Investigations into the Dip of the Horizon

H. C. Freiesleben
(German Hydrographic Institute)

1. INTRODUCTION. The dip of the horizon resulting from the observer's height of eye can be calculated from the formula Dip $= 1'{\cdot}06\sqrt{H}$, where H is the height of eye in feet, or by Dip $= 1'{\cdot}93\sqrt{h}$, where h is the height in metres. In this paper the latter form will be used so that comparison can be made with studies in international publications.

Taking into account terrestrial refraction, which under normal atmospheric conditions reduces the dip, the formula becomes Dip $= 1'{\cdot}93\sqrt{\{h(1-k)\}}$, where k is the coefficient of refraction used in geodesy. A mean value of 0·13 is assumed for k, and the formula thus becomes Dip $= 1'{\cdot}80\sqrt{h}$. This value, or one very near it, is used in most nautical tables.

The normal atmospheric conditions for which this figure holds good exist when the density of the air decreases uniformly with increasing height above the Earth's surface. As the actual density of air decreases, so also the optical density decreases; and the same holds good for the refractive index, which amounts to 1 *in vacuo*. If the density of the air is the same along the whole line of sight, the line of sight becomes a straight line; and this may occasionally happen. It is even possible for the density of the air to increase with height. In this case the line of sight becomes convex to the Earth's surface and the distance of the horizon is decreased.

The fall in density with increasing height may, however, sometimes be much greater than under normal atmospheric conditions. In this case the dip observed is unusually small and the distance of the horizon considerably increased. Since the density of the air decreases when the air gets warmer, this phenomenon occurs most often when warm air lies above colder water.

These phenomena are familiar to the navigator. He will, however, find it difficult, if not impossible, to ascertain the type of irregularity with which he is confronted. Elevations of the horizon can, for instance, be expected during the spring in the Newfoundland Banks area, and at all times of the year in the Red Sea and off Cape Guardafui. Occasionally, however, the dip of the horizon differs from its standard value elsewhere. The problem is whether such abnormalities can be predicted either from theory or as a result of experiment.

2. PRACTICAL DIFFICULTIES OF THE THEORY. The optical density of the air depends on air pressure, temperature and humidity. It can be shown

from theory that the temperature gradient is a particularly important factor in the dip; and this is confirmed by the fact that irregularities occur in places where warm air lies over colder water. The dip is normal in *conditions where the temperature decreases 1° C. for every 100 m. height.*

Nevertheless it is difficult to prove more than a general conformity between the theory and observations.

If the properties of an air mass could be known along the whole path of a ray of light, then that path could be accurately determined. However, such measurements imply a prior knowledge of the track, and there are practical difficulties of temperature measurement in particular, and a number of imponderables such as the effect of a rough sea on the layer of air immediately above it, which makes the problem by no means simple. It is not surprising therefore that the various observations that have been made should have given rise to a certain amount of discussion.

3. HISTORICAL DEVELOPMENT. The earliest observations were made on the south coast of France towards the end of the eighteenth and the beginning of the nineteenth centuries.[1] Further observations were made by Sabine[2] across the Gulf Stream, and here an attempt was made to establish numerically the relationship between the dip and the difference of temperature of the air and water. Systematic measurements combined with thorough meteorological observations were also made from the French warship *La Gallissonière* in east Asian waters.[3] A further series of observations were carried out on the Lake of Geneva by Forell.[4]

In the nineteenth century a great number of observations were made by the Austrian officer Koss in a series carried out in the Red Sea from 1897–8 aboard S.M.S. *Pola*, and later in an extensive series of theodolite observations carried out in the Adriatic.[5]

The dip of the horizon can be measured from on board by comparing the computed and observed altitudes of stars taken from a known position of the ship; alternatively it can be deduced from measurements of the altitudes of a star above the front and back horizons. A special instrument was developed for measuring the dip as a result of Koss's experiments, and a description of this is given later.

More accurate measurements can be made from on shore by means of theodolites, but difficulties in observing the horizon accurately and systematic observational errors, which become inseparable from other anomalies, give less accurate results for the method than one would expect.

