H.A.C.van Asten

Frederick Noonan Precomputed a Running Sunset Fix for Amelia Earhart's Flight from New Guinea to Howland, July 2, 1937

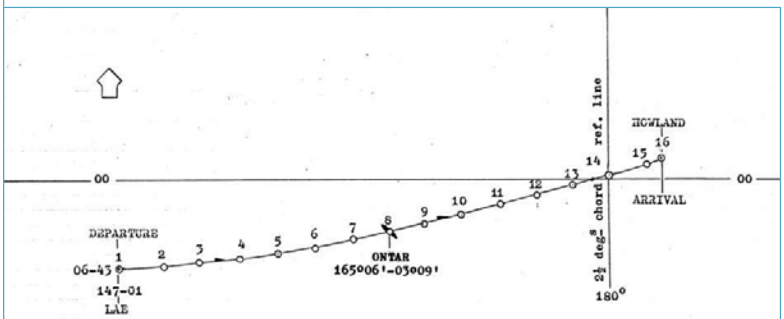
In the July, 2008, issue of EJN [1] it was introduced how navigator Noonan decided not to follow the great circle tracks for Howland, Pacific, to Lae, New Guinea as interpolated by consultant C. Williams for the world flight´s first attempt and here recomputed, fig.1, for the in reverse sense actual trip of July 2. This additional article will pursue the matter further to accurately bring to light all sources of figures and decisions F.Noonan made use of, to establish his geographical position before nightfall at the end of the first 850 miles track after takeoff. When one was over Gagan, latitude 05°14´on Buka Island, arriving from Lae on course 79 T, the no.3 great circle rhumb line ran 2.9 mls northwards, a decline within acceptable pilot error. When however, the aircraft was over the 0720 GMT position in 04°33´.5 south latitude, the no.5 circle chord attained 04°21´.0 with a 14.4 mls decline. This is a strong indication that, at least from the longitude of Gagan, the great circle option has been definitely abandoned, to be replaced by the first alternative of a row of five: the loxodromic curve Gagan to Nukumanu Island with the famous 0720 GMT fixed position on the S-04°33´.5 latitude circle.

We will hereafter try to exactly shadow Noonan when he precomputed the 0720 GMT fix which will show to have been a Sunset Fix on the Gagan – Nukumanu rhumb line, for a place not covered by great circle coordinates.

1. The Great Circle line abandoned

There was good reason to abandon a great circle plan: the anticipated more than ten hours flight in darkness would presumably catch adverse weather and overcast, your precise groundspeed and drift unknown, whereas even in ideal daytime circumstances it is not easy to

Fig. 1. The Howland to Lae great circle chords recomputed for the in reverse sense flight of July 2



Long	150.0	152.5	155.0	157.5	160.0	162.5	165.0	167.5	170.0	172.5	175.0	177.5	180.0	177.5	176.7
Nº	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Lat	06-10	05-41	05-12	04-52	04-12	03-40	03-09	02-37	02-05	01-32	01-00	00-27	00-06	00-39	00-49

Vertex Long.90°27'.5-W ; Lat.12°21'5-N . For re-computation use $\tan L_1 = \tan L_v \cdot \cos\theta$; θ = angle to vertex for longitude ; L_v = Latitude of Vertex ; L_1 = Latitude of interpolated position

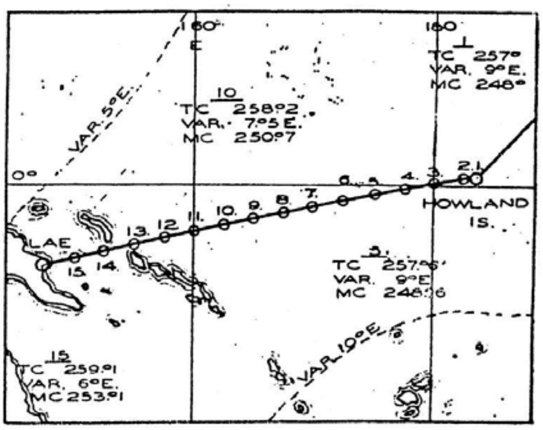
- Scale 2°.5 = .50 in -

Frederick Noonan
The Baltimore Sun, Mar 22, 1936

HOWLAND I. TO LAE
2556 MI. 17 H – 01 M.

