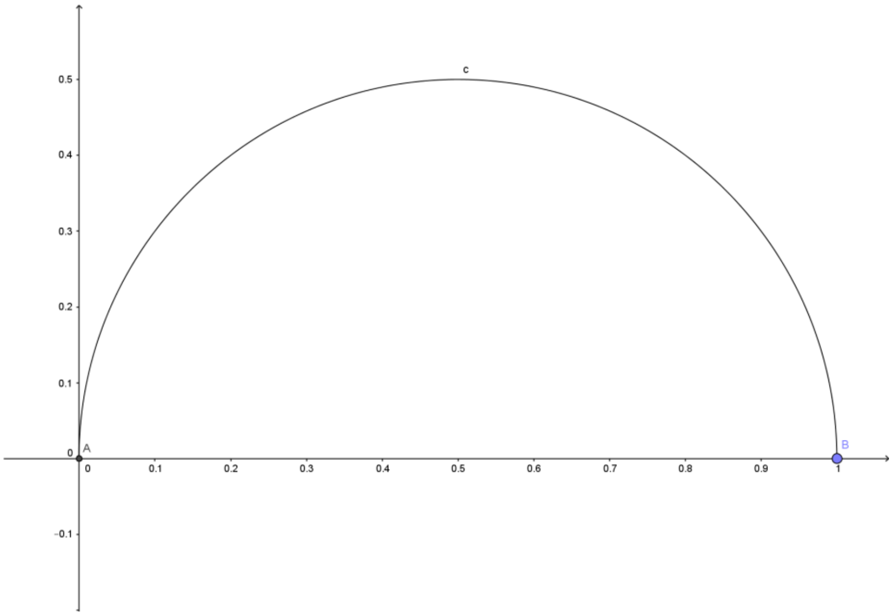
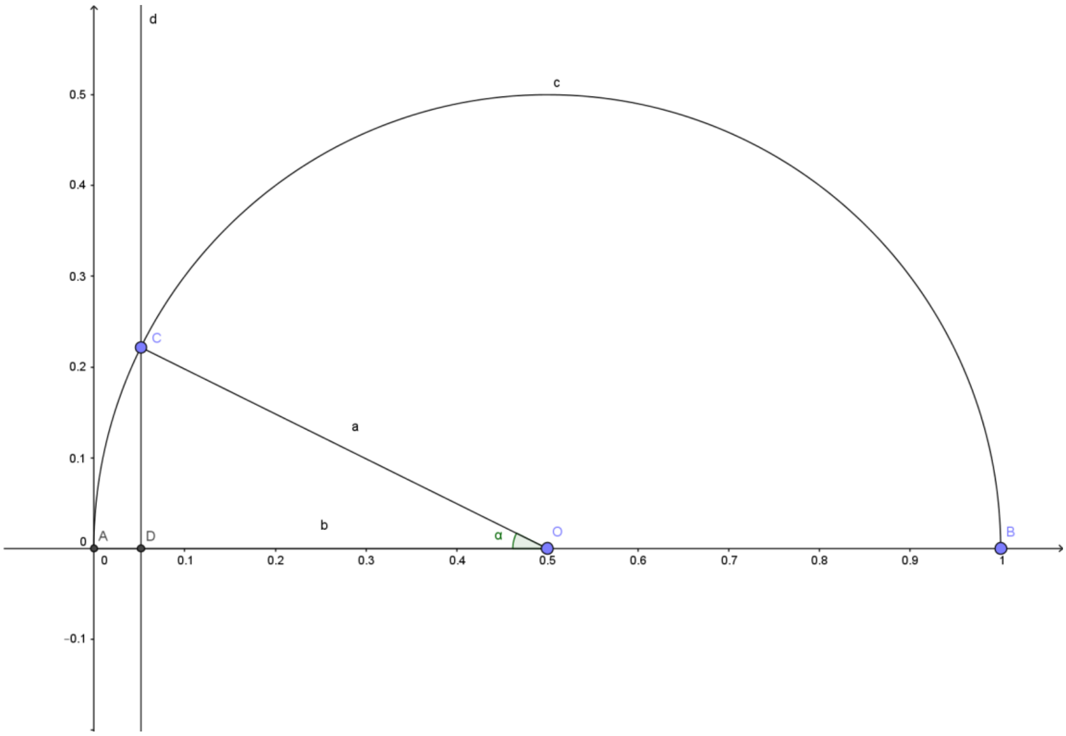
Determining Hc by graphical use of the all-haversine formula

Triggered by the worksheet for graphical sight reduction on Erik de Man's nautical pages [http://www.siranah.de/html/sail008h.htm] I set out to find if it would be possible to see if it could be done for the all haversine formula also.

