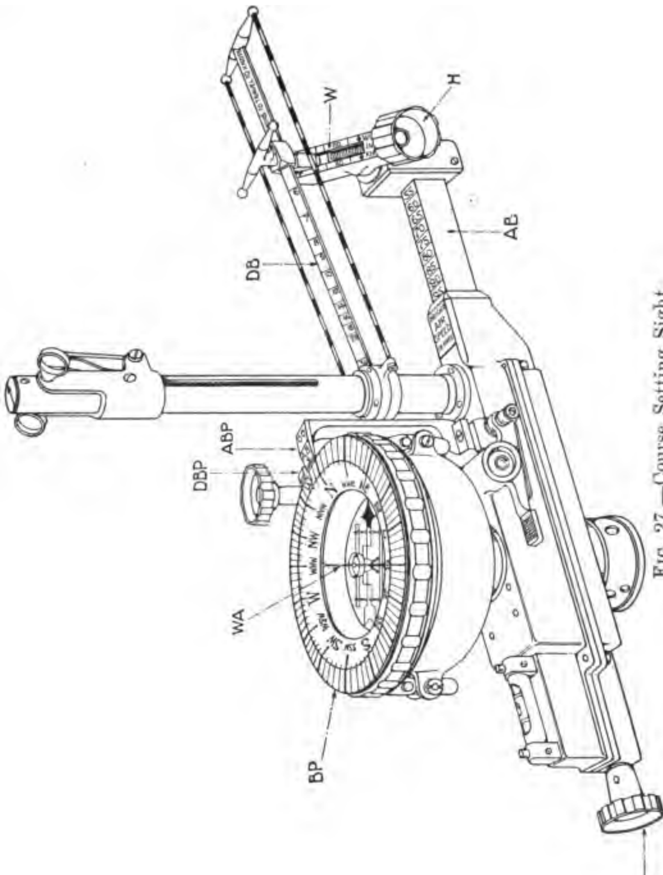


Fig. 27.—Course Setting Sight.

sight is mounted in the nose of the aircraft, and has certain automatic features which render it easy of use.

The instrument is shown in Fig. 27, and as will be seen consists of a vector triangle linkage combined with a bearing plate, usually containing within it a small compass (though this is not essential). When a compass is incorporated the method of use is as follows: First set the instrument for air speed, then fly up wind and note the direction; then fly across the wind and set drift bar to line of drift; the wind velocity can then be read off the wind scale. In detail, the procedure is this:

First, read air-speed indicator and set index to this reading on air-speed bar (A B) by turning handle (A). Next turn the tail of the wind arrow under air-speed bar pointer (A B P). Fly directly up wind, so that no lateral drift is discernible. Unclamp bearing plate, so that it is free from the wind arrow (W A), and turn it until the north (red) end of the compass needle comes under the red zero mark on bearing plate, when reclamp. Next, turn aircraft through about 90 degrees port or starboard, fly steadily, and turn bearing plate so that its red zero mark comes over red end of compass needle. Now adjust wind scale (W) by handle (H) until direction of drift is seen to be along drift wires. The sight is now correctly set for the wind, and the wind velocity and direction can be read off.

To use this information so as to make good a desired course. Turn the bearing plate (B P) until drift bar pointer (D B P) indicates desired course. The air-speed bar pointer (A B P) reads course to be steered. This information is given to the pilot who steers accordingly by his own compass. The ground speed in knots while on this course is shown by one of the two scales marked on the drift bar (D B); on the second scale is shown the number of minutes for ten nautical miles. The corresponding figures for the return journey can be similarly predicted, and thus the total time of flight ascertained.

WORKED EXAMPLE

Air speed 80 knots: wind 30 knots from E.: course desired to be made good, 235 degrees. Setting the sight accordingly shows that course steered should be 223 degrees, and that speed over ground will be 103 knots, whilst the time taken to cover 10 nautical miles will be 5·8 minutes.

On the homeward flight: Course required to be made good=235 −180=55 degrees. The course to be steered is indicated by the instrument as 67 degrees. Ground speed=54 knots. Time for 10 nautical miles=11·2 minutes. Thus the time to cover 100 nautical miles would be 58 minutes on the outward journey, and 112 on the homeward, giving a total time of 170 minutes. Had there been no wind, the total time would have been 200×60÷80, or 150 minutes. Whatever its direction, wind always has the effect of lengthening the total time of flight on an out-and-home journey.

60. Other methods of measuring the drift angle.—Sometimes—especially in an ocean flight—a suitable object for drift observations will be seen on the earth's surface, but it may not be so placed as to be exactly in the aircraft's path, or it may not be noticed until it is too late to alter course. For such cases the relative path (see Fig. 28) instead of being A B may prove to be C D. How is one then to determine the course being made good?

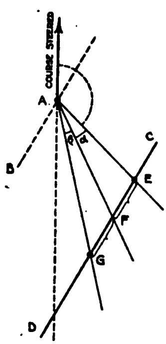


FIG. 28.

Perhaps the best method is to take times and bearings of the object as it passes through the points E, F and G, such that the time from E to F is equal to the time from F to G. Then if the angles α and β are small, i.e. not more than 20 degrees, it is easy to find C D, the course-made-good, by marking off A E proportional to the angle β and A G proportional to α, and then to join F G. Thus, if α be 15½ degrees and β say 13 degrees, we mark off A E at a length of 1·3 inches and A G at a length of 1·55 inches; this gives us the points E and G. Draw a line through these points and we have the bearing of the course-made-good; its inclination to the course being steered will give the drift angle.

With airships the ability to hover can be made use of. If an airship heads up wind and reduces speed until some object below appears stationary, the speed of the airship relative to the air must be equal and opposite to the wind. Hence, the velocity of the wind can be read on the air-speed indicator and its direction noted by reading the course being steered.

Whether this method was actually used by the raiding Zeppelin airships is not known, but it appears that use was made by them of an alternative and less attractive method. This alternative plan was to steer up wind and suddenly to put the ship on a wide circular uniform turn; on completing 360 degrees change of course the ship would, if the wind were zero, be back over the starting-point of the turn—in the other event it would be in the rear of this point by a distance equal to the space covered by the wind in the time of the turn. The additional time taken to recover this lost ground at a known air speed is related to the total of the two times in the same proportion as the wind velocity is to the air speed.

Thus, if T_1 and T_2 be the two times and V the air speed, then

$$\text{wind velocity} = T_2V/(T_1 + T_2)$$

61. Measurement of wind velocity from ground.—It is easy to measure the velocity of the surface wind with an ordinary anemometer, but the determination of the velocities occurring in the upper air needs other methods. One highly successful and simple means is to fire a shell fused to burst at the altitude where the wind velocity is to be determined and then to time the passage of the smoke puff as reflected in a horizontal mirror across a measured space on the face of the mirror; this determines the wind velocity. Its direction is given by the line followed by the image of the small puff across the face of the mirror. Nothing could be simpler, easier, or more accurate for the purpose; but it suffers from the disadvantage that it cannot be done when the sky is entirely overcast by clouds at a level below that at which it is desired to measure the wind velocity. In such cases a kite balloon carrying the requisite instruments may be employed; but there are risks inseparable from the use of a kite balloon and its tethering cable, especially when the latter passes through a cloud-bank, in which position it may wreck an unsuspecting aircraft which happens to be passing through the same cloud. In such cases it is better to send up an aeroplane to make the necessary observations, which are conducted in the following way. A.A. shells are fired at regular intervals and burst above the clouds. The puffs of smoke drift with the wind, and the speed and direction of the wind may be discovered by flying along the line of the puffs, and noting the compass course and the time taken to pass from one puff to the next. The first clearly gives the direction of the wind; and from the second, and the observed air speed of the aeroplane, the speed of the wind may be deduced.

62. Calculators.—For the calculations as to course to be steered and time of flight, which are necessary when the drift instrument itself does not itself give this information, or when examples are required to be worked out on the ground, a **Campbell-Harrison Course and Distance Calculator** is used. For subsequent plotting on a chart, the **Bigsworth Chart Board** is convenient, and both will be described.

63. Course and Distance Calculator (Campbell-Harrison) with Time and Distance Dials (Appleyard).—The " Battenberg Course and Distance Indicator " is the basis of this instrument, which is merely a simplified form of the naval pattern, adapted for use in aircraft.

The instrument (see Fig. 30) consists of a ring, marked every 5 degrees from 0 to 360—the points of the compass being marked; a central rotating disc, squared, of diameter representing 240 miles; two arms pivoted at the centre of the disc marked to the same scale as the disc; two movable pointers working on the two arms; a central clamp for holding instrument rigid when set; and on the circumference is the Appleyard computing dial.

The principle employed is the theory of relative velocity. If two bodies be moving in definite directions at fixed speed, and if through any point lines be drawn equal in length to those speeds, to any scale, and parallel to the directions in which the bodies are moving, and then the ends of these two lines be joined, a triangle will be formed, the third side of which represents in direction and length (to the same scale) the relative velocity of these bodies to each other.

For example (Fig. 29): A body A is moving along line A B with a velocity represented by the length of A B

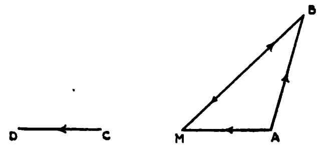


FIG. 29.

to any scale. A body C is moving along line C D with a velocity represented by the length of C D to the same scale. Through A draw a line parallel and equal to C D; call it A M. Then M B represents in direction and length, to the same scale, the relative velocity of the two bodies to each other.

WORKED EXAMPLE

To find what "allowance to make for a wind," or the amount of "drift" (see Fig. 30).

(*a*) Set the arrow on disc to desired track angle.
(*b*) Set arm and pointer B to direction from which wind is blowing and to speed of wind.
(*c*) Set pointer A to speed of machine.
(*d*) Revolve arm A till pointer A is on same line (parallel to arrow) as pointer B.

Arm A points to the course to steer.

The distance between pointers a and b will be ground speed.

Thus if wind be from 45 degrees (true), and of a velocity of 40 knots; machine air speed, 70 knots; desired track angle or course to make good, 270 degrees. What is the course to steer?

(*a*) Set arrow on the disc to 270 degrees.
(*b*) Set arm and pointer B to 45 degrees and 40.
(*c*) Set pointer A to 70.
(*d*) Revolve arm A as in (*d*).

Arm A points to 295 degrees. Length A B=91.

Answer.—Course to steer 295 degrees. Ground speed along track in direction 270 degrees 91 knots.

64. Bigsworth Protractor, Parallels and Chart Board.—The chart board, which is made in two sizes, 17 inches square for large aircraft and 14 inches for craft where space is more limited, is designed to accommodate at one time four or five charts or maps. If it is to be fixed in the aircraft it should be so arranged that there is a space

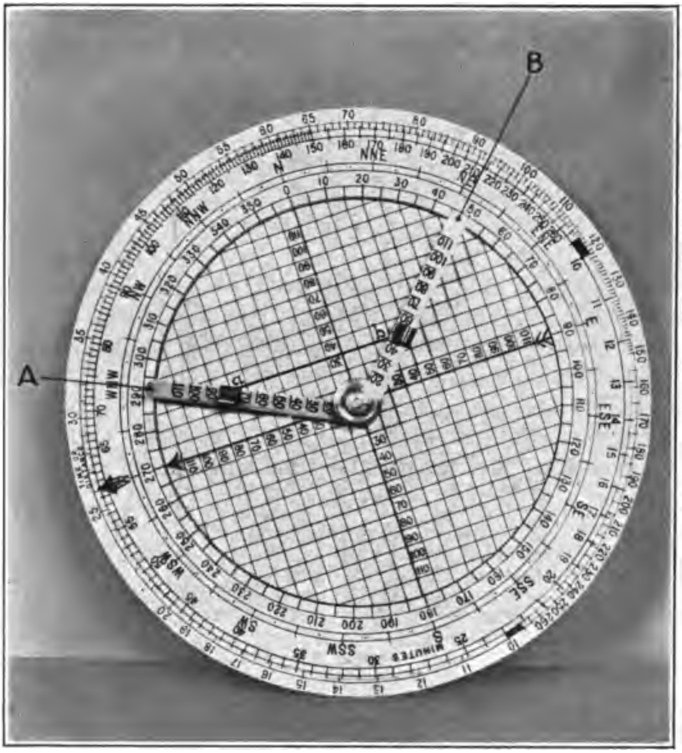


FIG. 30.—C. D. C.

of about 6 inches on the right-hand side, to allow the elbow of the brass parallels to protrude this amount when working the protractor on the extreme edge of the board and also to facilitate insertion or removal of charts. It can best be used as a loose chart board, to be placed on one side, or preferably in a canvas pocket, when not in use. To use the instrument clamp the parallels by means of a clamping screw at the bottom right-hand corner of the board (Fig. 31). The protractor can then be used over any portion of the board provided there is room for the elbow to work down-

FIG. 31.—Bigsworth Chart Board.

wards. The parallels are fitted with a hinge so that they and the protractor can be lifted off the board if so desired. Should it happen that the portion of the chart to be used is in the bottom right-hand corner of the board, it will be desirable to unclamp the parallels, slide them off one edge and on to the other edge of the board, clamping them in the left-hand corner. When this is done, care must be taken to reset the pointer of the protractor, which will otherwise be indicating the reciprocal of the course or bearing. To use protractor, turn it by the large milled-head screw till the engraved arrow points to the northward and place one side of it coincident with the magnetic meridian of the compass rose on the chart. Ease back side screw. Set the head of the metal pointer to zero. Clamp side screw. The metal pointer will now indicate magnetic direction only. To maintain any particular direction on the protractor, say 176 degrees, turn it until the pointer indicates 176 degrees and then clamp centre screw. It will now show 176 degrees magnetic wherever it may be moved over the chart board. The protractor will thus be acting as a parallel rule and will enable parallel lines to be drawn anywhere over the board. If the protractor is being used on a map where there is no "compass rose," but the amount of the variation is known, set the side of the protractor coincident with the true meridian and clamp centre screw. Set the pointer the number of degrees required, to the left if the variation is westerly and to the right if the variation is easterly. Then clamp side screw, ease up centre screw, and the protractor will now show magnetic direction. In the event of true direction being required, proceed as above but set the pointer to zero. All indications by the protractor will then be true. A special pencil for drawing on the celluloid cover is supplied with each board; the marks can be easily erased by the finger.

EXAMPLES.

1. With the following aeroplane speeds, compass courses, wind speeds, and wind directions (magnetic bearing of point from which wind is blowing), find the ground speed and drift angle.

