# An Historical Review of the Ex-Meridian Problem

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THE method of finding the latitude of a ship at sea from an observation of the Sun or other body near the meridian is practised extensively in the Merchant Navy. The history of the method, which dates from the middle of the eighteenth century, is full of interest: and the ex-meridian problem is almost as celebrated in the history of nautical astronomy as the double altitude problem. Considerable attention was devoted to the ex-meridian problem during the last century—a period which was, in truth, the golden age of astronomical navigation. Many ingenious solutions were contrived and a diversity of ex-meridian tables were constructed, all aimed at facilitating the problem of finding latitude at sea.

It may be argued that the ex-meridian method became obsolescent as soon as it had been invented. The increasing use and reliability of the chronometer, due largely to improvements in manufacturing techniques, and the introduction of position-line navigation resulting from the discoveries of Sumner and St. Hilaire, brought astronomical navigation to a stage of near-perfection. Had logic prevailed in the chartroom, all existing methods of astronomical navigation would have been swept aside, before the dawn of the twentieth century, by the all-embracing intercept method of Marcq St. Hilaire. This was not the case, and the old methods have remained to this very day. In this paper an attempt is made to trace briefly the history of the ex-meridian problem. A mathematical treatment, comparing the several methods, is beyond the intention of the author.

*A seaman always knows his latitude*, is an ancient proverb which, although seldom uttered in this age of electronic navigation, emphasizes the importance the old-time seaman attached to knowledge of latitude. After the advent of position-line navigation due to Captain Sumner's discovery in 1837, astronomical navigation was brought to a state of excellence by the French naval officer Marcq. St. Hilaire. Thereafter the problem of finding latitude at sea by astronomical navigation became merely a special case of the general method of obtaining an astronomical position line. The end product of every sight for position finding at sea is a position line which may be, but seldom is in practice, drawn on a chart or plotting sheet.

Old-established practices die hard at sea: so much so that in spite of the fact that position-line navigation was introduced no less than a century and a quarter ago, many present-day navigators seldom think in terms of position lines. Many think in terms of 'latitude by meridian altitude'; 'latitude by ex-meridian'; 'latitude by Pole Star'; or 'longitude by chronometer': as their seafaring fathers and grandfathers did before them.

The astronomical position line obtained with the least effort is that which results from a meridian altitude observation of the Sun. This method for finding latitude became available to European navigators as soon as tables of the Sun's declination and a *Regiment for the Sun* were devised for, and used by, the Portuguese navigators of five centuries ago. Since that far-off time the altitude observation of the culminating Sun has been a tradition among seamen. There is a distinct sense of lost hope, even in these enlightened times, when a cloudy sky or indistinct horizon precludes the observation of the Sun's meridian altitude. There is something sacrosanct about the noon position by observation: and the normal way of getting it, when out of sight of land, is by observing the noonday height of the Sun. The noon position by observation, being a running fix, is usually unreliable. Nevertheless it is regarded as being the pre-eminent fix of the whole day: and no dedicated and self-respecting navigator fails to obtain one. The morning-sight-run-up-to-noon, with its associated ritual, remains, as it has done for generations, the general practice on board merchant ships.

The troublesome disadvantage of the midday Sun sight is that the Sun and the horizon vertically below him must be visible at one particular instant of time during the day. It is not surprising, therefore, that in pre-position line days the enterprising seaman aimed to provide himself with alternative methods for finding latitude. Latitude, together with Lead, Log, and Lookout, had become one of the four L's of the navigator long before the Mariner's Creed had been penned by that well-known Secretary to the Board of Trade, Thomas Gray.

Any visible star, the Moon or a planet, when at meridian passage, provides a navigator with the opportunity for observing for latitude. It appears, however, from what has been written on the subject, that the generality of seamen did not use these bodies for navigational purposes until comparatively recent times: the Sun has always been the firm favourite of the conservative navigator. Finding latitude from an observation of the seaman's star *Polaris* was a method that was used by the earliest ocean navigators. The *Regiment of the North Star* formed part of the stock-in-trade of seamen of the fifteenth to seventeenth centuries. Their right ascensions being approximately the same, the stars at the head and foot of the Southern Cross culminate at about the same time. It was an easy matter, therefore, to provide the early seaman who ventured south of the line with rules for finding latitude by the Crozier. These rules are to be found in many early navigation manuals.