I came up with a circle (or semi-circle for compactness) between 0 and 1.



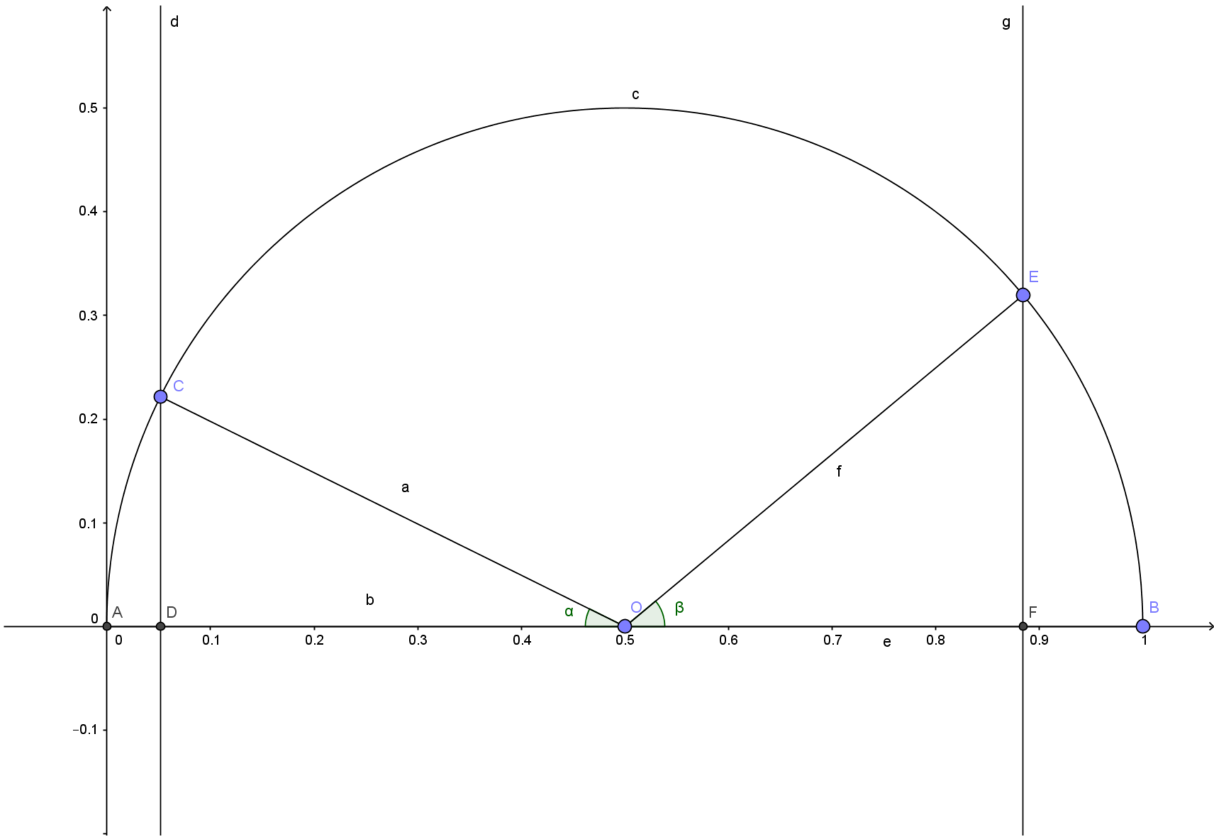
When a line is drawn from the center of the (semi-)circle at angle of α as the included angle of AOC, the distance AD with D being the projection of C on the x-axis is the haversine of α.



Can this be used to model the formula

Hav (|B - Dec|) + (1 – hav (|B - Dec|) – hav (|B + Dec|)) * hav (LHA) ?