Aeroplane speed.	*Compass course.*	*Wind speed.*	*Wind direction.*
(i) 80 knots	43°	20 knots	163°
(ii) 100 ,,	170°	10 ,,	254°
(iii) 90 ,,	350°	40 ,,	207°
(iv) 60 ,,	36°	32 ,,	64°

2. An aeroplane flying at 72 knots observed drift angles on two compass courses thus:

Compass Course	200°	Compass Course	140°
Drift Angle	17° to Port.	Drift Angle	12° to Port.

Find the speed and direction of the wind.

3. With indicated air speed 70 knots an aeroplane at 5000 feet found the following drift angles:

Compass Course	145°	Compass Course	190°
Drift Angle	15° to Port.	Drift Angle	15° to Port.

Find the speed and direction of the wind.

4. The wind is at 24 knots from 274°. An aeroplane flying at 6000 feet at 80 knots indicated air speed leaves Kenley aerodrome at 10.15 a.m. for Bircham Newton aerodrome. What is the compass course for the flight, and at what time does it arrive at Bircham Newton ?

5. An aeroplane whose air speed is 85 knots flies to an objective whose bearing is 80° and distant 1107 miles. The prevailing wind is from 120° at 30 knots. With the criterion of air mileage of Chapter I, page 6, can it reach its objective without landing for petrol ? And if it can, how much petrol (in percentage of weight of petrol complement on setting out) remains in the tank on completing the journey.

6. The air speed of an aeroplane is 100 knots, and it flies at 6000 feet on a compass bearing 326°. (Magnetic variation is 15° W.). At this height the wind is at 28 knots from 41° true. What is the track angle ?

(Use table of Chapter III, page 39, to get ground speed in still air from indicated air speed.)

7. A navigator takes drift observations and finds the wind at 15 knots from 223° magnetic. The indicated air speed is 94 knots at 5000 feet. The desired track angle is 110° true. (Magnetic variation 15° W.) What compass bearing does he indicate to the pilot ?

8. An aeroplane carrying enough petrol for 3 hrs. flying at 80 knots flies to an objective distant 84 miles, whose bearing is 300° true and returns to its starting-point. The wind is at 40 knots from 98° true. What percentage of its original complement of petrol remains in the tanks ?

CHAPTER IV

DIRECTIONAL WIRELESS TELEGRAPHY

Direction-finding wireless telegraphy—At beacon stations—On aircraft —Plotting position lines on Mercator Charts—Weir Azimuth diagram.

65. Direction-Finding Wireless Telegraphy.—During recent years—1914 onwards—yet another means of obtaining the geographical position of an aircraft, or of any other moving vessel, has been developed. This invention followed from the introduction of a method of determining the direction from which wireless waves reach the receiving instrument.

An analogy with wireless waves is afforded by the circular ripples which spread out in a pond when a stone is thrown into the midst of it. At any point in the pond the water level rises and falls rhythmically as the train of little waves passes it. The speed with which the waves move is but a few feet per second. Wireless waves have the enormous velocity of 185,000 miles per. second. They are, however, similar to water waves in that the direction of the displacement is perpendicular to the direction of motion. For as in water the waves move horizontally while the actual direction of motion of a particle of water, or of anything floating on the surface, is up and down, so in electric waves the electric intensity which is the " displacement " for this kind of wave, is perpendicular to the direction in which the waves are travelling. Now if a rectangular coil is placed in the path of the rays, as in Fig. 32, there will be an oscillating electric intensity along D C and along A B as the waves pass. If the coil is facing the direction of travel of the waves, clearly, the electric field will always be in the same direction at the same instant along both. Consequently no current will flow. If the coil is not facing the direction of travel, there will be a phase difference between the effect of the field along D C and A B, and an oscillating current will flow around the coil, and this phase difference will clearly be greatest when the coil faces at right angles to the direction of travel. Similarly a spirit level attached to a piece of wood floating in water waves will show no oscillations when the axis of the level is at right angles to the direction of travel of the waves, but in other positions will show a motion of the bubble which will increase to a maximum when the level is in the direction of travel of the waves. Once, therefore, a search coil of this kind is mounted on a vertical axis it can be turned until the current received is either a maximum or a minimum and the direction of the wave direction be determined. It will be noted that the direction of a sending station due west could not be distinguished from one due east, but other considerations usually enable a distinction to be made.

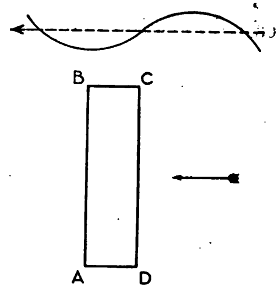


FIG. 32.

In practice it is much easier to determine the coil position for minimum current than the position of maximum, because the relative distance of the two sides of the coil from the sending station changes more rapidly, for a small rotation of the coil, when the latter faces the waves than when it is in line with them.

With this simple form of the coil no great accuracy in direction finding would be obtainable, but with the introduction of auxiliary electrical detecting apparatus of great sensitiveness the direction can be found within a degree or two.

66. Alternative methods of use.—In applying this apparatus to aircraft, two methods are available :—

1. Direction finding at the beacon stations.
2. Direction finding on the aircraft itself.

In the former case the aircraft calls up the beacon stations within range, and any two of these acting in collaboration can determine the direction of the aircraft, and by combining their results ascertain and report to the aircraft its latitude and longitude. Thus, if A and B, in Fig. 33, be the two stations and the former finds the waves reaching it along the direction C A and the latter along D B, it is clear that the craft must actually be at E, and its position thus plotted on a chart gives the desired information. If three stations collaborate the chance of avoiding errors in the determination is improved.

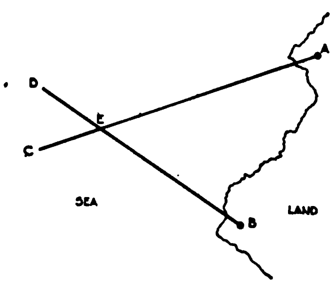
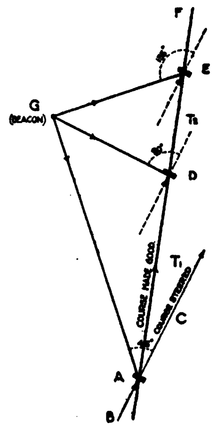


FIG. 33.

In the second alternative the aircraft itself carries a search coil, and by rotating it finds the direction of A and B, plots the lines on a chart, and so determines its position. This method has the advantage that the aircraft gives less work to the beacon stations, and is able, if it wishes, to keep its movements free from publicity.

If only one beacon station is within range, a single "position line" is obtained which can be combined with one or more obtained by astronomical means. Care must be taken in plotting wireless position lines on a Mercator Chart, since wireless waves travel along great circles, and great circles on such charts are represented by curved lines. The essential point to bear in mind in air navigation is that the dead reckoning position must always be logged ; position lines derived from astronomical observations or directional wireless should be obtained as a check on the position so logged.

A point which may perhaps be worth bringing out is that a combination of bearing and timing observations on a wireless station which is being passed to port or starboard enables the drift angle to be determined without the earth itself being visible. Thus, if in Fig. 34 an aeroplane A is steering a course B C and making good a course A D F, and the time taken to pass from A where the beacon is on the port bow to D where it is abeam to port, and from D to E where it is on the port quarter, then the tangent of the angle of drift (the angle D A C) is equal to the difference of these times divided by their sums. This may prove a convenient method of checking the estimated drift angle.



NOTE. Tan drift angle $= \frac{T_1 - T_2}{T_1 + T_2}$

FIG. 34.

A method of wireless navigation which has been proposed is for an aircraft to set its head steadily towards a wireless beacon and so make sure of arriving at the desired haven, whether the course steered is otherwise a scientific one or not. This will be quite satisfactory in still air, but when there is a high wind blowing it may lead to a somewhat sinuous path. For the existence of wind means that the air ocean has motion relative to the earth; or we may regard the air as still and the earth moving under it at the same speed as the wind, but in the opposite direction—the relative motion is the same. Adopting this convention, we have a wireless beacon, towards which the aircraft is headed, itself in rapid motion relative to the air. This sets the aircraft on a curved course—the path of a dog running after its master, studied by mathematicians as the "curve of pursuit." In Fig. 35 is seen the path of an aircraft relative to the air under these conditions, with the wind at starting coming over the port quarter with a velocity equal to half the air speed; and in addition its path relative to the ground. In dotted lines is shown the route which would have been followed had the machine been headed with exactly the right allowance for the wind. The " wireless " air path A E C is longer than the correct path A D by 10 per cent., and this requires a correspondingly increased fuel capacity if this method of navigation were adopted under high wind conditions. Similar curves can be drawn for other wind velocities and other wind directions.

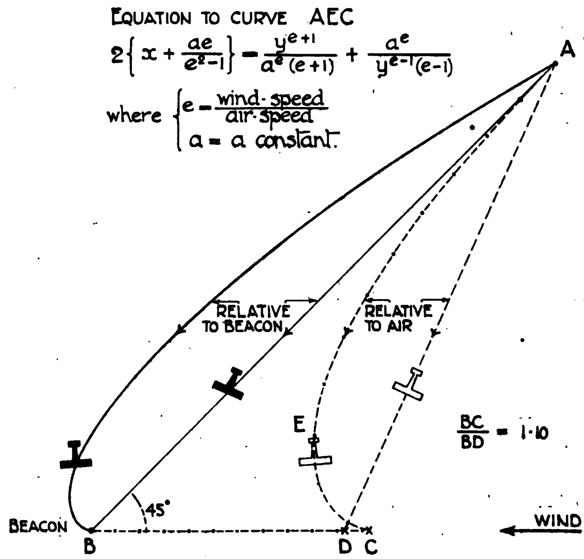
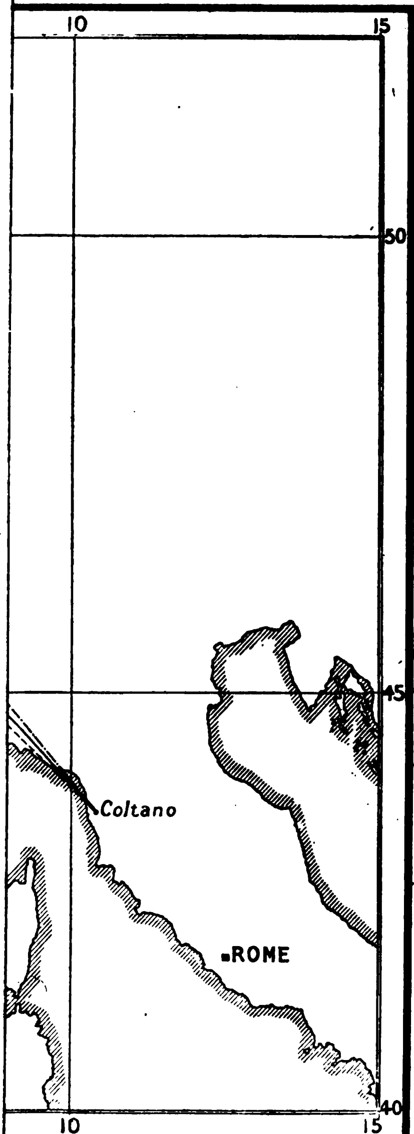


FIG. 35.

67. Plotting position lines on Mercator Charts.—In Fig. 36 is shown a Mercator Chart, on which are marked two points London (L) and Coltano (C). The rhumb line which joins L and C by a straight line is shown, but it is *not* the path followed by the wireless waves, as it does not form part of a great circle. The great circle path joining L and C is shown by the upper dotted line, and it is the path taken by the wireless waves.

Evidently therefore a beacon station at C which wishes



ercator Chart.

to draw the position line for an aircraft at L must draw a curve which will arrive at C in the correct direction and which will be an arc of a great circle. Similarly for an aircraft at L using directional wireless to find the arc of a great circle which shall pass through C and arrive in the correct direction at L. Usually one does not need to draw the whole arc L C but only that portion of it which comes near to the D.R. position of the aircraft.

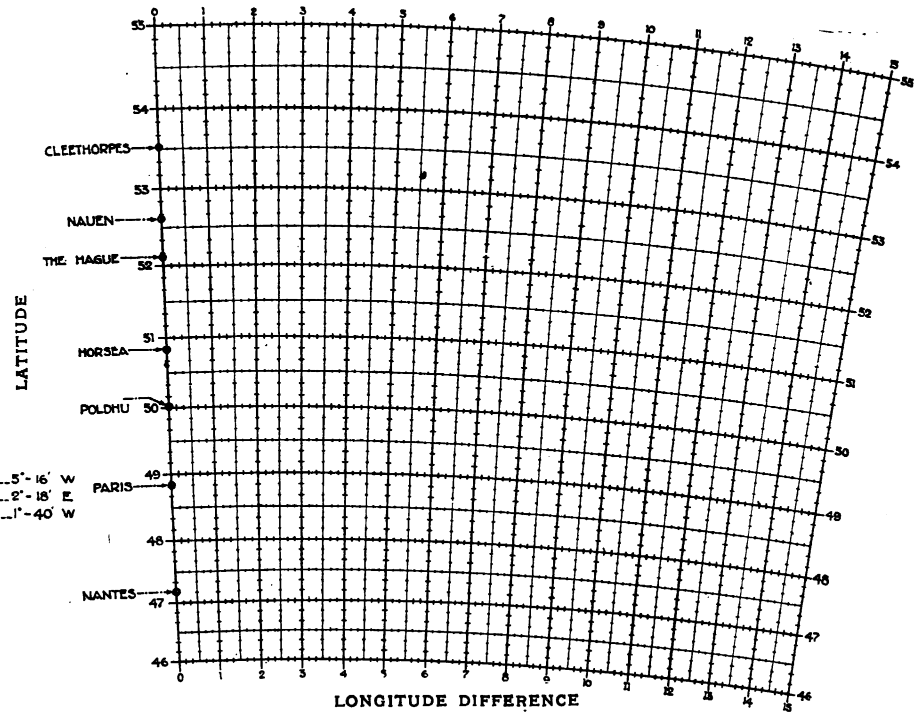
68. Weir Azimuth diagram.—Probably the simplest way to draw the line L C, or that small portion of it which we need in the neighbourhood of the D.R. position, is by the use of the **Littrow** projection of the sphere, better known as Captain Weir's diagram. In Fig. 37 such a diagram is reproduced. It covers a range of latitude of 46 degrees north to 55 degrees north, with an hour angle, or longitude difference, of from 0 to 15 degrees.

The usefulness of this projection lies in the fact that lines joining places having the same bearing from any place on the zero hour angle line are straight lines. This makes them easy to draw.