It is a curious fact that the Pole Star, as a means for finding latitude, was not favoured during the eighteenth and early part of the nineteenth centuries. It cannot be a mere coincidence that this was the period when the double altitude problem, using the Sun, was popular. Numerous solutions to the double altitude problem were devised for the seaman's attention: and the method remained fashionable, despite complex rules (which seamen learnt by rote) and the tedious computations associated with it, long after it had outlived its usefulness. The earliest method for finding latitude by ex-meridian observation applied to the Pole Star. The azimuth of the Pole Star, on account of the star's small polar distance, is never very large; and its rate of change of altitude is always very small. The Pole Star is, therefore, an admirable object for finding latitude by the ex-meridian method at any time when it is visible.

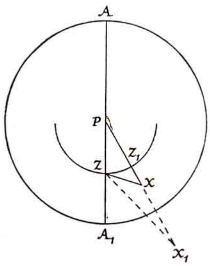


FIG. 1

The essential problem in the ex-meridian method for finding latitude is the comparison of the altitude of a celestial body at a place where the body is culminating (the latitude of the place being the same as that of the observer), with its altitude at the same instant at the observer's position. In Fig. 1 the celestial sphere is projected on to the plane of the equinoctial. Z is the projection of the zenith of the observer, and $Z_1$ is that of the zenith of a place whose latitude is the same as that of the observer, and over whose meridian the body X is passing. P is the projection of the celestial pole, and the circle is that of the equinoctial.

If the arc $Z_1X$ can be found, the latitude of the place whose zenith is at $Z_1$ and, therefore, the observer's latitude, can also be found.

If $l$, $d$, $z$ and $h$ denote the observer's latitude, the body's declination, the body's zenith distance and the time from meridian passage of the body, respectively, we have, from the triangle PZX

$$\cos h = \frac{\cos z - \sin l \sin d}{\cos l \cos d}$$

when $l$ and $d$ have the same name, and

$$\cos h = \frac{\cos z + \sin l \sin d}{\cos l \cos d}$$

when $l$ and $d$ have different names.

When $l$ and $d$ have the same name, $\cos z - \sin l \sin d = \cos l \cos d \cos h$ and it can be shown that $\text{vers}\,(l \sim d) = \text{vers}\, z - \cos l \cos d \,\text{vers}\, h$. Similarly, when $l$ and $d$ have different names, $\text{vers}\,(l + d) = \text{vers}\, z - \cos l \cos d \,\text{vers}\, h$. In general,

$$\text{vers}\,(l \underset{\sim}{+} d) = \text{vers}\, z - \cos l \cos d \,\text{vers}\, h \qquad (1)$$

Now $(l \underset{\sim}{+} d)$ is the meridian zenith distance of X at the place whose zenith is at $Z_1$. Let this be denoted by $z_1$. The latitude of this place and, therefore, the observer's latitude is thus given by $l = z_1 \underset{\sim}{+} d$

The above treatment is a modified form of that first given in 1754 by Cornelius Douwes, whose name figures prominently in the history of the double altitude problem. Douwes' investigation was modified by the Rev. James Inman, D.D., whose famous nautical tables were first published in 1821. Inman's modification, used above, consisted in adapting the formula to the tables of natural versines and haversines.

The ex-meridian method described requires the use of a latitude by account, which should approximate to the observer's actual but unknown latitude. If the latitude found differs materially from that used, it is necessary to repeat the computation, this time using the calculated latitude in place of the one initially used. Moreover, it is necessary for the observer to know his longitude, knowledge of this being required to find $h$ which figures in the computation.

It may readily be shown that an error in $z_1$ (and therefore in calculated latitude) is proportional to $\cos l \sin \text{Az.} \times$ error in $h$. It follows, therefore, that the smaller the latitude, and/or the nearer the azimuth to 90°, the greater will be the error in latitude consequent upon an error in $h$. Knowledge of correct time is all-important in the ex-meridian problem.

It was early realized that, when using stars for finding latitude by the ex-meridian method, those with big declinations gave the best results, because of their relatively slow rates of change of altitude. The Pole Star, therefore, is admirably suited for the purpose, and accurate Pole Star tables have been available for seamen since the early nineteenth century—although they were not included in the British *Nautical Almanac* until 1834.