Koss's measurements by theodolite in the Adriatic were made from various heights of eye: 6 m., 10 m., 16 m. and 42 m. (most of them from 16 m.). This procedure was no doubt correct, since the height of eye, in addition to its simple geometric application, can also be significant for more complex conditions when it influences the curvature of the ray.

The shore observations were supplemented by temperature measurements both from sea and on shore. Air temperature was taken at various heights down to 1 m. and ½ m. above the sea surface and the surface temperature of the sea was recorded.

The results of the observations were described by Koss himself, and later by Kohlschütter.[6] Kohlschütter's study was mathematically the more thorough, but the conclusions do not differ essentially from those drawn by Koss.

According to the analysis the dip of the horizon and its dependence on the temperature distribution can be established from the formula

$$\text{Dip} = 1'{\cdot}82\sqrt{h} - 0'{\cdot}37\Delta \qquad (1)$$

where Δ is the difference in degrees centigrade between the surface temperature and the temperature of the air at the height of eye.

Koss introduced into his original formula the difference between the surface temperature and that of the air at a height of 1 m. This was modified by Kohlschütter, who had derived from Koss's measurements a law governing temperature gradients

$$\frac{dT}{dh} = 0^{\circ}{\cdot}016 + \frac{\Delta}{\sqrt{h}}0{\cdot}029$$

This regular distribution however only obtained in conditions when a wind of at least force 2 was blowing to mix the layers of air. Laboratory experiments by Brehmer [7] confirmed this law.

The law for the gradient of temperature is empirical only. The necessity of excluding all observations made during calms restricts the applicability of the two formulae. This had already been pointed out by Kohlschütter, who regarded it as a serious objection that the temperature gradient immediately above the surface of the water should work out as infinity. Also the formulae were based on observations from heights between 6 and 42 m., and are not necessarily universally valid; frequently they will not be in accordance with fact.

Despite this, the results of Koss's observations have been included in several nautical tables as the basis of correction tables. In fact the corrections seem to be inappropriate as often as not; for instance, they do not fit the observations made on board *La Gallissonière*, and were not confirmed by many measurements made by various ships of the German Navy between 1926 and 1929.[8]

4. THE PULFRICH INSTRUMENT. When he was working up the results of Koss's observations, Kohlschütter had a special instrument designed for measuring the dip. The principles of this instrument, named after its constructing engineer, Pulfrich, are illustrated in Fig. 1. Parts of the horizon separated in azimuth by 180° are lined up in the instrument by means of a fixed mirror A and a rotatable mirror B. The mirrors are at right angles to each other in conditions of zero dip when the visible and true horizons are identical, and the degree of rotation of B which is necessary to produce coincidence for any other condition will indicate the arithmetical mean value of the dip of the horizon and the counter-horizon. Possible errors in the indication of B's movement can be overcome by averaging the readings taken on opposite horizons.

The investigation of the dip in a particular direction can only serve any purpose if the temperatures along the path of a ray of light from that direction can be correctly ascertained. It is permissible for the purpose of examining the relationship between the dip and the temperature surrounding the ship to assume a uniform field of temperature, and therefore a uniform depression, all round; this is particularly the case when some distance off the coast.

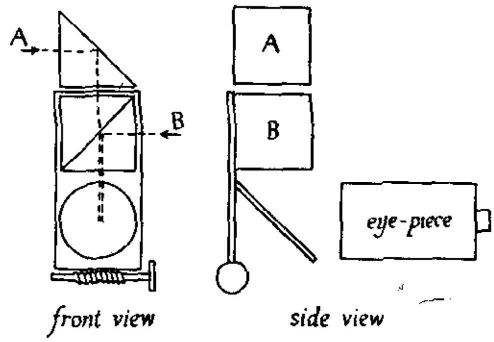

Fig. 1. The Pulfrich instrument.

Observations with the Pulfrich instrument were expected to confirm the results obtained by Koss and Kohlschütter; in fact this was only partly so, and formula (1), which until then had been regarded as the most accurate formula, was in fact contradicted by them.