NO.	TIME ON CRSE.	TIME TO CHGE.	TIME TO GO AT ST.	TOTAL TIME AT ST.	DIST. ON CRSE.	MAG. CRSE
1	:22	00:00	17:01	00:00	56	248°
2	1:11	00:22	16:39	00:22	177	248°
3	1:11	1:33	15:28	1:33	177	248°
4	1:11	2:44	14:17	2:44	177	248°
5	1:11	3:55	13:06	3:55	177	249°
6	1:11	5:06	11:55	5:06	177	249°
7	1:10	6:17	10:44	6:17	176	250°
8	1:10	7:27	9:34	7:27	176	250°
9	1:10	8:37	8:24	8:37	176	250°
10	1:10	9:47	7:14	9:47	178	251°
11	1:10	10:57	6:04	10:57	178	251°
12	1:10	12:07	4:54	12:07	175	251°
13	1:10	13:17	3:44	13:17	175	252°
14	1:10	14:27	2:34	14:27	175	253°
15	1:24	15:37	1:24	15:37	210	253°
1	17:01	TOTALS			2556	

FLIGHT DATA BY Clarence S. Williams
CONSULTANT IN NAVIGATION
LOS ANGELES, CALIFORNIA



The Howland to Lae Plan by C.S.Williams

keep to a great circle by slightly changing course, to set for a next chord every few hours and if a track is for any reason lost it is hardly possible to proceed for that next so that long distance great circle over ocean flying might easily become suicidal, if not the most deadly enterprise navigators could venture at the time, and they were aware of. Against common usage and somewhat mysteriously the crew left no flight plan behind at the airfield of departure: possibly it was beforehand for safety, see the conclusions in § 6, decided not to follow the great circle, without divulging since a direct 2,550 miles [spherics distance 2,547; sum of 15 chords 2,556] flight would generate more publicity than one via a variety of aids to navigation: Noonan who had just remarried considered the foundation of a navigation school. The first sign of the great circle plan having been abandoned came from serendipity: the discovery, early in the preliminary research, that the coordinates of the 0720 GCT fix exactly match the rhumb line 154°37´-E to 159°30´-E from 05°14´-S to 04°30´-S, the coordinates of Gagan and Nukumanu, specifically for the case that the from radio communications fixed time group figures of 0720 GMT and in addition ETA Nukumanu 0726:33 GAT, the latter as found from comparison of the metro forecast with the actual weather, are inserted, as we will presently see. All given time figures are basically in GAT; it has already in the mentioned 2008 article been remarked that for an isolated part of one day the difference with the uniform acceleration of mean time may be readily neglected to avoid time equation differences [here 3^m42^s to 3^m45^s, true sun slow] to sneak in. The aircraft took off Lae at 1000 Local Zone Time, 0000 mean time at Greenwich and was evidently flown on Zulu Time schedule. As measured in Greenwich Apparent Time, the departure was at July 1, 2356:18 GAT, the July time equation [3^m42^s] being subtractive from GMT.

2. An Astronomical Triangle recovered

The difference of longitude between Gagan and Nukumanu is 4°53´, or 293´.0 for the loxodromic distance 341 miles to compass point 081.5 T [True]. Since the distance is rather small we respectively find equal triangle sides with each algorithm: Difference of Latitude, Pythagoras, Direct Spherics or the Plane Traverse Table in H.O.208 Navigation Tables for Mariners and Aviators [Dreisonstok], always used by Noonan from its first, 1928, edition. All tables however, need some time consuming ´intricate´ work for arcmin interpolations with additional rounding so that, probably, it was preferred working with plane triangulation. For reason of history we nevertheless draw diagram 1 with mean figures extracted from H.O.208, p.108 of the 1937 edition , course 81°.5, column 2 for 200´ (200 nm) distance.