A method for finding latitude by ex-meridian observation of the Sun using 'direct spherics' was given in the later editions of James Robertson's *Elements of Navigation*.[1] This method is explained with reference to Fig. 2, which is a projection of the celestial sphere on to the plane of the celestial horizon of an observer whose zenith is projected at Z. P is the projection of the celestial pole, WQE the equinoctial, and NZS the observer's celestial meridian. The arc XM is a perpendicular from X on to the observer's celestial meridian at M.

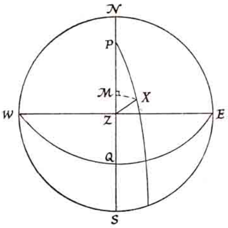


FIG. 2

In PMX

$$\tan \text{PM} = \tan \text{PX} \cos \text{P} \tag{2}$$

$$\cos \text{PX} = \cos \text{MX} \cos \text{PM} \tag{3}$$

In XMZ

$$\cos ZX = \cos ZM \cos MX \qquad (4)$$

from which we can obtain

$$\cos ZM = \cos ZX \cos PM \sec PX \qquad (5)$$

From (2) and (5), arcs PM and ZM may be found, and these arcs, when combined, will give arc PZ which is equivalent to the observer's co-latitude.

The direct method is independent of the latitude, and may be used to good effect even when the observed object has a large hour angle and azimuth, provided that the angle P is known with accuracy. A disadvantage of the method applies to cases in which the object's declination is small. In this event the arc PX is near 90°. Because the tangent of PX is required in (2), it is necessary, in this case, to extract the logarithm with great care. Moreover any small error in the polar distance used will cause, in this circumstance, a relatively large error in sec PX used in (5); and this will lead to error in arc ZM.

An interesting case of the above method described by W. R. Martin[2] applies when the declination of the observed object is less than about 1° and the hour angle P is less than about half an hour. In this case for all practical purposes cos ZM = cos ZX sec P. The latitude is found by combining the computed arc ZM with the object's declination.

A method alternative to the direct method described is known as the *Reduction to the Meridian* method. This method involves the calculation of a correction to be applied to the altitude of the body when out of the meridian to find its altitude at meridian passage. The reduction method, like that attributed to Douwes, requires the use of a latitude by account. If this proves to be materially different from the computed latitude it is necessary to repeat the calculation using the computed latitude in place of the latitude by account. The reduction to the meridian method is attributed to the renowned French astronomer Delambre, who published it in 1814. The method is described with reference to Fig. 3.

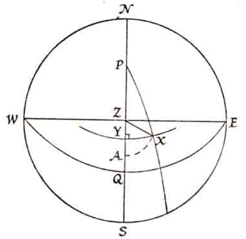

FIG. 3

Fig. 3 represents the celestial sphere projected on to the plane of the celestial horizon of an observer whose zenith is projected at Z. X is the projection of a celestial body whose declination is $d$ and whose hour angle is $h$. Y represents the body's position when it is at meridian passage relative to the observer. The arc ZA is drawn equal to arc ZX and YA = ZX − ZY.

The arc YA is referred to as the 'Reduction', here denoted by $r$.

It can be shown that in PZX

$$\cos z = \cos (l \sim d) - 2 \cos l \cos d \sin^2 h/2 \qquad (6)$$

Now $ZX = ZA = (ZY + r)$ and therefore $\cos ZX = \cos ZY \cos r - \sin ZY \sin r$. Since $r$ is a small quantity and $ZY = (l \underset{\sim}{+} d)$,

$$\cos z = (1 - r^2/2) \cos (l \underset{\sim}{+} d) - r \sin (l \underset{\sim}{+} d) \qquad (7)$$

By equating the values for $\cos z$ from (6) and (7) we have

$$(1 - r^2/2) \cos (l \underset{\sim}{+} d) - r \sin (l \underset{\sim}{+} d) = \cos (l \underset{\sim}{+} d) - 2 \cos l \cos d \sin^2 h/2$$

from which

$$r^2/2 \cos (l \underset{\sim}{+} d) + r \sin (l \underset{\sim}{+} d) = 2 \cos l \cos d \sin^2 h/2 \qquad (8)$$

The first term in (8) is small when the object is near the meridian. It may, therefore, be neglected, and for practical purposes,

$$r = 2 \frac{\cos l \cos d}{\sin (l \underset{\sim}{+} d)} \sin^2 h/2 \qquad (9)$$

Formula (6) may be transposed thus:

$$\cos (l \sim d) = \cos z + \cos l \cos d \; 2 \sin^2 h/2$$

therefore,

$$\cos z_1 = \cos z + 2 \cos l \cos d \times \sin^2 h/2 \qquad (10)$$

$z_1$ may, therefore, be found using a latitude by account provided that $h$ is known accurately.