To plot the position line of a wireless observation of bearing B, place the sending station on the zero hour angle line according to its latitude and draw from the sending station a straight line, making an angle B with the south direction of the zero line 00. Place the points when this line cuts hour angles and latitude circles on the Mercator Chart and join the points thus found. This is the position line for the observation. It is only necessary in practice of course to draw the position line in the immediate neighbourhood of the D.R. position.

The position of the usual beacon stations would be marked on the chart at their individual latitudes, and with their longitude clearly marked in the margin.

69. Conversion angle method.—An approximate method, which, however, gives sufficiently accurate results up to distances of 1000 miles, employs the "conversion angle." This is defined as one half the difference of longitude of the sending and receiving station multiplied by the sine



LONGITUDES

CLEETHORPES	0°-02′ W	POLDHU	5°-16′ W
NAUEN	12°-51′ E	PARIS	2°-18′ E
THE HAGUE	4°-15′ E	NANTES	1°-40′ W
HORSEA	1°-06′ W		

FIG. 37.—Weir diagram for D.F./W.T.
(Method of use suggested by Commander Veater, R.N.

of their mean latitude, and it is approximately the angle between the rhumb line and the great circle through the two stations. Hence, if we subtract the conversion angle from the observed bearing B, using the D.R. latitude and longitude of the receiving station to determine the conversion angle, we get the angle P R A (see Fig. 38), which is equal to Q S A, and if a line is drawn through the sending station S on the Mercator map making this angle (B—conversion angle) the line will pass close to the receiving station. Another such line, determined from another sending station, will give a cut which must be very close to the true position of the receiving station. It should be noticed, however, that neither the great circle through R and S, nor the rhumb line, is really the position line. The position line is clearly a line such that all great circles through S, at points on it make the same angle with the meridians through the points. Actually it is found that the position line and the great circle through S and R make equal angles with the rhumb line S R. The position line is shown approximately by the dotted line in the figure.

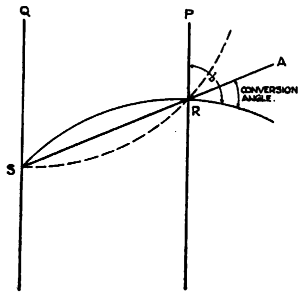


FIG. 38.

Since, however, we do not know the precise position of the receiving station R, we cannot draw the position line, for though we can find the angle it makes with the meridian we do not know through what point to draw it, and it is found that the use of the rhumb line gives a sufficient approximation.

EXAMPLES

1. Find the conversion angles for the following stations and D.R. positions :

Station.	*D.R. Position.*
(i) Cleethorpes— (Lat. 53° 31′ N. Long. 0° 02′ W.)	Lat. 50° 12′ N. Long. 3° 49′ E.
(ii) The Hague— (Lat. 52° 08′ N. Long. 4° 15′ E.)	Lat. 50° 02′ N. Long. 6° 18′ E.

(iii) Poldhu—
(Lat. 50° 01′ N. Long. 5° 16′ W.) Lat. 53° 56′ N. Long. 8° 17′ E.
(iv) Nantes—
(Lat. 47° 11′ N. Long. 1° 40′ W.) Lat. 53° 12′ N. Long. 1° 10′ W.

2. Show that, if the great circle bearing of a wireless signal from Horsea (Lat. 50° 50′ Long. 1° 06′ W.) is 38°, the position line passes through the position Lat. 49° N. Long. 1° 14′ W. (Use the Littrow Projection.)

3. Show, both by the conversion angle method and the Littrow projection method that the position line for a great circle bearing 151° for a wireless signal from Cleethorpes (Lat. 53° 31′ N. Long. 0° 02′ W.) passes within two or three miles of Dorchester (Lat. 50° 42′ N. Long. 2° 28′ W.).

4. Show, both by the conversion angle method and the Littrow projection method, that the position line for a great circle bearing 93° for a wireless signal from The Hague (Lat. 52° 7′ N. Long. 4° 15′ E.) passes within two miles of Ipswich (Lat. 52° 4′ N. Long. 1° 8′ E.).

5. An aeroplane whose D.R. position is Lat. 51° 21′ N. Long. 0° 12′ W. receives wireless signals from Paris and Nauen. The great circle bearing of the former is 35° and of the latter 105°. Plot the position lines and find the position (fix) by

(i) The Littrow Projection method;
(ii) The Conversion Angle method.

6. An aeroplane whose D.R. position is Lat. 50° 51′ N. Long. 1° 24′ W. receives a wireless signal of great circle bearing 52° from Paris. Twenty-five minutes later it receives a signal from Horsea, of great circle bearing 76°. The ground speed of the aeroplane is 85 knots, and its track angle 282°. Find the fix at the instant of reception of the second signal.

CHAPTER V

ASTRONOMICAL OBSERVATIONS

Purpose—Procedure—Sun's azimuth—Reduction of observations—Two-star altitude tables—Naval pattern sextant—Baker sextant—R.A.E. bubble sextant.

70. Purpose.—The purpose of astronomical observations whether from a ship at sea or from an aircraft in the air, is to afford some independent check of the D.R. position. Any *one* such observation enables a " position line " to be drawn on the chart ; two such lines crossing at an angle locate the position definitely.

At any one moment there is always some place on the earth which is exactly under any given heavenly body : this is called the " **geographical position** " of that body. An individual at the geographical position would of course find this heavenly body exactly at his zenith ; but if he were say 20 degrees (i.e. 1200 nautical miles) away, measured on a great circle on the earth, then he would find the heavenly body would be 20 degrees away from his new zenith. Tests would soon indicate that there were numbers of positions on the earth from all of which this heavenly body had this same zenith distance of 20 degrees, and that all these points lay in a circle having the geographical position as centre ; and apart from such tests it is easy to see that, if the earth be a sphere, this must be the case.

Thus an observation that the zenith distance was exactly 20 degrees (i.e. that the altitude was exactly 70 degrees) would show that the observer must be *somewhere* on a circle of exactly 20 degrees radius about the geographical position. Thanks, however, to the D.R. reckoning he will know approximately where he is, so that there is no need to draw the whole of the position circle. Only that part of it which crosses the chart in the neighbourhood of the D.R. position need be drawn, and for all ordinary purposes a straight line suffices—strictly it is slightly curved (to whatever curve represents a small circle on the chart employed), but the curvature is slight and may be neglected in practice. The direction it takes on the map must clearly be at right angles to the radius line joining the navigator's position to the geographical position of the heavenly body, in other words at right angles to the true bearing, or azimuth, of the heavenly body at the instant at which the observation was made.

71. Procedure.—To lay down the position line on the chart it is necessary to take the D.R. position, or some point close to it, and to compute for it the zenith distance and azimuth of the observed body at the time at which the observation was made; then if the measured zenith distance agrees exactly with the computed, it is only necessary to draw from the D.R. position a line A B (see Fig. 39) at the computed azimuth and a line C D at right angles to it passing through A. Then C D is the required position line.

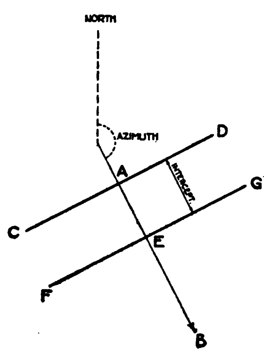


Fig. 39.

But if it be found that the observed zenith distance is less than the computed, say by 25 minutes of arc (showing that the geographical position of the heavenly body is nearer to the observer than the D.R. observations would have suggested), then the position line needs moving in closer to the geographical position and arrives at F G, such that the intercept A E is exactly 25 nautical miles, allowing, of course, for the scale of the map. F G is then the correct position line.

If two such position lines are found the actual position, assuming no errors in the measurements, must be at the intersection. If three position lines A B, C D and E F can be drawn—see Fig. 40—then the true position may be

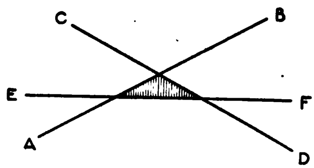


FIG. 40.

reckoned to be somewhere within the shaded triangle, or " cocked hat " as it is usually termed.

Obtaining a number of position lines usually means taking nearly simultaneous observations of different heavenly bodies, or by taking wireless bearings of known " beacon stations." It would appear as though in the absence of wireless observations it would not be possible to get a " cut " from observations of the sun (except on comparatively rare occasions when the moon or planets were also visible), but this is not so. A position line derived from a forenoon observation of the sun can be made to cut the position line corresponding to an afternoon observation by moving the former parallel to itself by a distance equal to the estimated " run " between observations—but, of course, this needs to be done with care, and it is scarcely an independent check of the D.R. readings.

72. Sun's azimuth.—A single observation of the sun—or of any other heavenly body—would give the actual position if it were possible to measure the azimuth as well as the altitude. The altitude observation would give one position line, and the line of equal azimuth would give another, where these two intersected would be the true position. This leads to the generalization that any sort of observation of a heavenly body can be used to give a position line or curve of some sort, and that the traditional procedure is far from being the only one possible even if it happens to be the most convenient.

Reduction of observations. The computations above mentioned are described in the following chapter. Many suggestions have been made for avoiding them, but none have proved accurate enough for sea work—as witness the fact that calculations with logarithmic and other tables are still universal at sea. The alternative methods, when not too tedious, are insufficiently accurate for sea use ; but this does not mean they are unsuitable for use in the air, indeed it is probable that the methods of reduction customary at sea are not destined for adoption by air navigators.

Four such alternatives for air use are :—

(1) **The Veater and the d'Ocagne diagrams.**
(2) **The Baker navigation machine.**
(3) **The use of two-star tables.**
(4) **The use of special forms of slide-rule.**

The first, second, and fourth of these methods will be described in the following chapter. The third follows directly from what has just been said about position lines.

73. Two-star altitude tables.—If the altitudes of two different stars are measured simultaneously (in such a case "simultaneously" means that the star observed first is observed again as a third reading and the mean of the first and third readings taken) it is possible by looking up a special book of tables to read off the latitude of the observer's position and the sidereal time of the observation.

To explain the principle of this method it is best to take actual examples. Let it be assumed that the reader has taken an observation of a given star and found its altitude to be 55 degrees. Let it be assumed also that he has at his side a celestial sphere on which he is free to draw lines. Since the altitude is 55 degrees the zenith distance must be 35 degrees. On the sphere draw a circle having this star as centre and 35 degrees as radius. He knows at once that his own zenith must lie at some point in the circumference of this circle but he does not at present know where.

Now if he had taken not one observation of one star only, but two simultaneous observations of two different stars, he could draw two such circles on the sphere. If these two circles fail to cut one another the observations are at fault or the wrong star has been chosen on the sphere as the centre of one of the circles. If they are correctly observed and correctly drawn they must cut at two points, as shown

FIG 41.

on the diagram, Fig. 41; it follows that the observer's zenith must be at either the one or the other and his D.R. reckoning will tell him which (unless they are very close together—in which case a different pair of stars should be chosen, for a glancing cut of this kind would lack in accuracy even if the choice of the points were certain) Now, on the celestial sphere the stars are located by their declination and right ascension, and the positions of the points of intersection can be read off on the scales of the sphere in declination and R.A. Now the declination of the zenith of any place on the earth is of course equal to its latitude, and since the zenith is necessarily on the meridian of the place the R.A. of the zenith is equal to the sidereal time at the moment of observation. If the sidereal time at Greenwich, as read from a watch keeping sidereal time, is noted at the time the observation is made, the difference between the two times converted to degrees and minutes gives the longitude of the position. Thus the place of the aircraft is determined.

Since it is possible to calculate the declination and R.A. of the cut for any given pair of stars at any given altitudes, it is possible to construct a table having the possible altitudes of one named star along the top, the possible altitudes of another down the side, and in the body of the table figures for latitude and sidereal time. The short table shown on page 81 is a sample.

74. Sextants.—Three types of sextants are available for these observations :—

(1) **The Naval Pattern Sextant.**
(2) **The Cloud Horizon Sextant (Baker).**
(3) **The Bubble or Pendulum Sextant.**

75. Naval Pattern Sextant.—The general principle of this sextant has already been described in Chapter I. This type of sextant has but limited use for air purposes, but the most suitable form is probably that shown in Fig. 42. It will be observed that readings are given by a tangent screw—a vernier would be difficult to read in the air and would give a greater degree of accuracy than is required.

76. Baker and Hughes' Cloud Horizon Sextant.—At varying heights above the earth's surface the "dip" or depression of the visible horizon below the true horizontal will change in amount. Approximately the value of this depression in minutes of arc is equal to the square root of the observer's height in feet. If it be assumed that the dip is of the same amount all round the horizon, then the true vertical is half-way between the two horizons seen in two opposite directions. This principle is employed in the

ALTITUDE OF BETELGEUSE.

Altitude of Regulus	41°	42°	43°	44°	45°	46°	47°	48°
20°	55°–56′ *h. m.* 5–14·0	54°–47′ *h. m.* 5–12·9	53°–46′ *h. m.* 5–12·2	52°–41′ *h. m.* 5–11·4	51°–39′ *h. m.* 5–10·8	50°–35′ *h. m.* 5–10·2	49°–31′ *h. m.* 5–9·7	48°–29′ *h. m.* 5–9·2
20° 30′	56–3 5–17·8	54–55 5–16·6	53–54 5–15·7	52–47 5–14·8	51–44 5–14·1	50–43 5–13·5	49–38 5–12·8	48–38 5–12·3
21°	56–9 5–21·5	55–1 5–20·3	54–1 5–19·3	52–54 5–18·3	51–51 5–17·4	50–50 5–16·7	49–46 5–16·0	48–47 5–15·4
21° 30′	56–14 5–25·2	55–7 5–23·9	54–7 5–22·8	53–0 5–21·6	51–56 5–20·7	50–57 5–20·0	49–53 5–19·2	48–55 5–18·6
22°	56–18 5–29·0	55–13 5–27·5	54–13 5–26·4	53–6 5–25·1	52–1 5–24·0	51–3 5–23·2	50–0 5–22·3	49–2 5–21·6
22° 30′	56–22 5–32·8	55–18 5–31·3	54–19 5–30·0	53–10 5–28·5	52–6 5–27·4	51–9 5–26·4	50–6 5–25·5	49–9 5–24·8
23°	56–25 5–36·6	55–21 5–34·9	54–21 5–33·4	53–14 5–32·0	52–11 5–30·8	51–13 5–29·8	50–12 5–28·8	49–15 5–27·9

TWO-STAR ALTITUDE TABLE.

(EXAMPLE.—If the altitude of Betelgeuse be 43°, and of Regulus be 22°–30′; then the latitude is 54°–19′, and the sidereal time is 5 h. 30·0 m.; the second cut comes outside the possible latitude range).