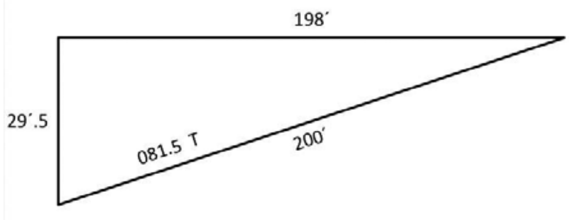


Diagram 1 . Plane Traverse Table of H.O. 208 for 200´

Since the true difference of longitude is 293´, not 198´, the figures of diagram 1 need linear upscaling with a factor 293/198 resulting in diagram 2 revealing that the latitude of the fix in 159°07´-E was

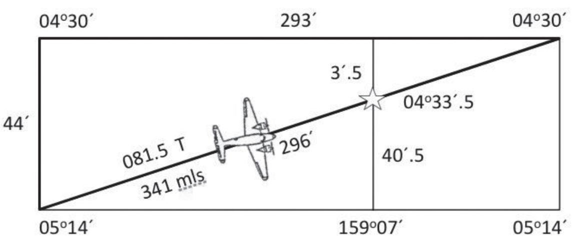


Diagram 2 . H.O.208 Plane Traverse Table for 200´ upscaled by factor 293/198 and linear subdivision

5°14´ - (270´/293´ x 44´) = 4°33´.5 in accordance with the 0720 GMT radio message *"Position latitude 4 degrees 33´.5 south – Longitude 159 degrees 07´ east"*. Diagram 2 also demonstrates the above mentioned Pythagorean triangulation, easiest to handle and giving identical results, acquired without any severe rounding off.

3. The approximate longitude at on board sunset

Over Gagan at 0439:53 GAT sun´s hour angle versus aircraft´s meridian was 44°33´15, or 44°33´ for air navigation of the era, leaving 43°13´ after subtraction from 87°45´53", or 87°46´ which is the approximate local our angle for central sunset on the 05°14´ parallel. At 121 mph track speed, or 01°44´ due east convey the angle between aircraft´s meridian and the subsolar point protracts by 16°44´ per hour. The *approximate* time of sunset on board is now found (43°13´):16°44´/hr = $2^h34^m58^s$ after passing over Gagan, or against 0714:51 GAT, 0718:36 GMT watch time [2]. At that point of time aircraft´s assessed longitude would be 154°37´+ (0714:53 - 0439:53)hrs x 1°44´/hr = 159°05´.5-East. A position versus time group was especially in the era sensible for rapid westwards and [slower] eastwards deviation since the speed of aircraft as measured in arc was far below sun´s speed in orbit: for our case with a factor 7. It was therefore necessary to precompute for expected sundown in a row of ETA coordinates for an expected geographical position as well as for a row of sunset time points: in this event called a *Sunset Running Fix* and fly the aircraft steadily as possible to assure the chosen speed to remain constant.

4. Inserting the exact time of on board sunset

The *exact* time of central sunset in any geographical latitude L is a function of the local hour angle P measuring the arc between the meridian and the subsolar point and sun´s declination d as well as the latitude with prescription cos P = tan d. tan L for declination [+] and latitude [-] of different name. Since we have found 04°33´.5-E the approximate latitude of sunrise on board, the next step is: calculate the exact time point of sunset on sequential latitudes about this parallel, by which the belonging longitudes, here in the 159°06´-E vicinity, will be also established. We (re)compute the local hour angles at central sunset for latitudes south 4°35´.5 through 4°31´.5 in 0´.5 steps, a range totaling 28 st.mls on the aircraft progression line APL in diagram 2.

4.1. Local Hour Angle by H.O.208

[73]	L 04°33´.5	lg cot x 10^3	D	1098.5
[77]	d 23°04´.0	lg cot x 10^3	D	371
[73]	90°- P	lg csc x 10^3	B	1469.5

The in brackets lower number [73] is the corresponding page in H.O.208 [3], Table II, where we find 90° – P = 1°56´39" by linear interpolation, giving the complement P = 88°03´21". In latitude 4°33´.5-S the time of sunset is T_{GAT} = 0715:45, so that the belonging longitude follows from

$$[P + 180°] – GAT_{arc} = Long_{arc}$$

giving 159°07´, or $10^h36^m28^s$-E. Since we must enter into a part and expound, our calculation is extensive and seemingly intricate, but for an experienced navigator going straight to the point it was not: he probably checked for longitude by the difference of Greenwich and Local Apparent Time which was a professional routine operation of the era, given in § 9, H.O.no.9 "Useful Tables" belonging to The [Bowditch] American Practical Navigator.