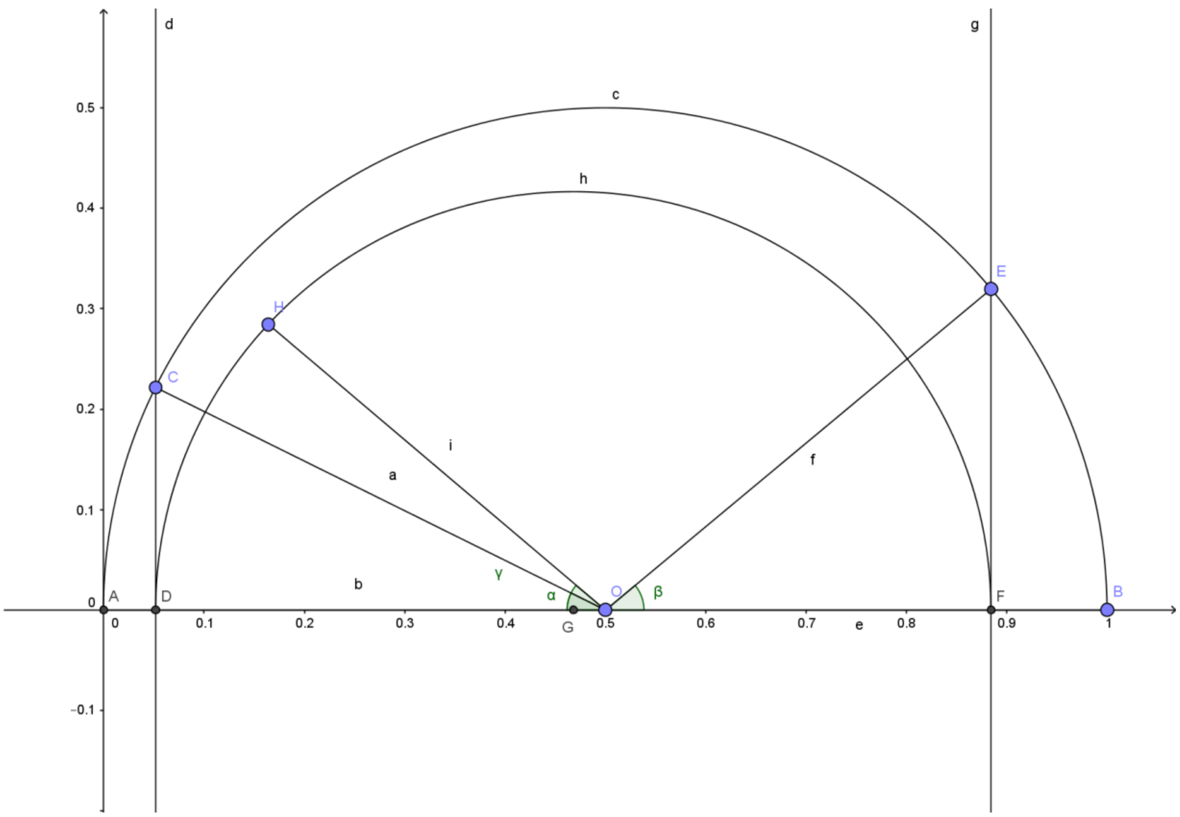
I think so. As the total distance between A and B is 1 (by definition) and we take α = |B – Dec|, it is then also possible to define β as |B + Dec| and put it in the same figure.