Baker sextant to avoid the dip correction; and if the "horizon" is really a bank of mist of uniform thickness, instead of the true horizon, the instrument still reads correctly. Its general design is shown in Figs. 43 and 44. A telescope has the main part of its axis vertical and has a right-angle bend produced by the prism at the top. The object glass of the telescope is at the upper end of the system, and the four lenses of the eye-piece are arranged as shown in the figure. The eye looks into the eye-piece of

FIG. 42.—Naval pattern sextant (Hughes).

the telescope in a horizontal direction, as shown. Above the O. G. is a plate, upon which are mounted two prisms whose functions are to collect light from the front horizon under the sun by the front horizon prism, and from the back horizon (directly behind the observer) by the back horizon prism. Above the horizontal prisms is the third prism, the index prism. This prism rotates around an axis perpendicular to the plane of the paper, and is used like the index mirror of an ordinary sextant to reflect the image of the sun into the field of view. Light from the sun, after reflection at this prism, passes into the telescope system through a small circular hole drilled centrally through the two horizon prisms.

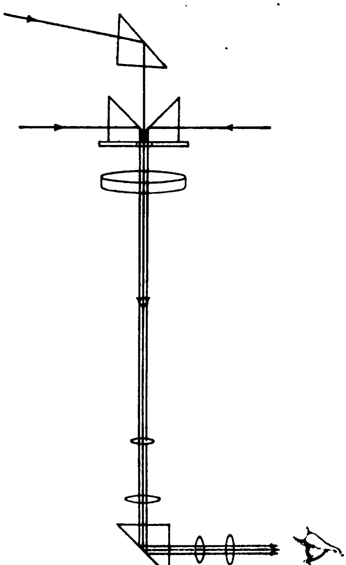


FIG. 43.—Baker sextant.

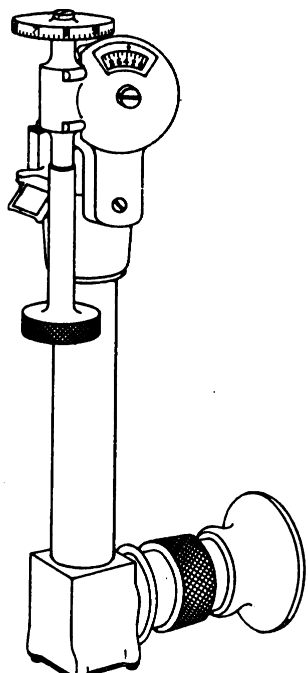


FIG. 44.

In this manner an image of the sun or other heavenly body can also be brought into the field of view and, by adjustment of the index prism, so placed as to bisect the space between the two visible horizons. If it does this exactly, then the sun's centre has been brought down to the true horizontal, and the scale on the instrument reads off the correct altitude.

Obviously by this method, acceleration errors are avoided : but at some cost in difficulty of observation and in complete loss of utility on dark nights. Moreover, when a bank of mist or cloud is used as an horizon, it is impossible to be certain that the layer is at a uniform height in all directions ; furthermore, on some days the clouds are too tumbled and broken to give any suitable line. The instrument, however, has its uses and it was

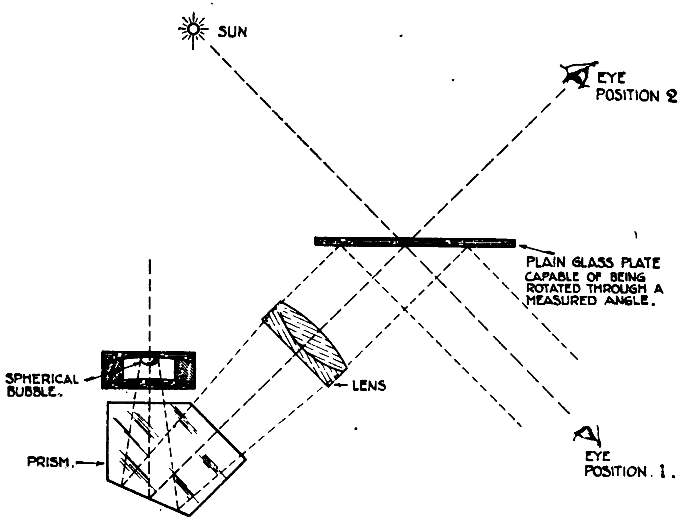


FIG. 45.—Principle of R.A.E. sextant.

first introduced at a time when no suitable alternative type of sextant was available.

77. R.A.E. Bubble Sextant.—The naval pattern sextant shown in Fig. 42, on p. 82, is used to obtain the altitude of any heavenly body above the visible horizon, whether that horizon line be formed by sea, land, fog or cloud bank. The Baker sextant is capable of use when two such horizons in opposite directions are equally visible and may be presumed to be at the same height. But for use at night, or when among broken clouds, it is necessary to have some alternative instrument available which will not depend on the existence of visible horizons of any sort. Such an instrument is the **R.A.E. bubble sextant.***

In this sextant the vertical is given by the position of the bubble in a spherical level, which can be illuminated for night use. The instrument can therefore be used under all circumstances and is therefore of more utility than any of the other sextants mentioned in this book. It is illustrated diagrammatically in Fig. 45. The eye may take up either of positions (1) or (2) shown. Position (1) is best for star observations and position (2) for sun observations—but, theoretically, there is no reason why either position should not be used for any or all observations ; it is merely a matter of convenience which is chosen. It is easier to find a star by direct vision, whilst it is quite easy to find an object so bright as the sun by reflection on the glass plate and much less dazzling to look downwards than upwards.

As the lens is chosen to have a focal length equal to its optical distance from the bubble, and as the curvature of the upper surface of the latter is also equal to this distance, it follows that the bubble will always be in focus and will appear to move *with* the sun or star if the instrument should rock in the hand—this enormously facilitates observations.

* A bubble sextant designed by Prof. Willson has been used on aircraft in U.S.A. with success.

CHAPTER VI

REDUCTION OF ASTRONOMICAL OBSERVATIONS

Requirements—Dip—Refraction — Parallax — Semi-diameter — Greenwich date—Methods employed—D'Ocagne nomogram—Veater diagram—Spherical triangle slide rules—Baker machine—Logarithmic computation.

78. Astronomical observations.—Astronomical observations need to be corrected in various ways before they can be " reduced " to give the navigator his position. Thus if an altitude has been measured above an apparent horizon, a correction for the dip of the apparent horizon below the true horizontal plane requires to be made. All altitudes are liable to require correction for refraction, i.e. the bending of a ray of light owing to the change of density of the air encountered by the ray as it enters the atmosphere—this correction, fortunately, is of no great moment in air navigation. Similarly an effect called parallax affects such observations, but fortunately also it scarcely troubles the air navigator unless a sight on the moon is being taken. These corrections will now be explained.

79. Requirements.—When the altitude, and perhaps the azimuth, of some heavenly body has been measured at an instant of time noted by a watch or chronometer, the next step is to use this information to obtain the position, or position line, of the aircraft. This is called " reducing the observations " ; but before the readings are ready for " reduction " they have to be corrected first for any index error in the observing instrument and then for :—

Dip.
Refraction.
Parallax.
Semi-diameter.

An observation of the moon's altitude when just above the visible horizon would bring in all four corrections; but a bubble sextant observation of a star would not need to be corrected for any of them unless the altitude were below 20 degrees, which would be unusual, since whenever possible such small angles would be avoided.

80. Correction for Dip.—Owing to the spherical shape of the earth, a line joining an observer in an aircraft to the visible horizon will be sloped below the horizontal. Thus the true horizontal from an aircraft at A (see Fig. 46) is A B, whereas the line to the horizon is A C. The angle B A C is the "**dip.**" It is increased by the refraction caused by the change in density of the atmosphere with altitude, and its actual value, taking both factors into account, is equal to the square root of the height in feet. This gives the "dip" in minutes of arc. Thus at 10,000 feet the dip is 100 minutes of arc.

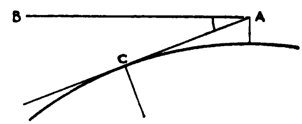


FIG. 46.

81. Correction for Refraction.—The correction for refraction as affecting the "dip" is included above. Refraction also has the effect of making the altitude of all the stars above the *true* horizon appear to be a little greater than it really is. For the degree of accuracy required in air navigation it may be usually neglected, since it is never more than 3 minutes of arc for all altitudes of 20 degrees and over. A table of refraction corrections is given in Inman.

82. Correction for Parallax.—Astronomical tables are calculated on the assumption that the observer is at the centre of the earth. Owing to the small size of the earth in comparison with the distances separating it from the heavenly bodies, the correction for the earth's diameter is small, and it is easily calculated. Thus in Fig. 47 an observer at A would read the zenith distance of the body S as the angle Z A S, whereas if at the centre of the earth at E he would read the angle as Z E S, which is less than

Z A S by the angle A S E. The angle A S E is the correction for parallax. Its largest value—called the "horizontal parallax"—would occur when A E was at right angles to E S. If S were the sun, E S would be 93,000,000 miles and as A E is but 4000 miles the maximum value the parallax could have would be an angle whose sine was 4000÷93,000,000, which is but 9 seconds of arc, and for air navigation would be quite negligible. This, moreover, is the maximum value for the sun; in general it would be less than this in the ratio of the cosine of the altitude,

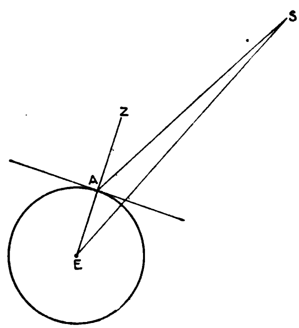


FIG. 47.

and the resulting angle is called the "parallax in altitude." The moon is the only body which has a large enough parallax to affect air navigation observations. At its maximum it is an angle whose size is 4000÷240,000, i.e. about one degree. A table of corrections for parallax is given in Inman.

83. Correction for semi-diameter.—Sextants, even when fitted with magnifying telescopes, do not show any disc for stars or planets. In the case of the moon or sun, however, the observing point may be the centre, or the upper or lower "limb." An appropriate correction must be made if the limb is used, the semi-diameter must be added or subtracted so that the final result applies to the centre, for which the tables are calculated. Since the angular diameter of both sun and moon are always approximately 30 minutes of arc, the correction for semi-diameter will be 15 minutes.

WORKED EXAMPLE

Thus if the altitude above the sea horizon, observed from 2500 feet, of the sun's lower limb were 46° 32′, then the true altitude of the sun's centre would be :—

Observed altitude	46°	32′	
Correction for dip		50′	subtract
	45°	42′	
Correction for semi-diameter . .		15′	add
True altitude	45°	57′	

Corrections for refraction and for parallax are negligible.

84. Greenwich date.—The Greenwich date (G.D.) for any observation is the approximate astronomical time at Greenwich at the moment of the observation. It is needed to enable data to be taken out of the Nautical Almanac ; and is most easily obtainable from the local astronomical time with the addition or subtraction of the D.R. longitude. The declination of the sun, for instance, is tabulated in the N.A. for various Greenwich times, so that the declination of the sun will depend not on local time but on Greenwich time.

Thus if the local time be 3.40 a.m. on Sept. 28th, 1920, and D.R. longitude is 10 h. 30 m. W. (i.e. 157° 30′ W.), the Greenwich date is found by addition :—

Local time	3 h. 40 m. a.m. on Sept. 28th.
Subtract to convert to astronomical local time	12 h. 0 m.
	15 h. 40 m. on Sept. 27th.
Add longitude W.	10 h. 30 m.
G.D.=	26 h. 10 m. on Sept. 27th.
or	2 h. 10 m. on Sept. 28th.

85. Methods of reducing observations.—There are three alternative methods available. The traditional procedure is the employment of logarithmic tables, but it is equally possible to proceed by graphical methods, or by the use of a calculating mechanism.

Reduction of Observations

Logarithmic or other tables.	Calculating machines such as special forms of slide rule.	Graphical methods employing curves.
(1)	(2)	(3)

The disadvantage of (1) for air navigation lies in the bulk of the reference books and their usual over elaboration for the

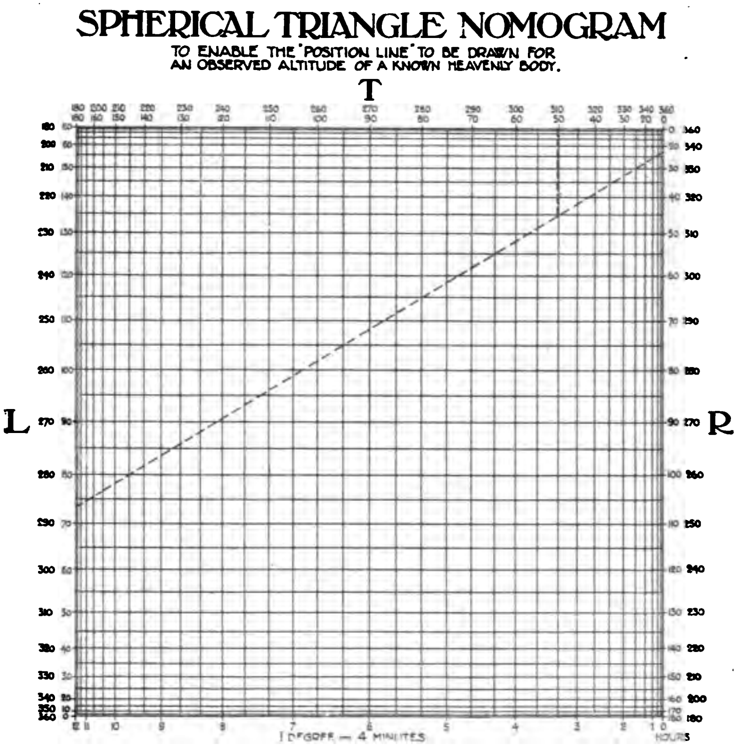


FIG. 48.

standard of accuracy fixed for air observations. It would be possible to prepare less elaborate editions for air use, but before selecting this alternative it is well to see what can be done by methods (2) and (3).

86. Graphical methods.—The easiest graphical method to understand is the **d'Ocagne Nomogram** shown* in Fig. 48. To use this chart proceed as follows :—

(*a*) Enter scale L with the sum of the latitude of the D.R. position and the declination. Enter scale R with the difference of the latitude and declination, and join these two points by a straight line. Enter scale T with the hour angle in degrees (or along the base, for hours and minutes) and drop a vertical to this line; run across from the intersection point to the scale R, and read off the zenith distance and so obtain the calculated altitude for the "D.R." position.