From formula (10) we have, by transposition

$$2 \sin^2 h/2 = (\cos z_1 - \cos z) \sec l \sec d \qquad (11)$$

John Hamilton Moore[3] described a method for finding 'latitude by one altitude of the Sun when the time is not more distant than one hour from noon'. In his description he gave one rule for finding the time from noon, based on formula (11); and another rule, based on formula (10), for finding the meridian zenith distance and thence the latitude.

The expression $2 \sin^2 h/2$ was known as the *rising*. It figured prominently in the double altitude problem, and existing tables of log-risings, therefore, were adapted to the ex-meridian problem.

Moore pointed out that the rule for finding time from noon should be applied only to sights taken when the Sun's altitude did not exceed 18°. He also noted that 'an error in the supposed latitude can make a very small difference in the change of altitude; and the nearer the altitude is taken to noon the better to find the change of altitude'. He also warned against using the method when the time from noon exceeded one hour; and stated that in cases where the Sun's meridian altitude exceeds 60°, or when the latitude is small, good results were not to be expected unless the time from noon was much less than one hour.

Norie[4] gave the same rule as Moore for finding latitude from a single observation of the Sun; but he did not specify any limit to time from noon. In the 1825 edition of Norie's *Epitome* the following rule is given: 'In this method . . . the time from noon should not exceed 30 minutes'. In the 1877 edition of the same work[5] the rule is: 'The number of minutes in the time from noon should not exceed the number of degrees in the Sun's meridian zenith distance'. This rule, often quoted by seamen of our time, appears to have stemmed from the celebrated Raper.

After the ex-meridian method had become established, rules gave way to tables giving limits of time from meridian passage. In Rosser's *Self Instructor*[6] we find a table, similar to those in other textbooks of the time, giving limits of time from meridian passage (meridian distances) computed to give the number of minutes of meridian distance, when an error of half a minute in time will produce an error of one minute of arc in the reduction. This is a reminder of Raper's definition of the term 'near the meridian'.[7] 'The term', he wrote, 'implies a meridian distance limited according to latitude, declination, and also the degree of precision with which the time is known'.

The treatment on limits for ex-meridian by reduction given by Merrifield[8] is interesting. After showing that error in lat = error in mer. dist. $(h) \times \sin$ Az. coslat. he concludes that '. . . as a rule, altitudes for latitude by circumeridianal altitudes [ex-meridians] should be taken within 20 minutes of the body's transit, or when the object's azimuth is not more than one point'. He then quotes the practical rule given by Norie (and others) given above.

Merrifield, in his discussion on ex-meridian sights, also pointed out (as other writers had done) that by finding latitude by observations of an object near the meridian both when it is east and west of the meridian, and then meaning the results, '. . . the method is susceptible of very great accuracy'.

The books of Andrew Mackay,[9] published during the early part of the nineteenth century, were among the more comprehensive works on astronomical navigation. Mackay made no mention of the ex-meridian problem, but he did describe a method for finding latitude by equal altitudes of the Sun. In this method half the elapsed interval between the times of the observations is equal to the time of either observation from noon, assuming the observer to have been stationary during the elapsed time. From formula (10), putting half elapsed time equal to $h$, it is possible to find the meridian zenith distance and hence the latitude.

The ex-meridian method for finding latitude became increasingly popular with the ever-growing number of ex-meridian tables that appeared from the middle of the nineteenth century onwards.

If $r$ in formula (9) is expressed in seconds of arc the formula becomes:

$$r = \left(\frac{\cos l \cos d}{\sin (l \underset{\sim}{+} d)}\right) \left(\frac{2 \sin^2 h/2}{\sin l''}\right) \qquad (12)$$

Values of $2 \sin^2 h/2/ \sin l''$ were tabulated for suitable values of $h$; and, by means of these tabulated values, the computation of the reduction $r$ could be performed with great facility. The table *Reduction to the Meridian* that appears in Riddle's *Treatise on Navigation*[10] gives values of the expression for values of $h$ in seconds up to 20 minutes.