In this figure AD = hav (|B – Dec|) , FB = hav (|B + Dec|) and thus

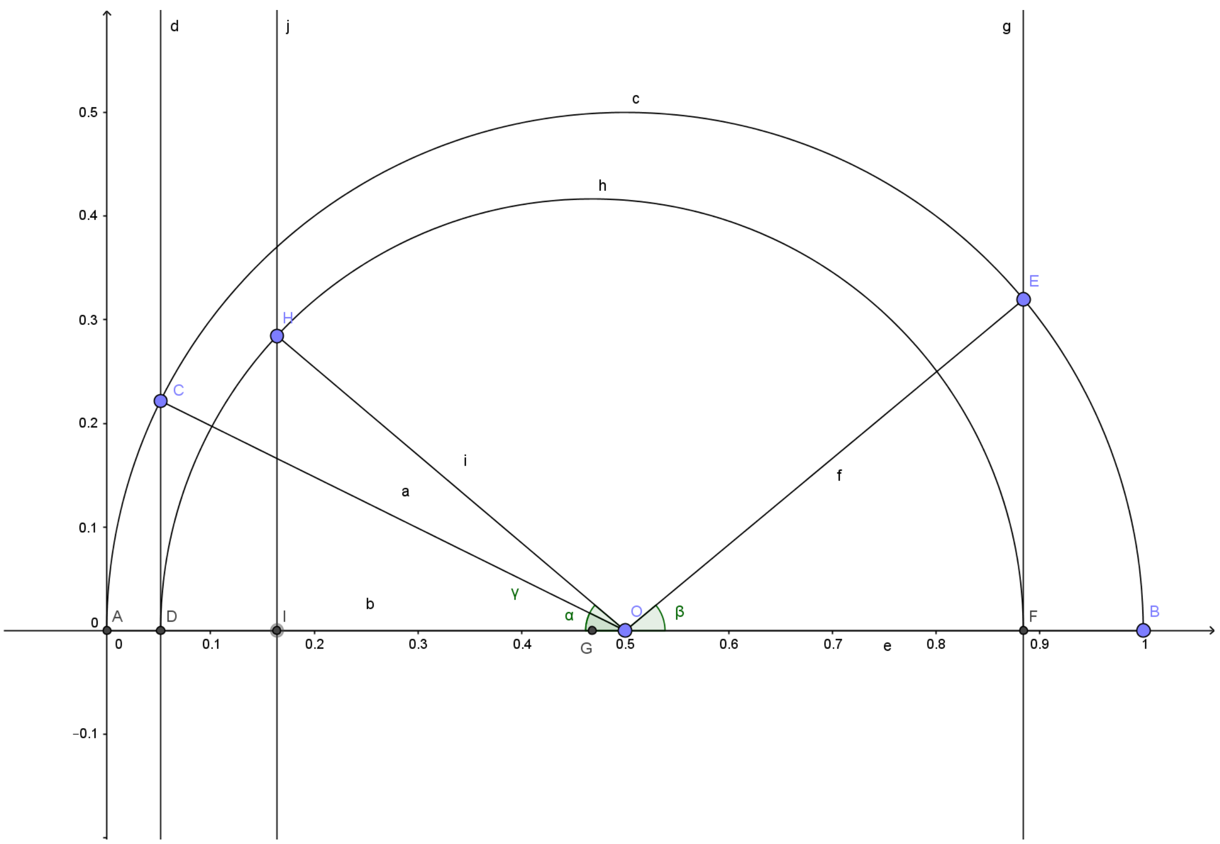
DF = (1 – hav (|B – Dec|) – hav (|B + Dec|)

Hav (LHA) is then constructed by drawing a new semi-circle between D and F. From the center G of this semi-circle a line GH is drawn so that the included angle γ in DGH equals LHA.



With I being the projection of H on the x-axis and DF being (1 – hav (|B – Dec|) – hav (|B + Dec|), DI becomes (1 – hav (|B – Dec|) – hav (|B + Dec|)) * hav (LHA) and