Table I. Conservative computation of longitude, GAT subtractive from LAT. GMT named Greenwich Civil Time from 1925 through 1951

GCT	Sunset	$7^h19^m30^s$
	Equation of Time	03^m45^s
GAT	Sunset	$7^h15^m45^s$
LHA	88°03'18"	
	180°	
LAT arc	268°03'18"	$17^h52^m13^s$
Long-E	159°07'00"	$10^h36^m28^s$

Table II. GMT of sunset, latitude and longitude for on board sunset. Read from below for aircraft heading east

Latitude	GCT Sunset	Longitude
4°31´.5	0718:45	159°19´
4°32´.0	0718:56	159°16´
4°32´.5	0719:07	159°13´
4°33´.0	0719:18	159°10´
4°33´.5	0719:30	159°07´
4°34´.0	0719:41	159°04´
4°34´.5	0716:07	159°01´
4°35´0	0720:03	158°58´
4°35´5	0720:14	158°55

For longitudes west of 159°07´ arrival for sunset is early. For longitudes east of 159°07´ arrival is late. Match is at simultaneous means of time and longitudes rows.

4.2. No Time error of Chronometers

Whatever algorithm is followed, be it H.O.208, any logarithmic gonio table, or e.g. the Douwes-Borda formula, the here recomputed endogenous outcomes remain constant: the Earhart to Herald Tribune Offices, New York, June 30, 1937 cable, reading in part: *"..In addition FN has been unable* [electric breakdown at Malabar radio station. auth.] *account radio difficulties to set his chronometers lack knowledge their fastness or slowness.."* later outdated since the 071930 GMT, 0720 by radio communicated time-coordinates group is inviolably interconnected by mathematical precomputation with heliographic time as exogenous parameter which made a structural time error impossible: the on board chronometers [and Longines hack watch] must have been perfectly synchronous with a record of the Greenwich time and for that matter: a navigator would never reset two [of three] chronometers on his own initiative, knowing thereby activating Spode´s Law [3] in its deadly configuration.

4.3. The accuracy of the Sunset Fix

In the course of time mathematicians have developed more precise algorithms for loxodrome sailing; an advanced prescription for true course Ct in Mercator sailing is:

$$Ct = \tan^{-1}\left[\frac{3.143\,(Long_1\text{-}Long_2)}{180\,\{[\ln\tan(45° + ½\,L_2) - \ln\tan(45° + ½\,L_1]\,\}}\right]$$

[4] with for distance $D_{nm} = 60[L_2 - L_1]$. Sec C_t and the latitude of arrival $L_2 = \{[D_{nm} \cdot \cos C_t] / 60\} + L_1$. By insertion of the here applicable figures we obtain Ct = 81°26´02", D_{nm} 295.4, D_{sm}341, Lat.of arrival 04°30´00", this time computations by electronic calculator, with outcome that for air navigation the of the era calculation methods were of a sufficient if not excellent degree of exactness: in the thirties the enclosed circle of uncertainty for an aeronautical sundown – sunrise fix had a 6 st.mls radius if the sextant was operated by an experienced air navigator, hence the precalculation´s quality was better than the obtainable precision of observation.

5. The Sunset Observation in Practice

Table II charts the practical solution of the sunset fix. Note that westwards of the 159°07´ meridian the aircraft´s velocity was too great, whereas eastwards it was too small for having sun´s centre in the crosshairs of the sextant´s artificial horizon at the instant of sunset for the longitude. Evidently, the aircraft piloted as steadily as possible, Noonan acquired collimation, the bubble sextant preset +53´ [5], green filter, at plusminus 071930 GMT watch time for the 159°07´- 04°33´.5 closest coordinates of his running fix diagram, upon which he advised Amelia to communicate his findings about the closest coordinates pair and she so did at 0720 GMT according to the radio logbook of Lae Airport held by Harry Balfour. Shortly afterwards the aircraft´s radio changed from 6210 to the ´night time´ 3105 kcs channel as a result of which the operator never heard KHAQQ again. It was neglected that the 864 miles eastwards, over New Guinea sun was still largely [10°08´] above the Lae horizon so that the wave front quenched [8] before reaching airport´s aerials [6]: the 0720 fixed position remains the one and only on record.