(*b*) Next enter scale L with the sum of the latitude and the altitude above obtained, and scale R with the difference of these two; join these two points by a straight line. Enter scale R with the co-declination and draw a horizontal line from this point. At the intersection raise a vertical and on scale T read off the calculated azimuth (E. of N.) corresponding to the "D.R." position.

The following example illustrates the method :—

Latitude by D.R.	=	48° N.
Declination of heavenly body	=	25° N.
Deduced hour angle of heavenly body	=	50° or 3 h. 20 m.
Corrected altitude, observed with sextant	=	45° 6′

By par. (*a*), sum of altitude and declination=48° plus 25°=73°, difference 48°−25°=23°. With hour angle 50°, zenith distance is found to be 45°·4, giving *calculated altitude* 90°−45°·4=44°·6 or 44°36′. By par. (*b*),

* This nomogram was brought to the author's notice by Mr. Gerrans of Worcester College, Oxford.

sum of latitude and altitude=48°·0 plus 44°·6=92°·6 and difference 48°·0−44°·6=3°·4. With co-declination 90°−25°=65°, this gives *calculated azimuth* as 258°·0 E. of N., or 78° W. of S. The position line can then be drawn in the usual way.

87. Accuracy of Nomogram.—The d'Ocagne Nomogram if drawn out about 15 inches by 15 inches enables the zenith distance to be obtained quickly, in a large aeroplane, with an average accuracy of about seven minutes of arc, but more than this must not be looked for. Air observations should, if possible, be reduced with an accuracy of two or three minutes, and the nomogram would become very large if drawn to a scale to enable this accuracy to be obtained.

88. Graphical solution of Spherical Triangles, by true Projections of the Sphere.—Spherical triangles could, of course, be solved by direct measurement on a suitable sphere, graduated with meridians of longitude and parallels of latitude to the required degree of accuracy. Such a sphere, graduated to each 10°, is shown in Fig. 49. The observer's position O is fixed on the observer's meridian E P by the dead reckoning latitude E O. The point Z, representing the geographical position of the observed heavenly body, is fixed by the hour-angle E P F and the declination F Z. It is now required to measure the position of the point Z with reference to the observer O, in terms of the zenith distance O Z and the azimuth angle E O Z. These measurements can be made directly by the rotation of the line O Z as a whole across the surface of the sphere, about an axis A B at right angles to the observer's meridian, until it takes up the position P Y. The rotation of the point O in moving to P is obviously equal to the co-latitude O P, and the rotation of the point Z in moving to Y is also equal in angle to the co-latitude, though the actual distance appears smaller owing to the point Z travelling on a smaller circle than O, about the common axis A B. As the observer's meridian is a great circle having A B for its axis, the angle between the line O Z and the meridian is not changed during the rotation to P Y, and the azimuth E O Z becomes the angle O P Y and can be directly read off against the graduations on the sphere. The zenith distance P Y (or the more useful complementary angle, the altitude, G Y) can also be read off. It will be seen that there is no need

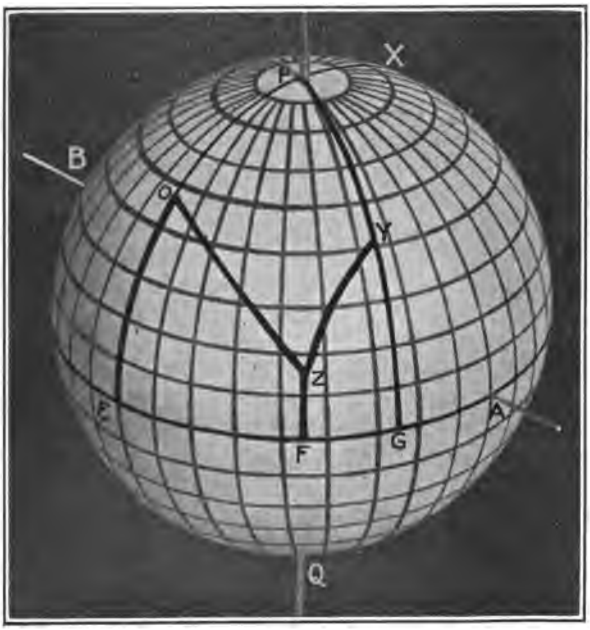


FIG. 49.

actually to construct the line O Z, as any problem could be solved by fixing the point Z from the hour-angle E P F and the declination F Z, rotating this point through an angular distance equal to the co-latitude to the point Y, and reading the altitude G Y and the azimuth O P Y against the parallels and meridians respectively.

As the graduation of the whole sphere is symmetrical, any problem could be solved on the octant E P A, provided that suitable rules were followed in its use. It would be inconvenient to use such an octant of a sphere, but any true projection of it can be employed. This has been done by Commander Veater.

89. Veater Diagram.—Commander Veater, R.N., first proposed the Mercator projection for this purpose, and this is now employed under the name of the Veater Diagram. The projection is made with the observer's meridian E P as the equator, and is shown in Fig. 50, lettered in a similar manner to the sphere. With this projection, rotation about the axis A B on the sphere becomes motion in a straight line parallel to the meridian E P, and the length of the line (the projected angular distance) is the same for a given angle on all parts of the diagram; for instance, O P and Z Y become parallel straight lines of equal length. It should be noted that the point A becomes projected to infinity, but this point is not required as a heavenly body having this geographical position would appear on the observer's horizon.

The diagram shown in Fig. 50 would have to be very large to solve spherical triangles with sufficient accuracy, and would therefore be inconvenient to use, but it can be divided up into a number of small sheets of suitable size, examples of which are given in Figs. 50A and 50B. The key diagram (Fig. 50) is used to obtain the number of the sheet of the enlarged diagram that is to be employed for entering with the hour-angle and declination.

In order to allow the position of the point Y to be determined on the enlarged diagrams, they are graduated with red co-ordinates, the horizontal distances being measured from the edge E K, Fig. 50, and the vertical distances from the observer's meridian E P. The point Z can therefore be read with reference to these co-ordinates, the co-latitude being added to the horizontal distance J Z to obtain the distance J Y, which is used in conjunction with the vertical distance to determine the position of the point Y on the second diagram, the sheet to be used being found without difficulty. In order to allow all possible triangles to be solved without having to subtract angles above 90 degrees from 180 the figuring appertaining to the adjacent octant X P A H (Fig. 49) is given in red on the diagrams.

Rules for solving all possible cases will now be given, and their derivation should be clear from the foregoing description.

In all cases the *hour angles* are measured east or west up to 180, and the *azimuth* from the pole of the opposite name to the latitude, east if the hour angle is east and west if the hour angle is west. When red figures are used for entering a diagram, the result must be taken out in red figures.

(*a*) Latitude and Declination same name. Enter diagram with declination and hour angle. Read vertical and horizontal co-ordinates. Add 90 degrees to horizontal co-ordinate and subtract latitude. Re-enter with the second horizontal co-ordinate so obtained and the vertical co-ordinate, and read the altitude and azimuth.

(*b*) Latitude and Declination opposite names. Enter diagram with declination and hour angle. Read vertical and horizontal co-ordinates. Add the latitude to the horizontal co-ordinate, and subtract the result from 90. Re-enter with the second horizontal co-ordinate so obtained and the vertical co-ordinate, and read the altitude and azimuth.

The following examples will illustrate the use of these rules, the Z or Y point being the same for all examples and marked by a small circle on each diagram (Fig. 50A and 50B). It will be noticed that the horizontal co-ordinate is read in degrees and minutes whilst the vertical co-ordinate is read in whole numbers and decimals to an arbitrary scale, as this avoids any doubt as to which co-ordinate has to be modified by subtracting or adding the latitude, etc.

Z Point.					Lat.	Y Point.				
Diagram.	Decln.	Hour Angle.	Co-ordinates.			Diagram.	Co-ordinates.		Altitude.	Azimuth.
	d	t	Horiz.	Vert.	l		Horiz.	Vert.	h	A
C. 2	N 15°–25′	E 45°–0′	21°–18′	2·86	N 47°–2′	C. 5	64°–16′	2·86	41°–14′	E 65°–0′
C. 2	S 15°–25′	W 45°–0′	21°–18′	2·86	N 4°–26′	C. 5	64°–16′	2·86	41°–14′	W 65°–0′
C. 5	N 41°–14′	W 65°–0′	64°–16′	2·86	N 38°–32′	C. 5	115°–44′	2·86	41°–14′	W 115°–0′
C. 5	S 41°–14′	E 115°–0′	115°–44′	2·86	S 47°–2′	C. 2	158°–42′	2·86	15°–25′	E 135°–0′
C. 2	N 15°–25′	E 45°–0′	21°–18′	2·86	S 47°–24′	C. 2	21°–18′	2·86	15°–25′	E 45°–0′
C. 5	S 41°–14′	W 65°–0′	64°–16′	2·86	N 4°–26′	C. 2	21°–18′	2·86	15°–25′	W 45°–0′

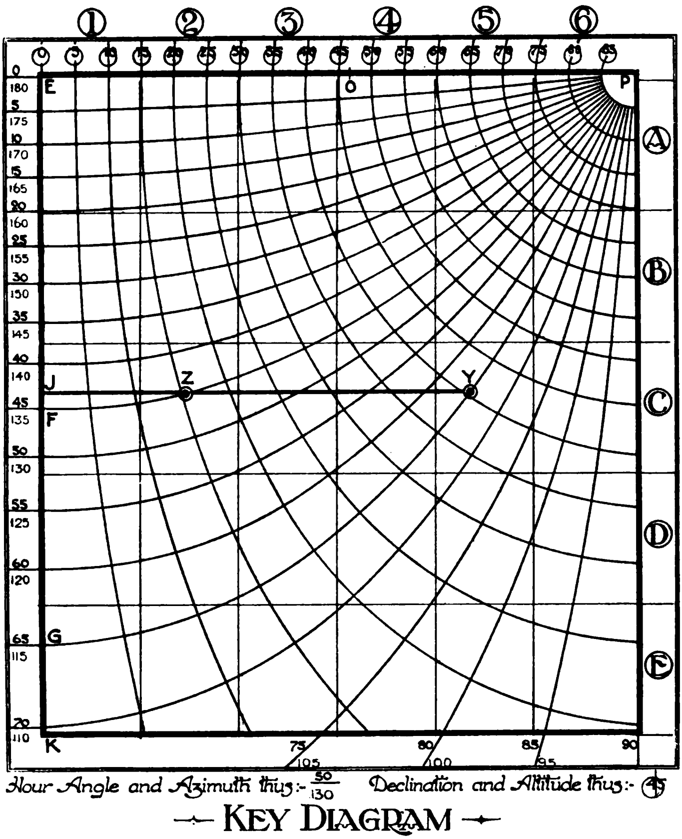
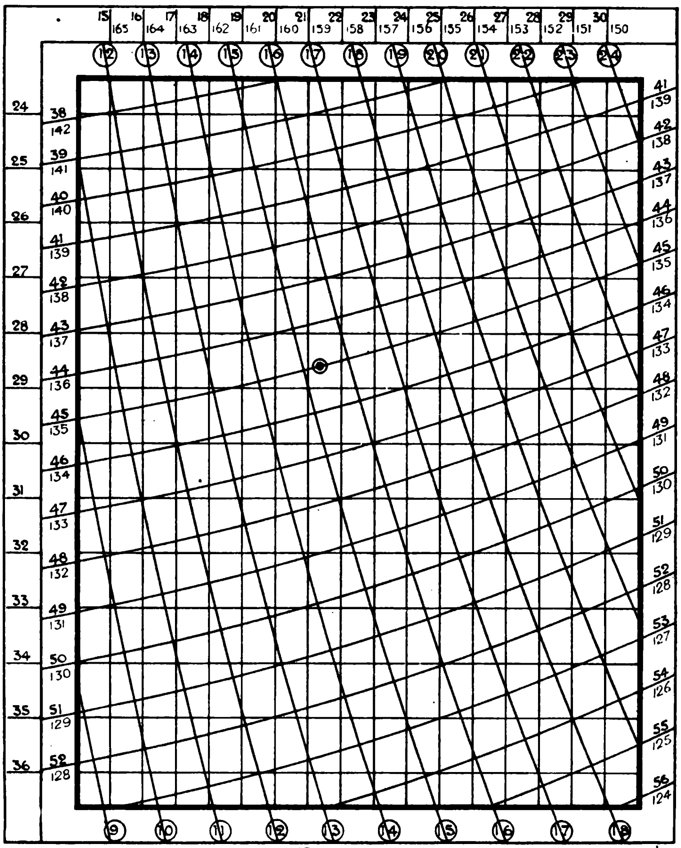


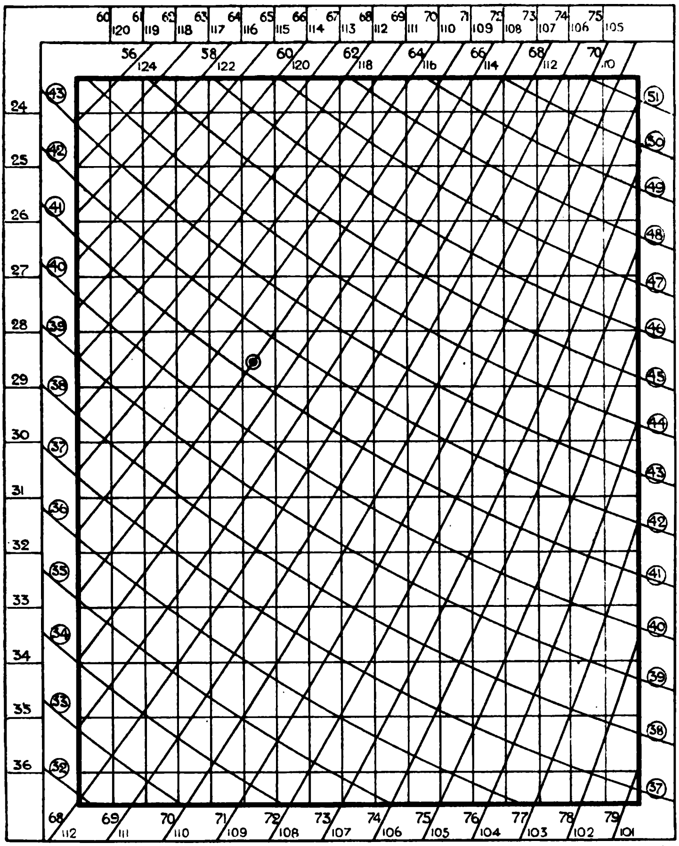
FIG. 50.—Veater Diagram.



Hour Angle and Azimuth thus:- $\frac{45}{135}$ *Declination and Altitude thus:-* (17)

C2

FIG. 50A.