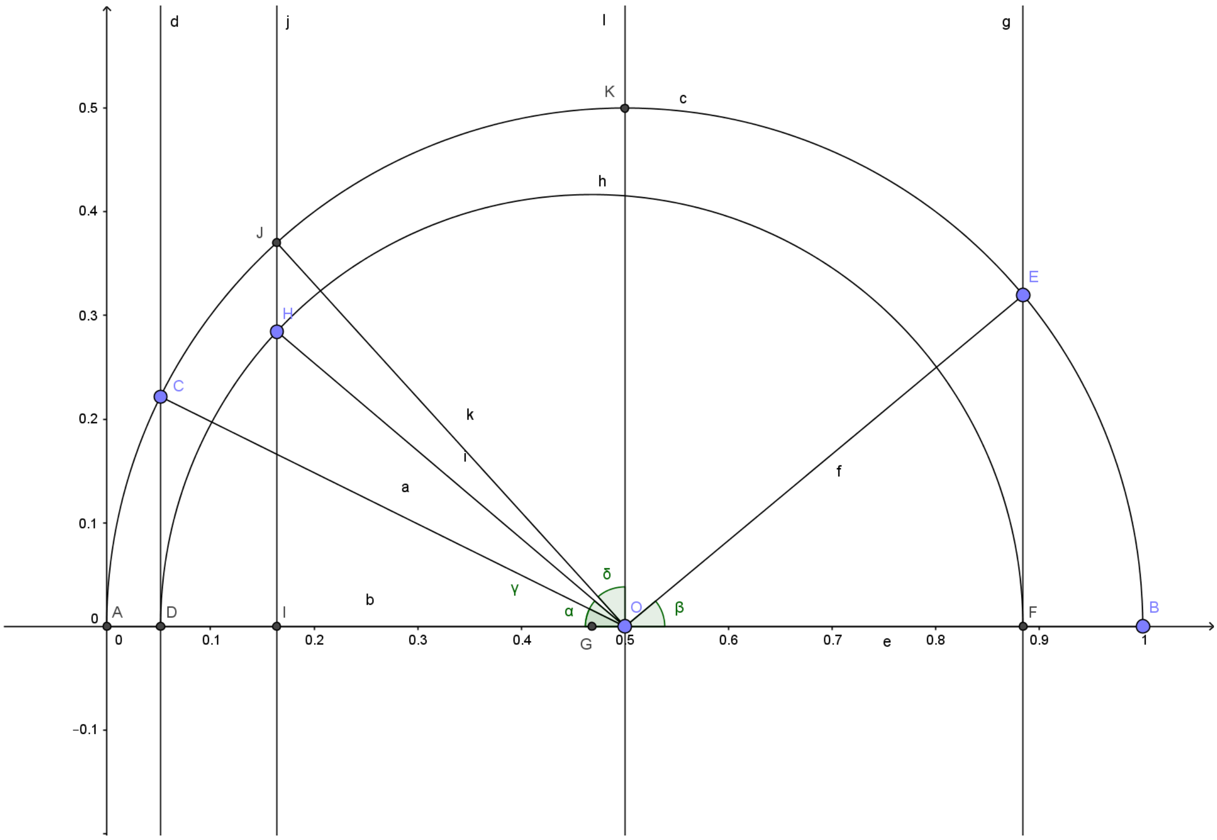
AI = hav (|B – Dec|) + (1 – hav (|B – Dec|) – hav (|B + Dec|)) * hav (LHA)



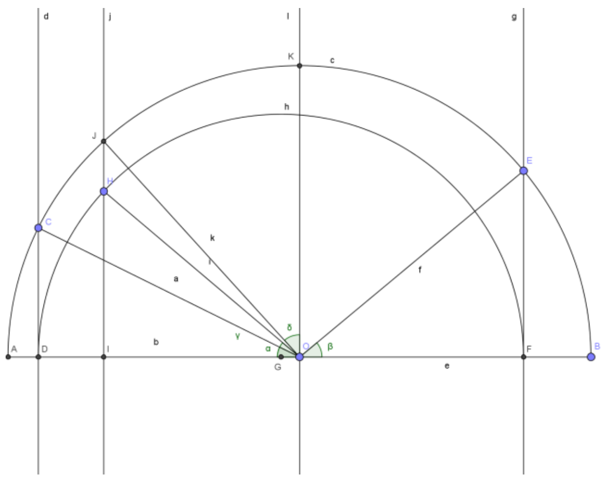
We then complete the determination of the zenith distance ZD, knowing

Hav (ZD) = hav (|B – Dec|) + (1 – hav (|B – Dec|) – hav (|B + Dec|)) * hav (LHA)

We can do this because we can see from the above hav (ZD) can be found by drawing a line from center C to a point J on the semi-circle AB such that the angle AOJ is the zenith distance. Hc being 90° - ZD means angle δ included in JOK is Hc.



As AB equals 1 by definition, it really isn't necessary to draw an axis with values. One could just start by drawing a line between to arbitrary points on an empty piece of paper and proceed from there.