5. INVESTIGATIONS BY F. CONRAD. Formula (1) had been obtained purely empirically, although a theoretical background had been provided by Kohlschütter to Koss's observations, making clear where the results had been obtained purely by experiment.

A stricter theoretical derivation was therefore required to explain the various anomalies. There was also the possibility that deviations between the formula and the observed values could have been caused by inaccurate measurements of temperature; and it seemed desirable to have observations made from greater heights than those used by Koss. It also remained to be established whether atmospheric conditions could be represented satisfactorily in terms of the difference between the air and surface temperatures.

These and similar considerations led the late Rear Admiral Dr. F. Conrad to institute a further series of observations of the dip, similar to those made by Koss. The observations were made from on shore by theodolite and the Pulfrich instrument, and every attempt was made to obtain the most accurate temperature measurements. The main object of the investigation was not to establish the validity of the Koss-Kohlschütter law, but to find out whether the exact relationship between atmospheric conditions and the dip of the horizon could be established in any individual circumstance.

The observations were carried out on Heligoland and Rügen in 1934–8 and were divided up into eight series, each series occupying 8–10 days. Observations on days when the horizon was not clear or when rough weather prevented it from being easily observed were discarded.

Theodolite observations were only made either into the wind or in directions from which land influence would not affect the path of the ray.

This may seem an excessive precaution since the influence of the land is only one of many factors which can affect the structure of the atmosphere in the lower layers. A similar caution was exercised in distributing the observations seasonally, though here again similar optical conditions can, in certain circumstances, prevail in winter and in summer.

The expense of these measurements was high, particularly as regards the number of vessels. Two steam boats and two motor launches were employed for measuring temperatures, and later a small captive balloon equipped with temperature-measuring gear. Automatic recording instruments were not used because of various difficulties in rough weather.

The value of the observations to astronomical navigation seemed, however, to justify the cost. The German Navy was primarily interested in small heights of eye and aviation in the large ones which exceeded Koss's measurements and were for the first time included in the investigations.

6. INSUFFICIENCY OF TEMPERATURE MEASUREMENTS AT SEA. Great difficulties were experienced in taking satisfactory temperature measurements at sea. Every vessel, by its mere presence, changes the structure of the air mass surrounding it. Quite apart from these air disturbances, the radiation of the ship is a source of error. The possibility of errors due to this may be somewhat reduced by carrying out the measurements on the weather side, but here also, the presence of the ship may, in certain circumstances, bring low-lying air masses up to the height of the upper deck or the bridge. Thus it is problematic whether the temperature measured at a certain height is in fact the true temperature of that layer of air. Even an inaccuracy of $0^\circ{\cdot}1$ in the temperature can have an appreciable effect and sometimes errors of almost 1° occur in the measurements.

That such errors exist was proved by comparing measurements carried out in different places at the same time. In many cases at least the probability of these errors was shown, and the observations were discarded for further investigations. It is disturbing to realize the number of errors that are possible in applying any formula giving the dependence of the dip of the horizon on temperature. Even if great pains are taken with the measurement of temperature, incorrect values are always liable to occur, not because the measurement is bad but because the temperature measured is not that of the air mass concerned. Consequently, even the application of the best formula for the dip to individual cases is valueless.

The omission of certain temperatures or individual series of measurements was, moreover, justified later. The deviations from the final results occurred so irregularly that they could not have been altered systematically, even if the omitted measurements had been taken into consideration. In the event, formula (1) was not confirmed, so that there could not have been prejudiced judgement.

7. AN EMPIRICAL FORMULA FOR THE DIP. If a formula of the type of (1) is used and the coefficient Δ is found to depend on height, then for a

mean height of eye of 4 m., 8 m. and 50 m. the best representations of the dips observed, on Heligoland, were

$$4 \text{ m. Dip} = 1'{\cdot}74\sqrt{h} - 0'{\cdot}47\Delta$$
$$8 \text{ m. Dip} = 1'{\cdot}74\sqrt{h} - 0'{\cdot}23\Delta$$
$$50 \text{ m. Dip} = 1'{\cdot}77\sqrt{h} - 0'{\cdot}04\Delta$$

The measurements on Rügen, which were fewer, showed similar results; Koss's measurements on the other hand had not indicated such a close relationship between temperature and height.

