Cook's observations of the eclipse 5 August 1766

The only reason for measuring the sun's zenith distance seems to me to be to find out the local apparent time. Although the sun is less than an hour from the meridian it is possible to get a rather decent time from the observation.

The sensitivity of the resulting LAT is -1.5^s per 0.1′ error in latitude, +1.6^s per 0,1′ error in both declination and zenith distance.

The declination for apparent noon at Greenwich for 5 August 1766 was N 16° 57′ 6″ (according to Frank's NA). This value has to be corrected first to noon at Eclipse Island (57.6°W). Using Maskelyne's Tables Requisite (2nd edition, 1781), table VI gives for 5 August -2′ 45″. Secondly you have to correct for the time after local noon (47.5^m); the same table gives -48″. Hence the declination should have been 16° 53′ 33″, giving the polar distance 73° 6′ 27″.

For calculating the local hour angle, let φ_c be the co-latitude, p the polar distance, z the zenith distance, and t the local hour angle. Maskelyne gives, in The British Mariner's Guide, 1763, the following formula

$\sin^2(t/2) = \csc \varphi_c \cdot \csc p \cdot \sin(s - \varphi_c) \cdot \sin(s-p)$,
where $s = (z + \varphi_c + p)/2$

z	32° 13′ 30″		
φ_c	42° 23′ 41″	log csc	10.17119
p	73° 6′ 27″	log csc	10.01915
$2 \cdot s$	147° 43′ 38″		
s	73° 51′ 49″		
$s-\varphi_c$	31° 28′ 8″	log sin	9.71770 1
$s-p$	0° 45′ 22″	log sin	8.12042 6
			18.02846 7
$t/2$	5° 55′ 51.8″	log sin	9.01423 4
t	11° 51′ 44″		
LAT	$0^h47^m27^s$		

The logarithms have been taken from table XIX in Tables Requisite and are not always corresponding to a modern calculator in the last digits. However, in this case there is no change on second-of-time-level in the final result.

Interpreting "just a minute after the beginning" literally, then the local apparent time of the start of the eclipse was $0^h46^m27^s$. That is twenty-one seconds off in time compared with Cook's result.

According to Tables Requisite the observed zenith distance should be corrected by +35″ for refraction and -4″ for parallax. To arrive at the stated true zenith distance of the centre this implies a semidiameter of 15′ 59″. This should be compared to 15′ 47″ as given by Frank's NA. This difference of 0.2′ would shift the LAT at the observation 3^s earlier in time.

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