Determining azimuth by graphical use of the all-haversine formula

Next up was the azimuth angle Zn. From rewriting the spherical law of cosines into an all haversine formula as has been done for Hc (ZD really) I found

$$\text{Hav (coDec)} = \text{hav} (|B - Hc|) + (1 - \text{hav} (|B - Hc|) - \text{hav} (|B + Hc|)) * \text{hav (Zn)}$$

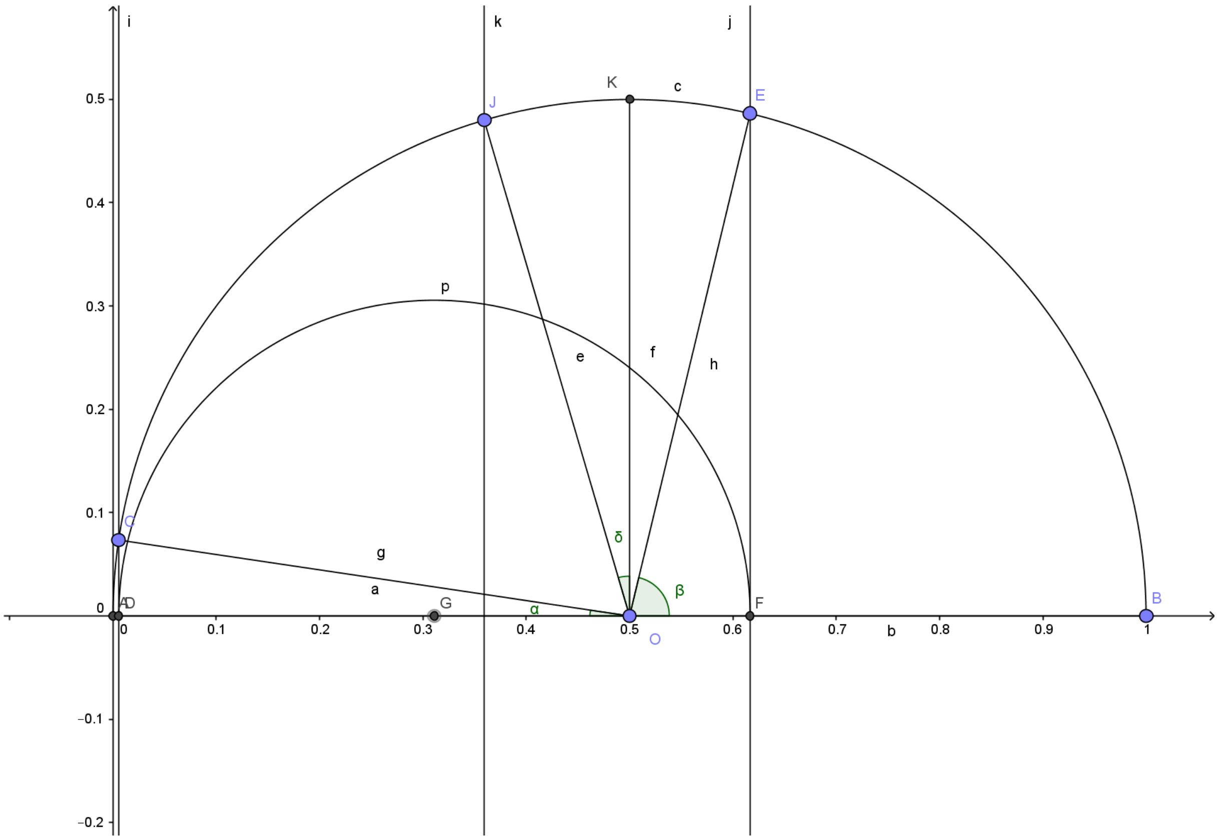
So this can be modelled in the same way as Hc above, δ being coDec, γ being azimuth and α and β being hav (|B – Hc|) and hav (|B + Hc|) respectively.

Normally the azimuth is calculated by rearranging the formula used. In this case, that would mean writing the haversine azimuth formula as

$$\text{Hav (Zn)} = (\text{hav} (|B - Hc|) - \text{hav (coDec)}) / (1 - \text{hav} (|B - Hc|) - \text{hav} (|B + Hc|))$$

But in case of using the diagram, that isn't necessary. The same is achieved by "working from two sides towards the middle".

The start is the same, points A, B and O, line AB, (semi-)circle AB -> points C, D, E, F, J and K, lines OC, OE, CD, EF -> semi-circle EF, point G and a line through point J, perpendicular to the x-axis. Now all the knows are in the diagram.



Next define point H at the intersection of the vertical line through J and inner circle DF. Then draw a line GH. The angle DGH (marked γ) is the azimuth Zn.