It is evident from these results that the influence of temperature depends largely on the height of eye. This influence is much smaller for great heights of eye than for small ones; but as a general rule, the dip of the horizon is more constant from great heights of eye than for small ones. Even in cases where the temperature conditions are equal or similar, the values differ for the latter.

The three formulae given are an incomplete representation of the dips observed. The random errors of the individual values in the formulae are considerable, the mean error being about $\pm 0'{\cdot}9/\sqrt{h}$, depending on the height of eye. The deviations are greatest for small heights of eye.

The Pulfrich instruments proved to be reliable and gave results that practically coincided with those from the theodolite measurements, and were well within the limits of the mean error given above. The instrument is suitable to navigational requirements and the application of the dip measured by it invariably gave a higher order of accuracy to astronomical position lines.

The Koss-Kohlschütter formula, being correct for a number of cases, must be an approximation. A more precise formula can be evolved as a result of the observations.

8. A NEW FORMULA FOR THE DIP. The dip of the horizon can be calculated in a theoretically more precise manner than before by assuming the atmosphere to be divided up into a number of concentric layers of air; an assumption similar to that made in the Admiralty Navigation Manual (1938)[9] for the derivation of the dip of the horizon and refraction (Fig. 2).

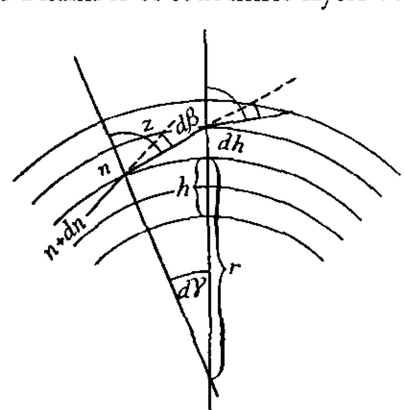

Fig. 2.

Considering a single layer of this kind, the depth can be denoted by dh, the height above sea-level by h, and the refractive index by n, the optical density being assumed constant. By Snell's law, then, the deflection of a ray of light passing from one layer to another of refractive index $n + dn$, will be given by $d\beta = (-dn/n) \tan z$, where $d\beta$ is the amount by which the zenith distance z is reduced by refraction.

The dip of the horizon does not, however, only contain the sum of these refractions from the lowest layer of air to that at the height of eye. Its main part depends on the curvature of the Earth's surface from the observer's position to the point where the ray strikes the surface; and this in turn depends on the total length of the light's path.

For a very small portion of the path of a ray of light, the amount $d\gamma$ by which the Earth's curvature alters the zenith distance can be represented by $d\gamma = (dh/r) \tan z$ where r = the Earth's radius + h.

The coefficient of refraction (k) used in geodesy is defined as

$$k = \frac{r}{n}\left(\frac{dn}{dh}\right).$$

In a first approximation, and at moderate heights (which are the only heights of importance for the problem of the dip), this coefficient depends on the temperature gradient and on numerical constants derived from the gas constant of the air, the refractive index at 760 mb and 0° C., and the radius of the Earth.

Thus
$$k = 6{\cdot}84\left(0{\cdot}034 + \frac{dT}{dh}\right)$$

On the other hand, the refractive index n of a single layer of air depends on its height h and its temperature T_h. Integrating the quantities $d\gamma - d\beta$ for the individual layers, the formula for the dip of the horizon in minutes of arc becomes

$$\text{Dip} = 5'{\cdot}04\sqrt{(0{\cdot}1123h + T_0 - T_h)} \qquad (2)$$

where h is the height in metres and T_0 the temperature of the lowest layer.

This formula, using only two temperatures, is also an approximation; but it does explain all the previous anomalies.