Hour Angle and Azimuth thus:- 64/116 Declination and Altitude thus:- (39)

C5

FIG. 50B.

90. Bygrave slide rule.—Attempts have been made to construct logarithmic slide rules for solving spherical triangles. The haversine-cosine formula has usually been favoured, as it is possible to construct a rule for obtaining the altitude from the latitude, declination and hour angle, by which the result is obtained without having to note down any intermediate figures. But it is difficult to obtain a high degree of accuracy owing to the comparatively short length of any one scale.

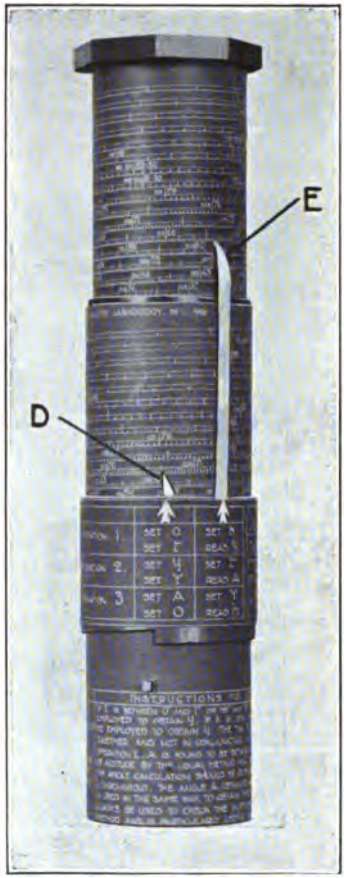


FIG. 51.
Bygrave spherical triangle slide rule.

The rule shown in the photograph (Fig. 51) employs the simple right-angled spherical triangle formulæ and is capable of a comparatively high degree of accuracy. It is necessary to note down an intermediate result, but as the operations are simple and quick, it compares favourably with other methods.

In Fig. 52 the method of dividing up the triangle to be solved into two right-angled spherical triangles is shown, O being the position of the observer, and Z the geographical position of the observed body. The bases of the two right-angled triangles are $(90°-y)$ and $(90°-Y)$, and $(90°-y)\pm(90°-Y)$ is equal to the co-latitude.

The following formulæ, then, connect the parts :—

1. $\tan y = \dfrac{\tan \delta}{\cos t}$
2a. If λ and δ are same name, $Y = y + 90° - \lambda$.
2b. If λ and δ are opposite names, $(Y = 90° - (y + \lambda))$.
3. $\tan A = \dfrac{\cos y \quad \tan t}{\cos Y}$
4. $\tan h = \cos A. \tan Y.$

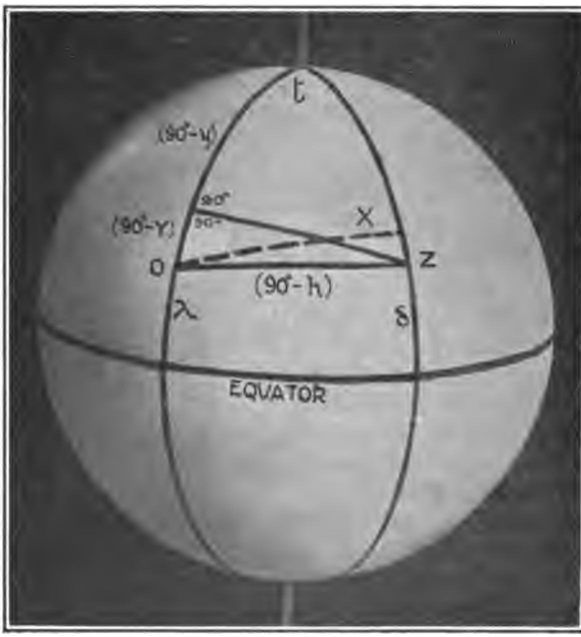


FIG. 52.

For the ordinary determination of azimuth and altitude for plotting the "position line" the procedure is as follows (see Fig. 51) :—

With pointers in the zero position, turn the inner cylinder until the declination is opposite E ; move the ring until D is set on the hour angle and read y at E. If declination and latitude are same name add 90° to y and subtract latitude to obtain Y, or if declination and latitude are opposite names add latitude to y and subtract from 90° to obtain Y. Set D to y, turn inner cylinder until the hour angle is at E, set D to Y and read the azimuth at E. Set D to azimuth, turn inner cylinder until Y is at E, move the pointers back to the zero position and read the altitude at E. The complete operations take but a few minutes and an accuracy of one minute of arc is usually obtained.

A special scale of short length is provided for dealing with hour angles between 89° and 91° and declinations less

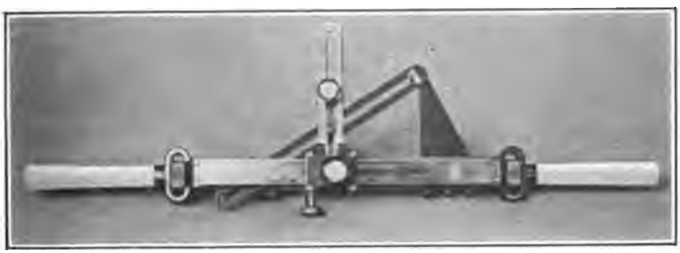


FIG. 53.—Nomogram slide rule.

than 1°. If the azimuth is found to be very near 90° the determination of the altitude by the usual method would not be very accurate, so the operations are repeated interchanging declination and latitude throughout. This is equivalent to dividing the triangle by X, and good results can then be obtained.

The diameter of the instrument is 2 inches—its length 6 inches.

91. Calculating mechanisms.—The d'Ocagne nomogram, since it consists only of straight lines can be made into a "slide rule" mechanism without a great deal of trouble, and such a slide rule is much easier to employ than the nomogram itself and may be expected—if very carefully made—to be more accurate. The uncertain factor is whether continued use would lead to a degree of wear on the sliding surfaces sufficient to interfere with the required accuracy being obtained. A slide rule (Wimperis and Horsley) of this kind is shown in Fig. 53.

Alternatively, the haversine formula may be solved in two steps by a double slide rule of either straight or circular form. Professor Poor has prepared a circular slide rule of this kind, but it is not very suitable for air use.

A combination of mechanical and graphical methods is afforded by Comdr. Baker's "Navigation Machine," in which the rotation of the earth is simulated by winding a

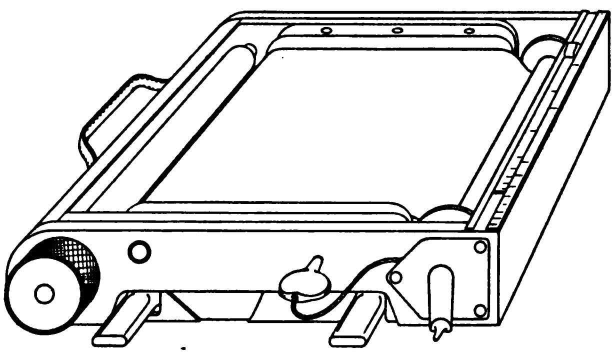


FIG. 54.—Baker Navigation Machine.

long chart from one roller to another, underneath a long tracing to represent the celestial sphere.

92. Baker Navigation Machine.—The purpose of this instrument is to simplify the calculation that has to be done after a sight has been taken in order to lay down the corresponding position line upon the Chart. It consists of a framework holding the chart and carrying a transparent diagram mounted on rollers, which slides over the surface of the chart. It is shown generally in Fig. 54. The direction of this sliding motion is along parallels of latitude, and the apparatus is so designed that a certain line on the transparent diagram always tracks upon a certain parallel of latitude.

Upon the transparent diagram are marked :—

1. A time scale (1° long = 4 min.) corresponding with the longitude scale of the chart.

2. A series of curves of constant altitude cut orthogonally by

3. A series of curves of constant Position Angle.

The method of construction of these scales and curves is as follows :—

At any instant the geographical position of the sun G (see Fig. 55) has latitude equal to the Sun's Declination at the moment and longitude equal to the Greenwich Apparent Time.

At any point X of the earth's surface the True Zenith Distance of the sun at the same instant is the distance G X, and therefore all points on the same circle through X, whose centre is G, will see the sun at the same True Zenith Distance as at X. Curves of constant altitude for every exact degree, upon the earth's surface, are therefore a series of concentric small circles with G as centre and radii 1°, 2°, 3°, etc.—90°, one degree being equal to sixty sea miles.

These circles are all cut at right angles by a series of great circles passing through G, the angle at G (e.g. N G X) being the Position Angle of the ordinary celestial triangle. As the earth turns round, these two sets of curves move to the westward on the earth's surface, and also to the north or south, according to the manner in which the declination is changing.

With a fixed star, the movement of the corresponding curves would be purely westwards.

In the Navigation Machine the motion of these two sets of curves is reproduced by sliding their Mercatorial projection over the surface of a Mercator Chart, but without any attempt to reproduce any daily change in declination. As the sun's declination alters, from 23½° S. to 23½° N., allowance is made for the change by constructing the curves for every fourth degree of declination from 24° S.

to 24° N., making thirteen diagrams in all. For any instant when the declination is different from any one of these thirteen values, the nearest one is used, and corresponding correction applied to the altitude.

In addition, for the thirteen diagrams for the Sun, whose declination changes, a set of curves is also supplied for use

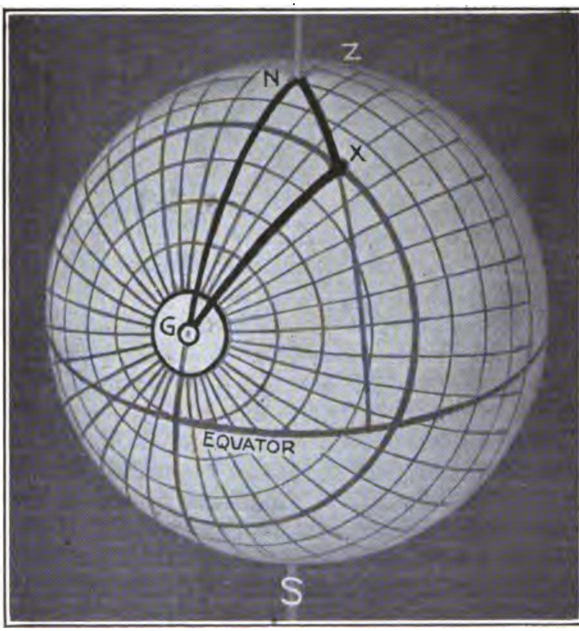


FIG. 55.

with certain fixed stars. In order to make the diagram not too confused, the star curves are drawn without the corresponding position angle curves and are shown, on the Transatlantic Chart, for each 2 degrees of altitude.

Position lines for intermediate angles can be laid down by drawing a line parallel to the nearest curve shown at a distance from it equal to the difference in altitude, measuring the distance from the latitude scale which has been put in on each meridian shown on the diagram. For example, if the navigator observed the altitude of Rigel to be 26° 50′ at a time when his approximate position was 52° N., 45° W., and his time by watch, keeping Greenwich Sidereal Time, was 6 hours 40 minutes, his position line would be drawn parallel to that for 26° N., and at a distance from it in a direction about SS.E. equal to 50′ taken from the latitude scale in the neighbourhood of 52° N.

93. Reduction by logarithmic computation.—This is the method hitherto invariably used at sea. It is mathematical, but the work is tedious rather than difficult.

The following symbols are in use :—

h	Altitude.	R.A.	Right Ascension.
z	Zenith distance.	E	Equation of Time (plus when added to A.T.)
A	Azimuth.		
d	Declination.	A.T.	Apparent Time.
p	Polar distance.	M.T.	Mean Time.
t	hour angle.	T_o.	Greenwich Time.
l	latitude.	T	Local Time.
c	co-latitude.	L	Longitude of place.

The computing work employs the standard trigonometrical relationships of **spherical trigonometry.** Those formulæ, and their proofs, are given in the usual mathematical text-books. For the purpose of navigation work the important relationships (see Fig. 56) are as follows :—

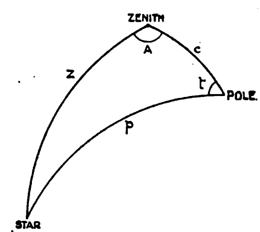


FIG. 56.

$$\cos z = \cos c \ \cos p + \sin c \ \sin p \ \cos t \ldots\ldots\ldots (1)$$

$$\frac{\sin A}{\sin p} = \frac{\sin t}{\sin z} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The former equation can also be written in **Haversine** form, as :—

$$\text{Hav } z = \text{Hav } (c \backsim p) + \text{Hav } t \{\text{Hav } (c+p) - \text{Hav } (c \backsim p)\} \quad \ldots (3)$$

and this is an important alternative as it forms the basis of the d'Ocagne nomogram. The Haversine of an angle (being equal to the square of the sine of half the angle) is never negative in sign; this facilitates its use and reduces the chance of a slip in the calculation. The following table illustrates the change of sign for the more common trigonometrical functions and the constancy of the sign of the haversine.

angle.	sin.	cos.	tan.	hav.
0	0	1·00	0	0
30	0·500	0·866	0·577	0·067
60	0·866	0·500	1·732	0·250
90	1·00	0	∝	0·500
120	0·866	−0·500	−1·732	0·750
150	0·500	−0·866	−0·577	0·933
180	0	−1·00	0	1·00
210	−0·500	−0·866	0·577	0·933
240	−0·866	−0·500	1·732	0·750
270	−1·00	0	∝	0·500
300	−0·866	0·500	−1·732	0·250
330	−0·500	0·866	−0·577	0·067
360	0	1·00	0	0

94. Calculation for Position Line.—For drawing the position line it is necessary first to ascertain the altitude and azimuth by calculation for the D.R. position. Then to mark off the intercept, allowing for the scale of the map, given by the difference between observed and calculated altitudes, in the direction given by the azimuth. The position line can then be drawn.

To calculate the altitude, it is necessary to find the third side of a spherical triangle given the other two sides and the included angle.

To obtain the azimuth,* it is required to find one additional angle.

95. Given two sides of a spherical triangle and the included angle, to find the third side.—Equation (1) may be written $\text{hav}\, z = \text{hav}\,(c \backsim p) + \sin c \;\sin p \;\text{hav}\, t$

* In marine navigation, the azimuth is usually obtained from special azimuth tables.

or hav [side required] = hav [diff. of sides]
+ sin [1st side]. sin [2nd side]. hav [angle].

In working with this formula it is usual to use an auxiliary angle θ such that

sin [1st side]. sin [2nd side]. hav [angle] = hav θ,

then hav [side required] = hav [diff. of sides] + hav θ.