An alternative solution was to use the log sine squared table; and to add the constant 5·615455 which is the log of $2/\sin l''$. Another similar solution was to use the log rising table and to add the constant corresponding to the log of $1/\sin l''$. It will be noticed that $2 \sin^2 h/2$ is equivalent to versine $h$. Inman's rule[11] for the ex-meridian problem was based on the formula: $\text{vers } z_1 = \text{vers } z - \text{vers } \theta$, in which $\text{hav } \theta = \text{hav } h \cos l \cos d$

One of the earliest of ex-meridian tables is that by J. T. Towson (of great circle sailing fame), first published by the Hydrographic Office in 1849. The principles of Towson's tables are explained with reference to Fig. 2. In the triangles PMX and ZMX it can be deduced from the Sine Rule and Napier's Rules that $\sin \text{MX} = \sin h \cos d$, $\cos \text{PM} = \sec \text{MX} \cos d$, and $\cos \text{ZM} = \sec \text{MX} \cos z$.

Values of MX are tabulated in columns labelled Index Number. Table I contains values of $(\text{PX} \sim \text{PM})$. Table II contains values of $(\text{ZX} - \text{ZM})$. To use the tables, Table I is entered using arguments $d$ and $h$ to extract Index Number and Augmentation 1. Augmentation 1 is added to the declination. Table II is then entered using arguments altitude and Index Number to extract Augmentation 2, which is the required reduction. Towson's tables were designed specifically for Sun ex-meridian observations, as were others, such as those of James Bairnson (*c.* 1880). The ex-meridian tables of Brent, Walter and Williams, which were first published in 1886, provided for star-sights as well as Sun-sights.

When $\theta$ is small $\sin \theta = \theta$ radians. Formula (12), therefore, may be expressed as:

$$r = \frac{\cos l \cos d(h^2 \sin^2 15')}{\sin (l+d)\ 2 \sin l''} \tag{13}$$

where $r$ is in seconds of arc and $h$ is in minutes of time. The tables of Brent, Walter and Williams were based on this formula.

It follows from formula (13) that the change of altitude of a body, when it is near meridian passage, varies as the square of the meridian distance. That is, $r \propto h^2$. This is the principle of an ingenious method for finding the reduction to the meridian graphically. The curve of $r$ against corresponding meridian distance $h$ is a parabola, which became known as *Foscolo's parabola* after Professor Foscolo of Venice who devised this graphical method for solving the ex-meridian problem. Foscolo's parabola was published by the British Hydrographic Office in 1867.

'Cloudy Weather' Johnson, in his *Brief and Simple Methods*,[12] gave a table for finding latitude by ex-meridian observation based on formula (1). He used the term 'reduced versine' for the natural versine corresponding to the log versine of $h$ diminished by the sum of the log secants of $l$ and $d$. Three small tables, all at a single double page opening, were provided for finding the reduced versine. Johnson's rule for finding the M.Z.D. was simple. 'The natural versine of the Ex-M.Z.D. diminished by the reduced versine is the natural versine of the M.Z.D.' In the same work, Johnson introduced his methods for finding longitude by ex-meridian and latitude and longitude by double ex-meridian. Johnson's explanation of the reduction to the meridian given in his well-known *On Finding Latitude and Longitude in Cloudy Weather*,[13] concludes with the formula

$$r = 2c \text{ hav } h \cos l \cos d \sec \text{Alt.}, \text{ where } c = 1 \text{ radian.}$$

The upper part of Johnson's ex-meridian table, in this work, gives values of $\cos l \sec$ Alt. or $N$. The lower part gives values of $2c$ hav $h \times N$. The factor $\cos d$ was ignored, presumably because when the table is used for the Sun $\cos d$ approximates to unity. Johnson justified himself by stating '. . . a further correction (for the declination) may be applied when both the reduction and declination are considerable'.

Formula (13) may be written:

$$r = A\,h^2$$

where

$$A = \frac{\cos l \cos d \sin^2 15'}{\sin (l \underset{\sim}{+} d) \sin 1''}$$

where $l$ and $d$ may be considered to have meridian values.