In the special case where k is constant dT/dh becomes $(T_h - T_0)/h$, which gives Dip $= 1'{\cdot}93\sqrt{\{h(1-k)\}}$. This gives values of Dip $= 1'{\cdot}93\sqrt{h}$, when $k = 0$, $(T_h - T_0)/h = 0{\cdot}034$. For $(T_h - T_0)/h = 0{\cdot}1123$, Dip $= 0$, i.e. it is a ray parallel to the Earth's surface. Beyond this value there would be an elevation of the horizon. If this does occur, either the condition of the concentric layers cannot be fulfilled, or there are two images of the horizon, for a ray of light which strikes the observer's eye with a zenith distance less than 90° must pass twice through the same height (Fig. 3).

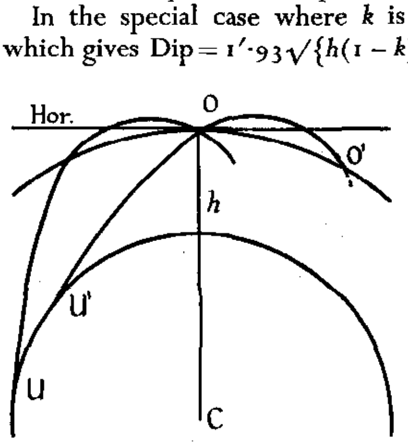

Fig. 3.

A graph (Fig. 4) of formula (2) shows that it can be adapted to certain heights of eye and temperatures by a formula resembling that of Koss and Kohlschütter. But the coefficient of Δ must become smaller when the height of eye increases, as was shown by the Heligoland observations. The coefficient of $\sqrt{h}$, on the other hand, will vary with the range of Δ.

Despite this, it is difficult from formula (2) to verify the figures determined by Koss and Kohlschütter. For $h = 16$ m., for instance, Dip $= 1'{\cdot}69\sqrt{h} - 1'{\cdot}88\ (T_h - T_0)$ can be derived. The temperature factor is about 6 times as great as that given by Koss and Kohlschütter; but the temperature of the surface of the water, instead of the temperature T_0 of the lowest layer of air was used for their formula, and the difference appears decisive.

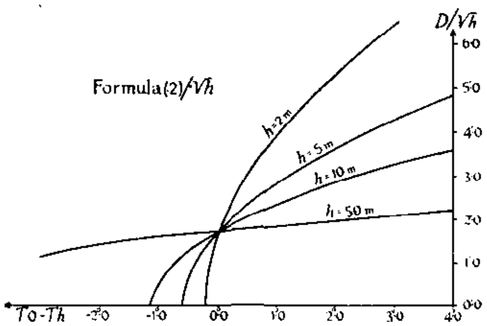

Fig. 4. A graph of formula (2) divided by $\sqrt{h}$.

9. THE INFLUENCE OF THE TEMPERATURE IN THE LOWEST LAYER OF AIR. It is difficult to measure the temperature of the water surface. The usual procedure is for the observer to obtain a bucketful of sea water from the surface and ascertain its temperature. This, however, will not be the temperature of the water of the uppermost stratum, but includes water from lower strata as well. There is, furthermore, no logical reason why the temperature of the uppermost stratum of water should be the same as that in the lowest layer of air. It may be so, particularly if the same meteorological conditions prevail for a very long time, but it is not necessarily so. In this case, the formula of Koss and Kohlschütter must fail.

From the observed dips of the horizon, therefore, the differences of temperature $T_0 - T_h$ were computed by formula (2). Many of them gave small values, and, like the dip itself, were fairly constant. This showed that the value T_0 which is essential for the dip is more closely related to the temperature (T_h) at the height of eye than to the temperature of the surface of the water. During the measurements on Heligoland and Rügen, temperatures had been measured at very small heights above the water, some at 1 m., and some at ½ m. A comparison of the optically defined values of T_0 with these values showed surprising conformities. Moreover, the differences between the two temperatures were not systematic in all cases of some series; but generally they looked like random errors, and remained within the permitted limits, amounting to $\pm 0^\circ{\cdot}3$. Accordingly, the dips computed by means of formula (2) corresponded much more closely to the observations if the temperatures measured at a height of ½ m. or 1 m. were taken as T_0 than if T_0 was replaced by the temperature of the surface water.