6. Conclusion

During the to Howland flight of the Earhart-Noonan crew, an observation of sunset with the bubble sextant, [A/c @ 7000 ft, no optical horizon] at 0719:30 GMT followed by the 0720 GMT radio announcement placed the aircraft close to if not spot on the coordinates pair 159°07´-E; 04°33´.5-S, 27 miles southwestwards of the Nukumanu Islands, which is unconditional proof that the originally for the March, 1937 Howland to Lae, New Guinea, precomputed great circle trail running 13 stat.miles northwards, prerequisited in many if not all current publications on the subject, was not flown during the in reverse sense voyage of July 2. The primary rationale for this statement is that the eastwards flight to a very small open ocean target was a notoriously more dangerous venture than the original one from Howland to Lae with the vast mainland of New Guinea, or occasionally the numerous Solomon Islands, extending ahead after an at random landfall. On the ship Ontario, on station halfway the great circle at 165°06´-E; 03°09´-S with watch kept and appointments made, no aircraft was heard and no radio call received whereas the aircraft´s alternative progression line ran 100 mls south-eastwards at 1000 GMT: continued mysteriousness. In addition the aircraft´s radio signals were on the air from 1030 GMT when they were heard at Nauru, but great circle chord coordinates, ranking first for safety, were not transmitted. Due to the rather elaborate precomputations, necessary to deal with a sunset fix and other navigational adaptations it is to be expected that the resolution for not flying the great circle has been taken a substantial time, at least days before takeoff, with exception for the case that an alternative [composite sailing via Nauru – Nikunau] was a priori deployed. It must have been with respect to the eventual flight plan that Noonan, answering superintendent of civil aviation for New Guinea J.Collopy´s question about expectations, said that he was at ease about finding the island. The optimism did not come true: most probably by using the marine sextant for the sunrise fix next morning in the roads of Howland, without 3^m50^s [7] correction for the with regard to the bubble sextant different reference line, whereas the mean sun was the watch time standard [1], an erroneous [westward] position line was flown along and Howland, at ETA 1912 GMT 16 mls on the port bow in lieu of below the APL and of another category than, e.g., the Hawaiian Islands, remained beyond visual range.

References [selection from 108 entries]

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Notes

[1] “Where to Search for the Earhart Lockheed Electra”. EJN Vol.6 no.2. July 2008.

[2] 1937 Nautical Almanac: Sun July 2, Decl. 83134”.8, shift per 24^{hrs} (-) 261”.9. Eq.of Time 0000 GMT $221^s.62$, shift per 24^{hrs} (+)$11^s.42$, 0000 GMT Appar.Right Asc. of sun $24129^s.01 = 6^h42^m09^s.01$, shift $247^s.97/24^h$, Lae 10 hrs ahead of, Nauru 11 hrs ahead of, Howland US Navy 11 ½ hrs slow on GMT.

[3] Spode´s Law: “Thrown away information will be the most needed in future”.

[4] I borrow this formula from S.P.Howell, viz.references.

[5] 37´ refraction, 16´ semi diam.

[6] Conflict of thought: due to lack of radio experience and knowledge Amelia took the night time channel indication for literal; Balfour hurriedly tried to hold back to transmit the latest weather forecast on 6210 kcs but he was too late.

[7] The equation of time, true sun slow, at sunrise happened to be 3^m50^s likewise, it is therefore possible that it was considered that the obligatory addition was already accounted for by this figure alone. As a result the course for the one line approach may have been set at 1754:53 GMT instead of at 1758:43 GMT, 10 mls westwards of the initial turning point.

[8] HF spectrum radio signals of the 2,000 – 4,000 kcs frequency suffer absorption by sunlight in the lower troposphere and up to the 60 miles level of the stratosphere.