The ex-meridian tables in present-day nautical tables are based on this relationship, the principle of which was described by H. B. Goodwin[14] as a mechanical principle ($s = \frac{1}{2}ft^2$ for bodies moving with uniform acceleration) applied to astronomical navigation. The same mechanical principle is used in the method of finding latitude known as the Short Double Altitude. Raper informs us that:

'The first work in which a method occurs of finding the latitude by two altitudes observed near the meridian (but restricted to the same side) with an interval of a few minutes, is the "Cours d'observations Nautiques" by Ducom.'

Robertson, in his *Elements of Navigation*,[15] drew attention to the fact that during the 18th century the problem of finding latitude by three ascending (or descending) altitudes of the Sun 'exercised the talents of many ingenious persons'. Numerous solutions to the problem were given, but the only case of practical value applied when the intervals between the first and second, and the second and third, observations were equal, and the observed object near the meridian. The Abbé de la Caille is credited with giving a solution to this problem as early as 1760.

Let $a$ be the meridian altitude, and $a_1$, $a_2$ and $a_3$ the ex-meridian altitudes observed at intervals of $t$. If $h$ is the time from meridian passage at which the middle of the three ex-meridian observations is made, then

$$a = a_1 + k(h \pm t)^2$$
$$a = a_2 + kh^2$$
$$a = a_3 + k(h \mp t)^2$$

where

$$k = \frac{\cos l \cos d \sin^2 15'}{\sin (l + d) \sin 1''}$$

From these relationships $a$, the meridian altitude, and hence the latitude, may be found algebraically.

In an interesting paper by John White, on the ex-meridian problem,[16] the following formula for the reduction, based on a formula by Godfray, was given

$$r = \frac{h^2}{2(\tan l \underset{\sim}{+} \tan d)\sin 1' \operatorname{cosec}^2 15'} \tag{14}$$

This formula suggested the construction of a table giving values of $2 \tan l \sin 1' \operatorname{cosec}^2 15'$, and $2 \tan d \sin 1' \operatorname{cosec}^2 15'$, against argument $h$ in minutes of time from unity to 60. By means of this table the denominator of the formula (14) could readily be found and the solution to the ex-meridian problem thereby facilitated.

The Admiralty published, in 1895, a diagram[17] devised by White for obtaining the reduction in an ex-meridian altitude observation. Later, in 1897, Messrs. J. D. Potter published a diagram[18] invented by F. Kitchin, a naval instructor on H.M.S. *Britannia*, based on White's tables. Kitchin described his diagram in an article in *The Nautical Magazine* of 1901.

Notable among a profusion of ex-meridian tables published during the present century are those of Captain H. S. Blackburne.[19] In a gentle rebuke, Blackburne stated in the 1918 edition of his *Tables for Azimuth, Great Circle Sailing and Reduction to the Meridian*, that 'the only ex-meridian tables which are at present allowed to candidates for the B.O.T. examination are those of Towson and Raper'. This restriction doubtless limited the use of other ex-meridian tables. Blackburne devised several ex-meridian tables each designed for a specific purpose. He was a great advocate of star-sights and, in his 'Excelsior' Tables,[20] he gave a comprehensive set of tables giving azimuths and reductions for 29 of the brightest stars.

That the ex-meridian method for finding latitude is a firm favourite in the Merchant Navy there can be no doubt. Ex-meridian tables, which facilitate the solution of this problem (of very limited usefulness), are widely used. One may wonder why it is that short-method tables of general utility are not more extensively used by Merchant Navy officers. The 'morning-sight-run-up-to-noon', in which the so-called longitude by chronometer method and the latitude by meridian or ex-meridian observation are employed, constitutes the principal part of astro-navigation as practised on board merchant ships. The all-embracing intercept method plays a secondary role.

The Rev. William Hall, writing at the beginning of this century,[21] compared the chronometer, ex-meridian and general or intercept methods of sight reduction in this way:

1. Longitude by Chronometer method:
   Good on large bearings—fails near the meridian.
2. Ex-meridian method:
   Good near the meridian—fails on large bearings.

3. General (intercept) method:
Good on ALL bearings—FAILS IN NO CASE.

The moral is clear.

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