

angle of an isosceles triangle is the perpendicular bisector of the opposite side, and divides the triangle into two equal right triangles.

A **scalene triangle** is one with no two sides equal. In such a triangle, no two angles are equal.

An **acute triangle** is one with three acute angles.

A **right triangle** is one with a right angle. The side opposite the right angle is called the **hypotenuse**. The other two sides may be called **legs**. A plane triangle can have only one right angle.

An **obtuse triangle** is one with an obtuse angle. A plane triangle can have only one obtuse angle.

An **oblique triangle** is one which does not contain a right angle.

The **altitude** of a triangle is a perpendicular line from any vertex to the opposite side, extended if necessary, or the length of this perpendicular line.

A **median** of a triangle is a line from any vertex to the center of the opposite side. The three medians of a triangle meet at a point called the **centroid** of the triangle. This point divides each median into two parts, that part between the centroid and the vertex being twice as long as the other part.

Lines bisecting the three angles of a triangle meet at a point which is equidistant from the three sides, and is the center of the **inscribed circle**, as shown in figure 128b. This point is of particular interest to navigators because it is the point often taken as the fix when three lines of position of equal weight and having only random errors do not meet at a common point.

The perpendicular bisectors of the three sides of a triangle meet at a point which is equidistant from the three vertices, and is the center of the **circumscribed circle**, the circle through the three vertices and therefore the smallest circle which can be drawn enclosing the triangle. The center of a circumscribed circle is within an acute triangle, on the hypotenuse of a right triangle, and outside an obtuse triangle.

A line connecting the mid points of two sides of a triangle is parallel to the third side and half as long. Also, a line parallel to one side of a triangle and intersecting the other two sides divides these sides proportionally. This principle can be used to divide a line into any number of equal or proportional parts. Refer to figure 128c. Suppose it is desired to divide line AB into four equal parts. From A draw any line AC . Along C measure four equal parts of any convenient lengths (AD , DE , EF , and FG). Draw GB , and through F , E , and D draw lines parallel to GB and intersecting AB . Then AD' , $D'E'$, $E'F'$, and $F'B$ are equal and AB is divided into four equal parts.

The sum of the angles of a plane triangle is 180° . Therefore, the sum of the acute angles of a right triangle is 90° , and the angles are complementary. If one side of a triangle is extended, the **exterior angle** thus formed is supplementary to the adjacent **interior angle** and, therefore, equal to the sum of the two nonadjacent angles. If two angles of one triangle are equal to two angles of another triangle, the third angles are also equal, and the triangles are **similar**. If the area of one triangle is equal to the area of another, the triangles are **equal**. Triangles having equal bases and altitudes have equal areas. Two figures are **congruent** if one can be placed over the other to make an exact fit. Congruent figures are both similar and equal. If any side of one triangle is equal to any side of a similar triangle, the triangles are congruent. For example, if two right triangles have equal sides, they are congruent; if two right triangles have

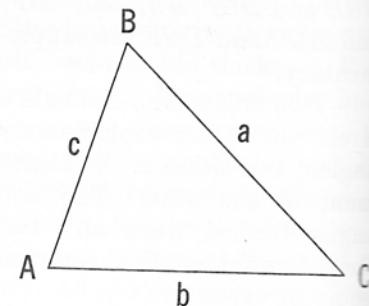


FIGURE 128a.—A triangle.



FIGURE 128b.—An inscribed circle.

If A = area of triangle, b = altitude, c = base, and S = the semiperimeter,

129. Properties of a triangle. A triangle is a **quadrilateral** and one whose interior angles are all **equiangular** and exterior angles are all **equianalogous**. It approaches a circle as its sides approach zero.

A **trapezoid** is a quadrilateral with one pair of parallel sides. A **parallelogram** is a quadrilateral with two pairs of parallel sides. A **rectangle** is a parallelogram with all interior angles right angles. A **square** is a rectangle with all four sides equal.

The sum of the interior angles of a triangle is 180° .

A **diagonal** of a triangle is a line segment connecting two vertices that are not adjacent.

The perimeter of a triangle is the sum of the lengths of its three sides.

If A = area of a triangle, b = base, and h = altitude of a triangle, then $A = \frac{1}{2}bh$, where b is the number of square units in the base and h is the number of square units in the altitude.