The value of θ is first obtained, making use of the formula :—

log sin [1st side]+log sin [2nd side]+log hav [angle]
=log hav θ.

EXAMPLE

In triangle Z P S (see Fig. 57) given Z P=42°, P S=65°, Z P S=50°; find Z S.

Angle Z P S		50° 0′	log hav	9·2519
Sides containing angle	P Z	42° 0′	log sin	9·8255
	P S	65° 0′	log sin	9·9573
	∽	23° 0′	log hav θ	9·0347
			hav θ	0·1083
			hav ∽	0·0398
			hav S Z	0·1481

S Z=45° 16′

To obtain the azimuth in addition, use may be made of equation (2) :— $\frac{\sin A}{\sin p} = \frac{\sin t}{\sin z}$

In this particular problem p=65°; t=50°; z=45° 16′. Hence,

log sin A=log sin t+log sin p−log sin z.

log sin 50° = 9·8843
log sin 65° = 9·9573

9·8416
log sin z = 9·8515

∴ log sin A = 9·9901

giving A = 77° 49′ or
102° 11′.

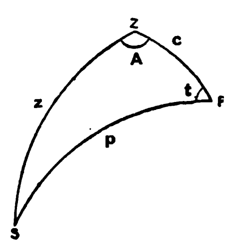


FIG. 57.

To distinguish between these, the following rules are used :—

(1) If the latitude and declination are one north, the other south, A is always greater than 90°.

(2) If they are both north, or both south, and sin d is greater than sin l sin h, then A is less than 90°, and vice versa.

Hence, in this case, we choose the value 102° 11′ (W. of N.)

The convention is often adopted, however, of measuring the azimuth from south through east or west, and since t is positive, the object whose altitude is observed is west. Hence, if we measure the azimuth in this way, the angle is 180°−102° 11′,

$$\text{i.e. } A=77^\circ\ 49'\ \text{W. of S.}$$

This it will be noticed is the example already worked out by the nomogram—see par. 86.

Worked Example.

96. Solar observation.

At $23^h\ 13^m\ 0^s$ G.M.T. on Nov. 13th, 1919, an observation of altitude of the sun is taken to a cloud horizon. The aeroplane is 4200 feet above the cloud layer.

D.R. Long. = 3° 12′ W.
G.M.T. = $23^h\ 13^m\ 0^s$
Eqn. of time = $+0^h\ 15^m\ 37^s$
G.A.T. = $23^h\ 28^m\ 37^s$
= 7° 51′ E.
D.R. Long. = 3° 12′ W.
t = 11° 3′ E.

Declr. = 18° 0′ S. D.R. lat. = 57° 2′ S.

From this we find

h (D.R.) = 21° 1′ A (D.R.) = 168° 44′ E. of S.

Observed altitude is

h = 22° 12′
Dip correction = 1° 5′
21° 7′
Semi-diam. = 0° 16′
h = 21° 23′

Diff. of altitudes

δ h = 22′

from which the position line may be plotted

Worked Example.

97. Cut of two position lines.

On Jan. 1st, 1919, at S.T. $4^h\ 6^m\ 15^s$ an observation of altitude of Polaris is taken with a bubble sextant, and $2^m\ 3^s$ (S.T.) later, an observation of altitude of β Andromedæ. The ground speed of the aeroplane is 80 knots, and the track angle 15°. The D.R. position for the first observation is Lat. 51° 12′ N.
Long. 4° 16′ E.

The run in the interval between the observations is 3 mls. (to nearest mile).

From the latitude difference and departure table, with the given track angle, the D.R. latitude and longitude for the second observation is found to be Lat. 51° 15′ N.
Long. 4° 17′ E.

Taking the right ascension (R.A.) and declination (d) from the Nautical Almanac for this date correct to the nearest second of time and minute of arc, we find the hour angle (t) thus:

Polaris.

$$
\begin{aligned}
\text{R.A.} &= 1^h\ 31^m\ 49^s \\
\text{S.T.} &= 4^h\ \ 6^m\ 15^s \\
\hline
\text{Diff.} &= 2^h\ 34^m\ 26^s \\
&= 38^\circ\ 36'\ \text{W.} \\
\text{D.R. Long.} &= \ 4^\circ\ 16'\ \text{E.} \\
\hline
t &= 42^\circ\ 52'\ (\text{W.})
\end{aligned}
$$

[W in brackets denotes that the star is west of the observer.]

With $d = 88^\circ\ 53'$ N.
and $l = 51^\circ\ 12'$ N.
we find h (D.R.) = 52° 1′ and A (D.R.) = 178° 46′ W. of S.

If the observed altitude is

$$h = 52^\circ\ 18'$$

we find difference of altitudes

$$\delta\ h = 17'$$

In the same way, we get the hour angle for β Andromedæ.

β. Andromedæ.

$$
\begin{aligned}
\text{R.A.} &= 1^h\ 5^m\ 13^s \\
\text{S.T.} &= 4^h\ 8^m\ 18^s \\
\hline
\text{Diff.} &= 3^h\ 3^m\ 5^s \\
&= 45^\circ\ 46'\ \text{W.} \\
\text{D.R. Long.} &= \ 4^\circ\ 17'\ \text{E.} \\
\hline
t &= 50^\circ\ \ 3'\ (\text{W.})
\end{aligned}
$$

With $d = 35^\circ\ 12'$ N.
and $l = 51^\circ\ 15'$ N.
we find h (D.R.) = 51° 5′ and A (D.R.) = 85° 35′ W. of S.

If the observed altitude is

$$h = 50^\circ\ 48'$$

we find difference of altitudes

$$\delta\ h = 17'$$

Proceeding as in the text, and taking the scale for plotting the altitude differences from the latitude scale of the Mercator map for the D.R. latitude, the cut is found to be at

Lat. 51° 41′ N.
Long. 4° 42′ E.

which is therefore the aeroplane's position.

EXAMPLES. CHAPTER VI

1. Find the altitude and azimuth, with the following values of hour angle, declination and latitude:

	Hour Angle.	*Declination.*	*Latitude.*
(i)	98° 11′	40° 12′ N.	48° 32′ N.
(ii)	0° 36′	6° 15′ S.	53° 25′ N.
(iii)	23° 27′	55° 36′ N.	3° 15′ S.
(iv)	73° 9′	49° 49′ N.	7° 18′ N.

2. The star α Andromedæ was observed on 1 Jan., 1919, at S.T $20^h\ 12^m\ 5^s$. Its R.A. at this date was found from the Nautical Almanac to be $0^h\ 4^m\ 13^s$ and its declination 28° 39′ N. The D.R. position was Lat. 28° 39′ N. Long. 3° 26′ W. Find the calculated altitude and azimuth.

3. The altitude of α Centauri was observed on 10 Aug., 1919, at S.T. $10^h\ 20^m\ 5^s$ and was found to be 14° 8′. The D.R. position, at the time of the observation, was Lat. 5° 8′ N. Long. 15° 6′ E. The R.A. of the star is $14^h\ 34^m\ 10^s$ and its declination is 60° 30′ S. Find the difference observed altitude—calculated altitude (after correcting the observed altitude for refraction), and plot the position line.

4. On 12 Nov., 1919, at S.T. $12^h\ 3^m\ 0^s$ the altitude of ε Ursae Majoris was observed to be 45° 8′. Its A.R. for this date was $12^h\ 50^m\ 30^s$ and its declination 56° 23′ N. The altitude was observed to a cloud horizon 1000 feet below the ship. The D.R. position was Lat. 12° 43′ N. Long. 2° 25′ E. Find the difference observed altitude—calculated altitude and plot the position line.

5. At D.R. position Lat. 42° 12′ N. Long. 28° 32′ W. the altitude of α Can. Maj. was observed to be 12° 15′, at S.T. $12^h\ 18^m\ 23^s$ on 9th July, 1919. The R.A. of the star for this date was found from the Nautical Almanac to be $6^h\ 41^m\ 35^s$, and its declination 16° 36′ S. Find the calculated altitude and azimuth, and plot the position line.

6. On 11th Aug., 1919, at $4^h\ 12^m\ 13^s$ G.M.T. the altitude of the lower limb of the sun is observed to be 28° 30′. The D.R. position is Lat. 51° 43′ N. Long. 2° 18′ W. Find the difference observed altitude—calculated altitude and the azimuth. (From the Nautical Almanac the equation of time is $5^m\ 12^s$ (to be subtracted from G.M.T.), the declination is 15° 27′ H., and the semi-diameter is 16′.)

7. Formalhaut (α Piscis Australis) gave observed altitude 21° 59′ on Aug. 10th, 1919, at S.T. $0^h\ 2^m\ 59^s$. Its R.A. was $22^h\ 53^m\ 15^s$ and its declination 30° 3′ S. on this date in the Nautical Almanac. The D.R. position was Lat. 4° 11′ S. Long. 83° 24′ E. At S.T. $0^h\ 6^m\ 15^s$ an altitude of α Cygni was taken, and gave altitude 32° 49′. The R.A. of this star was $20^h\ 38^m\ 39^s$ and the declination 45° 0′ N. The ground speed of the

ship was 100 knots, and the track angle 231°. Find the altitudes and azimuths, and hence plot the position.

8. α Cygni (R.A. $20^h\ 38^m\ 42^s$ Dec. 45° 0′ N.) gave altitude 32° 38′, at S.T. $0^h\ 12^m\ 6^s$, and β Ursae Minoris (R.A. $14^h\ 50^m\ 50^s$ Dec. 74° 29′ N.) gave altitude 18° 20′ at S.T. $0^h\ 15^m\ 1^s$ on 8th Dec., 1919. The D.R. position of the ship for the first observation was Lat. 6° 27′ N. Long. 101° 43′ W., and the ground speed was 60 knots and track angle 120°. Find the differences observed altitude—calculated altitude (correcting for refraction in the observed altitude of β Ursae Minoris) and the azimuths, and hence plot the position lines.

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APPENDIX I

OCCURRENCE OF FOG

Fog is so much the worst enemy the air navigator has to face that it may not be out of place to consider the conditions which govern its occurrence. Fortunately, meteorologists are usually able to provide warnings when this danger is likely to be met with, the conditions of its formation being fairly well known. It is known, for instance, that whenever the ground is exceptionally cold so that there is a sharp *rise* of temperature from the ground upwards, and at the same time a slow drift of air over it, there must be fog. This "reversed temperature gradient," as it is called, prevents the air below it from escaping upwards. The air below has a sort of "lid" on it. Thus, over a city a smoky pall will presently spread as the smoke finds itself unable to escape in an upward direction.

It might be expected that in a freely moving atmosphere the temperature would become equalized everywhere, i.e. that it would become isothermal. Owing, however, to the compressibility of the air, the law of temperature change is much less simple, being in fact for a well-churned atmosphere the same as that which governs the temperature of the air in the compression stroke of an internal combustion engine, i.e. the temperature changes are adiabatic. A well-churned atmosphere would have not a uniform temperature, but one falling off at the rate of one degree cent. for each 100 metres of altitude. This condition sometimes arises, and when it does the following curious phenomenon occurs, viz. that any body of air which gets slightly more warmed than its neighbour at once rises to the top, since at all positions it is slightly less dense than the surrounding air. Similarly, any portions slightly colder than their neighbours tend to descend to the bottom. Thus, on such occasions the whole atmosphere is very unstable.

When, however, the temperature falls off *less* rapidly than one degree cent. per 100 metres, then the atmosphere is stable. The *average* condition is a fall of 6½ degrees cent. per 1000 metres. It is very stable when the temperature is isothermal, and exceedingly so under a reversal of gradient. A steeper falling off in temperature than the adiabatic would seem to be impossible.

The bearing of this on fog formation has been pointed out by Sir Napier Shaw, following on many observations of the temperature of the air in fog, that "fog is always associated with an inversion of the lapse of temperature with height. The coldest stratum is at the ground and the temperature gradually increases upwards within the fog and for some additional height beyond it. The cloud of fog is the immediate effect of the mechanical convection of cold caused by the mixing of consecutive strata in the turbulence of the eddy-motion and in spite of the fact that the coldness of the lowest layers makes for stability and restrains convection."

Newfoundland fogs are proverbial, and since transatlantic flights may have that island as their starting-point the conditions for air navigation on that route seem at first sight somewhat unpropitious. It must be remembered, however, that these particular fogs are chiefly known by their effects on ships, and that the essential danger for air navigation is fog over land, not fog at sea. Fog at sea, like fog anywhere else, is always low-lying and can be flown above, but fog over a landing-ground is another matter.

APPENDIX II

THE EFFECT OF DIP AND ACCELERATION ON THE COMPASS

1. LET T be the product of the horizontal magnetic field of the earth into the magnetic moment of the compass; let Q be the same product for the vertical component of the field. Also write: g = acceleration of gravity,

h = distance of unbalanced mass from pivot,

m = unbalanced mass.

We may pair off the couple due to the vertical component of the earth's field and the couple due to gravity acting on the unbalanced mass. The resultant couple for these is zero for all positions of the compass card, and the condition for this is: $Q = m\,h\,g.$

The only forces left over which have a couple tending to turn the card about the normal to the card, i.e. which tend to alter the reading are:

(i) The couple due to the horizontal component of the field; and (ii) the couple due to accelerations (other than gravity) acting on the unbalanced mass.

If ϕ is the angle (assumed small) between the compass needle and the direction of the horizontal field, (i) gives $T\phi$ as the couple tending to reduce ϕ, acting about the normal to the plane through the needle and the direction of the horizontal field. If the compass card is parallel to the floor of the machine at all times, and the machine is flying with its fore and aft axis horizontal, the latter direction lies in the card, and the couple is therefore about the normal to the card. Owing to the short period of the card about an axis in its plane (5 seconds in the type 253 compass but much less in all other compasses tested) the card behaves, for rotations in an axis in its plane, as an infinitely short pendulum, and during banks therefore remains parallel to the floor (this may equally be stated to be a result of observation). Hence for a machine flying level, or on a *steady* glide, the conditions are fulfilled, and $T\phi$ is the couple about the normal to the card due to (i).

2. If A is the acceleration (other than gravity) resolved in the East-West direction *on the card*, the couple due to (ii) is $A\,h\,m$, and acts about the normal to the plane of the card.

Hence the couple tending to alter the reading of the card is $(A h m - T\phi)$, the couple $T\phi$ tending always to reduce the deflection due to $A\,h\,m$.