This result suggests that there is a border layer over the water, the determination of whose temperature is essential to knowledge of the path of a ray of light. This temperature is related to that of the air body through which the light passes, but it is not necessarily connected with the temperature of the water.

This conclusion is in accordance with those drawn from modern research on turbulence. Under certain stationary meteorological conditions, T_0 is also related to the temperature of the water surface because, in this case, T_0 adapts itself to the water temperature. Then the application of an empirical formula for the dip of the horizon similar to that of Koss-Kohlschütter becomes possible, although the numerical coefficients have to be adapted to the special circumstances. Detailed examination of the observations indeed make it possible to establish when such adaptations had taken place. As the measurements were taken in the vicinity of the coast where the influence of the land and the tidal streams often makes air and water masses which differ in temperature meet, non-stationary conditions were frequently found although they are comparatively rare on the open sea. (Probably they did not occur very often in the Adriatic either.) The dip of the horizon in this case as compared to the temperature difference Δ between the water and the air has an anomalous influence until stationary conditions are restored. It is only for such conditions that Δ can be considered representative of the structure of the air masses, and only then does a relationship exist between Δ and $T_0 - T_h$.

This relationship corresponds to modern conceptions of the thermal structure of the lowest layers of the atmosphere. In accordance with these ideas, attempts were also made to determine the height of the layer where a temperature T_0 prevails; these investigations only showed that it must certainly be below 1 m. As it is impossible to obtain accurate measurements of temperature at such low heights, it is impossible for practical navigators to compute reliably the dip of the horizon from measurements of temperature. Even a small error in $T_0 - T_h$ causes the dip of the horizon to be computed incorrectly, the latter value being extraordinarily susceptible to changes in the former.

10. Conclusion. These investigations into the dip of the horizon are offered as a considerable contribution toward the explanation of a much-debated scientific problem, and have proved for practical purposes that measurements of the temperature can only give approximate values for the dip of the horizon but will not yield correct results in individual cases.

11. Acknowledgement. Acknowledgement is made to Dr. G. Prüfer who, in cooperation with the author, submitted to the German Hydrographic Institute the results of the measurements of the dip of the horizon made by Dr. F. Conrad together with a detailed report which has not yet been published.

REFERENCES

[1] v. Zach, F. X., 1814, *L'Attraction des Montagnes,* Vol. 2, p. 501, Avignon.

[2] Raper, H., 1870, *The Practice of Navigation and Nautical Astronomy*, p. 61 *et seq.*, London: Potter.

[3] Perrin, E., 1886, Sur les dépressions de l'horizon de la mer, *Comptes Rendues hebdomadaires des Séances de l'Académie des Sciences*, Vol. 102, p. 495, Paris: Gauthier Villars.

[4] Forel, F. A., 1899, Les variations de l'horizon apparent, *Comptes Rendues hebdomadaires des Séances de l'Académie des Sciences,* Vol. 129, p. 272, Paris: Gauthier Villars.

[5] Koss, K., and Graf Thun-Hohenstein, E., 1900, Kimmtiefenbeobachtungen zu Verudella, *Denkschriften der Kais. Akademie d. Wissenschaft.*, Math. nat-Kl. Bd. 70, Vienna.

[6] Kohlschütter, E., 1903, Folgerungen aus den Kossschen Kimmtiefenbeobachtungen zu Verudella, *Annalen der Hydrographie u. mar. Meteorologie*, Vol. 31, p. 533, Berlin.

[7] Brehmer, K., 1909, Beitrag zur atmosphärischen Refraktion über Wasserflächen, *Annalen der Hydrographie und mar. Meteorologie*, Vol. 37, p. 306, Berlin.

[8] Hessen, K., Huber, A., und Thorade, H., 1930, Kimmtiefenmessungen an Bord von Schiffen der Reichsmarine, *Veröffentlichungen des Marine-Observatoriums in Wilhelmshaven* Nr. 15, Berlin: Mittler u. Sohn.

[9] *Admiralty Navigation Manual*, 1938, vol. 2, p. 80, London: H.M.S.O.