Replacing $h\,m$ from the identity $Q = h\,m\,g$, we get

$$\text{Couple} \qquad = \frac{A}{g}Q - T\phi$$

$$\text{If} \qquad \delta = \text{angle of dip, } \tan\delta = \frac{Q}{T}$$

$$\text{Hence Couple } = T\left(\frac{A}{g}\tan\,\delta - \phi\right)$$

Suppose the aeroplane to be flying due North and to begin to turn towards East, i.e. to starboard. Then the acceleration due to the turn is towards the West, and since the unbalanced mass is South of the pivot, the card begins to be deflected towards starboard also. As soon as the needle is deflected from North through this cause, the horizontal field begins to be effective, and tends to reduce the deflection.

3. *If the compass has a short period*, this means that the inertia is small compared with the controlling couples, and in consequence it moves rapidly to the equilibrium position. Hence we may find ϕ by equating the couple to zero. But if β is the angle through which the machine has turned and x is the compass reading (both supposed small if we consider only the commencement of the turn) then $\phi = \beta - x$. Moreover, if α is the angle of bank, $A = \alpha g$, since at the commencement of the turn we may consider the East-West direction of the card to be approximately athwartships, and we get

$$\alpha\tan\,\delta - \beta + x = 0$$

$$\text{i.e.} \qquad x = \beta - \alpha\tan\,\delta$$

Hence x, the angle of turn recorded on the card, is too small by $\alpha \tan\,\delta$. Since $\tan\,\delta = 2{\cdot}145$ in these latitudes, too small by about 2 degrees for every degree of bank. Now the rate of turn for a bank α is $\frac{\alpha g}{V}$, if V is the velocity of the aeroplane, and the rate of turn at the commencement was zero. Hence we may take the average rate of turn to be $\frac{\alpha g}{2V}$ while the bank α is being put on. Suppose the aeroplane is putting on bank at the rate a degrees per second. Then at the end of the first second, $\beta=\frac{ag}{2V}$ and therefore at this instant

$$x=a\left(\frac{g}{2V}-\tan\,\delta\right).$$

At a speed of 80 m.p.h., $V=117$ feet per second, and $g=32$, so that $x=a(\frac{1}{7}-2)$ approximately, i.e. the compass begins to show a turn in the wrong direction at $1\frac{6}{7}a$ degrees a second, i.e. if the rate of bank is 1 degree per second, at $1\frac{6}{7}$ degrees per second.

If the aeroplane is flying at an angle β_0 with the direction of North (β_0 measured towards East), then the acceleration resolved in an East-West direction is ag cos β_0.

Hence $x=\beta-ag\cos\beta_0\tan\delta.$

When the aeroplane is flying due East, $\beta_0=90$ degs., and $x=\beta$, i.e. the turn begins to be recorded correctly.

For turns off West, $\beta_0=270$ degs., and $\cos\beta_0=0$, and the same applies.

For turns off South, $\beta_0=180$ degs., and $x=\beta+ag\cos\beta_0\tan\delta$, i.e. the turn is measured by the compass just as much in excess of its true value on Southerly courses, as it was less than its true value on northerly courses.

4. *If the compass has a long period*, this means that the controlling couples are small compared with the inertia of the card. Hence the card lags behind the equilibrium position for the couples. Consequently, though a compass will always be deflected to East in a turn to East off North, the aeroplane may actually (if the compass period is long enough) turn more rapidly than the card, so that a turn in the right direction (though too small in amount) may be shown.

For turns to port off any course, clearly x, β and a change sign, the equations therefore remain the same, and precisely the same effects are observed.

Since the term tending to give x a positive value is $\frac{ag}{2V}$, as V increases x will be more likely to have a negative value. Consequently erroneous readings are more pronounced on a fast than on a slow machine.

R. S. C.

ANSWERS TO EXAMPLES

Chapter II.

	Distance.	*Track Angle.*
(i)	235 miles	358°
(ii)	150 ,,	268°
(iii)	350 ,,	348°
(iv)	54 ,,	38°
(v)	44 ,,	85°

Chapter III.

1.

	Ground Speed.	*Drift Angle.*
(i)	92 knots	11°
(ii)	99 ,,	6°
(iii)	124 ,,	11°
(iv)	35 ,,	25°

2. *Wind Speed.* 21 knots. *Direction.* 263°

3. *Wind Speed.* 20 knots. *Direction.* 242°

4.

5. Yes. 10 per cent.

6. 14° 24′.

7. 132° Magnetic.

8. 8 per cent.

Chapter IV.

1. (i) 1° 30′.
   0° 47′.
   5° 10′.
   0° 12′.

2.

3.

4.

5. (i) Lat. 51° 10′ N. Long. 0° 9′ W.
   (ii) Lat. 51° 6′ N. Long. 0° 8′ W.

6. Lat. 50° 52′ N.
   Long. 2° 25′ W.

CHAPTER VI.

1.

	Altitude.	*Azimuth.*
(i)	24° 19′	56° 4′
(ii)	30° 20′	179° 19′
(iii)	28° 5′	14° 48′
(iv)	16° 25′	40° 4′

2. Altitude 39° 31′.
Azimuth 91° 38′.

3. Difference of altitudes −16′.
Azimuth 157° 40′ E.

4. Difference of altitudes —21′.
Azimuth 11° 8′ W.

5. Altitude 12° 5′.
Azimuth 126° 3′ W.

6. Difference of altitudes = −16′.
Azimuth = 77° 47′ W.

7. *Formalhaut.* Altitude 22° 12′.
Azimuth 120° 49′ W.
a Cygni. Altitude 32° 39′.
Azimuth 27° 20′ W.

8. *a Cygni.* Alt. difference −29′.
Azimuth 39° 7′ E.
β Ursae Minoris Alt. difference +2′.
Azimuth 10° 20′ W.

TABLES OF CONSTANTS

LENGTH

	INCHES.	FEET.	YARDS.	CM.	METRES.
1 IN.		·0833	·02767	2·540	·0254
1 FT.	12·		·333	30·48	·3048
1 YD.	36·	3·		91·44	·9144
1 CM.	·3937	·03281	·01094		·01
1 M.	39·37	3·281	1·094	100·0	

AREA

	SQ.INCHES.	SQ.FEET.	SQ.YARDS.	SQ.CM.	SQ.M.
1 SQ.IN.		·006944	·0007716	6·452	·0006452
1 SQ.FT.	144		·1	929·0	·09290
1 SQ.YD.	1296	9		8361·	·8361
1 SQ.CM.	·1550	·001076	·0001196		·0001
1 SQ.M.	1550·	10·76	1·196	10,000	

VELOCITY

	FEET/SEC.	FT./MIN.	M.P.H.	KNOTS	CMS/SEC.	METRES/SEC.	KM/HR.
1 FOOT/SEC		60	·68182	·592105	30·4801	·304801	1·0973
1 FOOT/MIN.	·016667		·011364	·0098684	·50800	·00508	·018288
1 MILE/HR.	1·4667	88		·86842	44·704	·44704	1·60935
1 KNOT	1·6889	101·33	1·1515		51·478	·51478	1·8532
1 CM/SEC.	·032808	1·9685	·022369	·019426		·01	·036
1 M/SEC.	3·2808	196·85	2·2369	·9426	100		3·60
1 KM/HR.	·91134	54·68	·62137	·53961	27·778	·27778	

ANGLES & TIME

	SECONDS	MINUTES	DEGREES OR HOURS	RADIANS	RIGHT ANGLES
1 SEC.		·016667	10^{-4}×2·7778	10^{-6}×4·848	10^{-6}×3·0864
1 MIN.	60		·016667	10^{-4}×2·9089	10^{-4}×1·852
1 DEG. OR HR.	3600	60		·017453	·0111
1 RADIAN.	20626	3437·7	57·296		·63662
1 RT. ANGLE	324000	5400	90	1·5708	

FORCE

	DYNES	POUNDALS	GMS. WT.	KGMS. WT.	LB. WT.	TONS.
1 DYNE		10^{-5}×7·233	·0010194	10^{-6}×1·0194	10^{-6}×2·247	10^{-9}×1·0033
1 POUNDAL	13825		14·093	·014093	·03107	10^{-5}×1·387
1 GM. WT.	981	·07096		·001	·0022046	10^{-7}×9·842
1 KGM. WT.	981000·	70·96	1000		2·2046	10^{-4}×9·842
1 LB. WT.	445000	32·185	453·6	·4536		10^{-4}×4·464
1 TON.	10^{8}×9·967	72090	1016000	1016	2240	

PRESSURE & STRESS

	LBS/SQ IN.	LBS/SQ FT.	TONS/SQ IN.	KM/SQ M OR MM. OF WATER	KM./SQ CM	DYNES/SQ. CM	FEET OF WATER
1 LB. PER SQ. IN.		144	10^{-4}×4·4643	703·06	·070306	68971·0	2·3067
1 LB. PER SQ. FOOT.	10^{-3}×6·9443		10^{-6}×3·1002	4·883	10^{-4}×4·883	478·96	01602
1 TON PER SQ. INCH	2240	322560		10^{6}×1·575	157·5	10^{8}×1·5448	5167·0
1 KM/SQ M OR 1 MM. OF WATER	·0014224	·20482	10^{-7}×6·350		·0001	98·10	·003281
1 KM PER SQ CM.	14·224	2048·2	·006350	10000		981000	32·81
1 DYNE PER SQ. CM.	10^{-5}×1·4499	·0020878	10^{-9}×6·473	·010193	10^{-6}×1·0193		10^{-5}×3·3445
1 FOOT OF WATER	·43352	62·43	10^{-4}×1·9353	304·8	·03048	29900·	

POWER

	ERGS PER SEC.	CM.GM/SECS	WATTS.	FT LBS/SEC.	H.P.	FORCES DE CHEVAL	KILOWATTS
1 ERG PER SEC.	—	·0010194	10^{-7}	$10^{-8}\times7·373$	$10^{-10}\times1·3406$	$10^{-10}\times1·3592$	10^{-10}
1 CM.GM /SEC	981	—	$10^{-5}\times9·81$	$10^{-5}\times7·233$	$10^{-7}\times1·315$	$10^{-7}\times1·3333$	$10^{-8}\times9·81$
1 WATT	10^{7}	10194	—	·7373	·0013406	·0013592	·001
1 FT. LB/SEC	$10^{7}\times1·3563$	13825·	1·3563	—	·0018182	·0018434	·0013563
1 H.P.	$10^{9}\times7·4595$	$10^{6}\times7·604$	745·95	550	—	1·0139	·74595
1 FORCE DE CHEVAL	$10^{9}\times7·3575$	$10^{6}\times7·5$	735·80	542·48	·9863	—	·73575
1 KILO-WATT.	10^{10}	$10^{7}\times1·0194$	1000	737·30	1·3406	1·3592	—

WEIGHT & VOLUME OF WATER

	LB.	TONS.	GRM. or CU.CM. of W.	CU. INCHES.	C. FEET.	LITRES. (C. DECIMETRE)	GALLS.
1 LB.	—	$10^{-4}\times4·464$	453·6	27·68	·016017	·4536	·1
1 TON.	2240	—	1,016.047	62000	35·88	1016·	224
1 GRM. OR CU.CM.	·002205	$10^{-7}\times9·842$	—	·06102	$10^{-5}\times3·531$	·001	·00022
1 CU. INCH OF WATER.	·03613	$10^{-5}\times1·6128$	16·388	—	$10^{-4}\times5·790$	·01639	
1 CU. FOOT OF WATER	62·43	·02787	28317·	1728	—	28·32	
1 LITRE OF WATER	2·2046	$10^{-4}\times9·842$	1000	61·02	·03531	—	
1 GALLON OF WATER	10	·004464	4546				—

Energy & Moments.

	Feet × Lb Wt	Inches × Lb Wt	Inches × Tons Wt	Feet × Tons Wt	Cms × Gms Wt	Metres × Kg. Wt	Foot Poundals.	Cm × Dynes or Ergs.	Joules	Calories (or ·01 Gm water from 0° to 100°C)	B.T.U. ($\frac{1}{180}$ Lb water from 32° to 212°)
1 Ft × Lb Wt		12	005357	$10^{-4}\times4\ 4643$	13825	13825	32·185 (this was the value of g)	$10^{7}\times1{\cdot}3563$	1 3563	32416	0012863
1 Inch × Lb Wt	083		$10^{4}\times4{\cdot}4643$	$10^{-5}\times3\ 720$	1152 1	011521	2 6821	$10^{6}\times1\ 13025$	11302	027013	$10^{-4}\times1\ 072$
1 Inch × Ton Wt	186 6	2240		083	$10^{6}\times2{\cdot}58075$	25 808	6007·8	$10^{9}\times2{\cdot}5317$	253·17	60 51	24012
1 Foot × Ton Wt	2240	26880	12		$10^{7}\times3\ 0969$	309 69	72094	$10^{10}\times3\ 038$	3038	726 1	2 8814
1 Cm × Gm Wt	$10^{-5}\times7\ 233$	$10^{-4}\times8{\cdot}6796$	$10^{-7}\times3{\cdot}875$	$10^{-8}\times3\ 229$		00001	002328	981	$10^{-5}\times9{\cdot}81$	$10^{-5}\times2{\cdot}3446$	$10^{-8}\times9{\cdot}304$
1 Met × Kg Wt	7 233	86 796	038749	003229	10^{5}		232·8	$10^{5}\times981$	9 81	2·3446	009304
1 Ft Poundal	03107	37285	$10^{-4}\times1{\cdot}6645$	$10^{-5}\times1{\cdot}3871$	429·57	·0042957		421400	04214	·0100717	$10^{-5}\times3{\cdot}9967$
1 Cm Dyne or Erg	$10^{-8}\times7{\cdot}373$	$10^{-7}\times8{\cdot}848$	$10^{-10}\times3{\cdot}950$	$10^{-11}\times3{\cdot}2916$	·0010194	$10^{-8}\times1{\cdot}0194$	$10^{-6}\times2\ 373$		10^{-7}	$10^{-8}\times2{\cdot}3901$	$10^{-11}\times9\ 484$
1 Joule	7373	8·8477	$10^{-3}\times3{\cdot}950$	$10^{-4}\times3{\cdot}2916$	10194·	·10194	23 73	10^{7}		23901	$10^{-4}\times9{\cdot}484$
1 Calorie	3 0849	37·02	·016526	$10^{-3}\times1{\cdot}3772$	42650·	·4265	99·29	$10^{7}\times4{\cdot}184$	4·184		·003968
1 B.T.U.	777·4	9329	4·1645	·34704	$10^{7}\times1{\cdot}0748$	107·48	25020	$10^{10}\times1{\cdot}0543$	1054 3	252	

(Note B T U = British Thermal Unit. 1 Board of Trade Unit (Electr) = 3 6 × 10^{6} Joules or Watt Secs = 1 Kilowatt Hour)

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