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EARLY TABULAR, GRAPHICAL AND INSTRUMENTAL, METHODS FOR SOLVING PROBLEMS OF PLANE SAILING

Abstract: This paper traces the development of the techniques used by navigators for solving plane sailing problems up to the seventeenth century, by which time mariners were gradually becoming accustomed to employing computational techniques using tables of natural and artificial (logarithmic) trigonometrical functions. From the outset of the Age of Discoveries, initiated by the Portuguese in the fifteenth century, the mariner relied heavily on scholars ashore to provide him with solutions to his navigational problems. The scholars did not fail him, and a wide range of interesting tabular, graphical and instrumental, methods, designed to offset the tedium of computation, were suggested by the cosmographers for use by the non-mathematical sailor. To emphasize the international character of navigation the inventions of Portuguese, Spanish, French, Dutch and English, scholars are discussed.

1. INTRODUCTION

Although Ptolemy, in his *Geographia*, explained the principles of map projections, the seaman's map during mediaeval times was a projectionless map constructed on the basis of compass bearings and estimated distances sailed between the stations at which the bearings were observed. The concepts of latitude and longitude were of no importance in cartography, so far as the seaman was concerned, until the rudimentary nautical astronomy used by Portuguese maritime explorers in the fifteenth century led to significant improvements in hydrographic surveying and charting. In those days the latitudes of places discovered were found from astronomical observations, and, as Cortesão remarks, a new dimension was given to nautical cartography (1).

(1) Cortesão, A. (1969). *History of Portuguese Cartography*. Vol. I. Junta de Investigações do Ultramar. Coimbra.

There is every indication that the Portuguese explorations (1) southwards along the West African coast led to the graduation of a charted equator. The equator, like the meridians, is a great circle, and it was graduated in the same way as the latitude graduations on the meridians. This, in turn, doubtless paved the way for the 'plane chart', or 'carta plana quadrada', as the Portuguese called it.

The characteristic feature of the plane chart, as indicated by the Portuguese name, is the simple network of squares forming the graticule. Parallels of latitude at equal intervals are projected as equidistant parallel straight lines, and equi-angular spaced meridians are also projected as equidistant parallel straight lines, these cutting the projected parallels at right angles. The scale of distance along the projected equator is the same as that along the projected meridians so that the graticule is a network of squares. In books dealing with map projections (2) this graticule is described as a 'simple cylindrical' or 'plate carrée' conventional (non-perspective) projection. The scale along the equator and meridians is preserved but east-west distortions increase proportionally to the secant of the latitude.

To the mediaeval mariner the plane chart appeared to conform to 'latitude sailing': by maintaining a constant altitude of the celestial pole, or 'altura', one sailed along a parallel of latitude; which is, of course, parallel to the equator. And this is in exact accord with the plane chart. Moreover, meridians cut parallels at right angles; and, indeed, their projections on the plane chart also cross at right angles. But it was puzzling to mariners to be told that if two ships started on the same parallel and sailed along their respective meridians at the same speed, their distance apart would change. The plane chart, of course, made it appear that their distance apart would remain the same; and it was this that created the puzzle. Sailors have always known that the Earth is spherical — simple observations demonstrate this fact but, as Professor Taylor (3) pointed out:

> 'The recognition that the world was a globe is one thing but to understand the properties of a spherical surface ... was another...'

The inaccuracies resulting from the use of the plane chart were identical to those resulting from the use of the 'toleta de marteloio', and these stemmed from the difficulty of handling the convergency of the meridians. The errors

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(1) The Portuguese maritime expansion during the fifteenth century was due to the genius of Prince Henry surnamed the navigator. The first voyage of exploration sponsored by the *Infante* took place in 1419. From 1421 to 1433 regular expeditions were sent to the African coast and in 1434 the dreaded Cape Bojador was rounded for the first time. By the time of Henry's death in 1460, the Portuguese had reached Cape Palmas and in 1488 the expedition under Bartolomeu Diaz rounded the southern African Cape.

(2) *Vide* Cotter, C. H. (1966). *The Astronomical and Mathematical Foundations of Geography*. Hollis and Carter, London. p. 217.

(3) Taylor, E. G. R. (1956). *The Haven-Finding Art*. Hollis and Carter, London.

are not too serious when navigating in low latitudes; and, at the equator, where the meridians are parallel and convergency is zero, no error results. But serious errors occur when using a plane chart in high latitudes.

2. THE 'TOLETO DE MARTELOJO'

Early Mediterranean mariners relied heavily on sailing directions which described courses in terms of named winds, and distances in terms of 'days' sail'. Within the enclosed 'middle-earth' sea this information appears to have satisfied seamen right down to the time when ocean navigation began and when significant advances, particularly by the introduction of the magnetic compass and the portolan chart, were made.

The mediaeval sailor, armed with a portolan chart, a pair of dividers and a straight-edge, could readily find the compass course to steer to reach his destination. But finding the course from a portolan chart is one thing, whereas ensuring that the ship maintains that course is another. It was seldom possible to make a direct passage because the path is determined largely by the wind direction and changes in the wind direction. It was of special importance, therefore, that an accurate record of courses and distances was kept. This meant that throughout the voyage the navigator needed to estimate the distance made by the ship each hour, and to record this, as well as the course, in a systematic way. Periodically, every watch or day, he would refer to his record, or 'reckoning' as it became known, and work out what progress the ship had made towards her destination.

The nigh impossible task of keeping a sailing vessel on the same course for any length of time, meant that the course had to be altered frequently, so that the straight-line path connecting the ports of departure and destination was crossed and re-crossed several times during the voyage. The process of finding a ship's position in these circumstances involved a process which became known as 'resolving a traverse', and it appears that in the earliest method of so doing, the mariner employed some form of 'traverse table'. The existence of such a table is implied in the writings of Ramon Lull.

Ramon Lull (c. 1233-1315) was an eminent Catalan mathematician and philosopher of Majorca. In the history of navigation he holds an important place, for he is credited with having been the first to write about the science of navigation and to mention the nautical chart (1). He was a prolific author and produced works in Arabic as well as in Catalan. In his *Arbre de Sciencia* written in Catalan and produced in c. 1295, in a section headed 'De Marineria', Ramon deals with '...les questions de geometria', one of which is:

> 'los mariners còm mesuren les milles en la mer? (How do mariners measure distance at sea?)

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(1) *Vide* Cortesão, A. (1969). *op. cit.* p. 204.

In answer to this question we read:

'los mariners consiren iiij vents general ço is saber, grec exaloc lebeg e maestre: e consiren lo centre del cercle en lo qual los vents fan angles e aprés consiren per lo vent levant anant la nau lung C milles del centre, quantes milles ha tro el vent de exaloc, e doblen les milles tro a CC milles, e conexen quant son moltiplicades les milles que son CC del vent de levant tro al vent do exaloc, per moltiplicamente de les milles qui son del terme centenar de levant tro al terme de exaloc: e daçò han instrument carta e compas agulla e tremuntana...' (1)

The instrument referred to in the concluding sentence is, according to Taylor (2), a form of traverse table; and Cortesão (3) agrees with this interpretation.

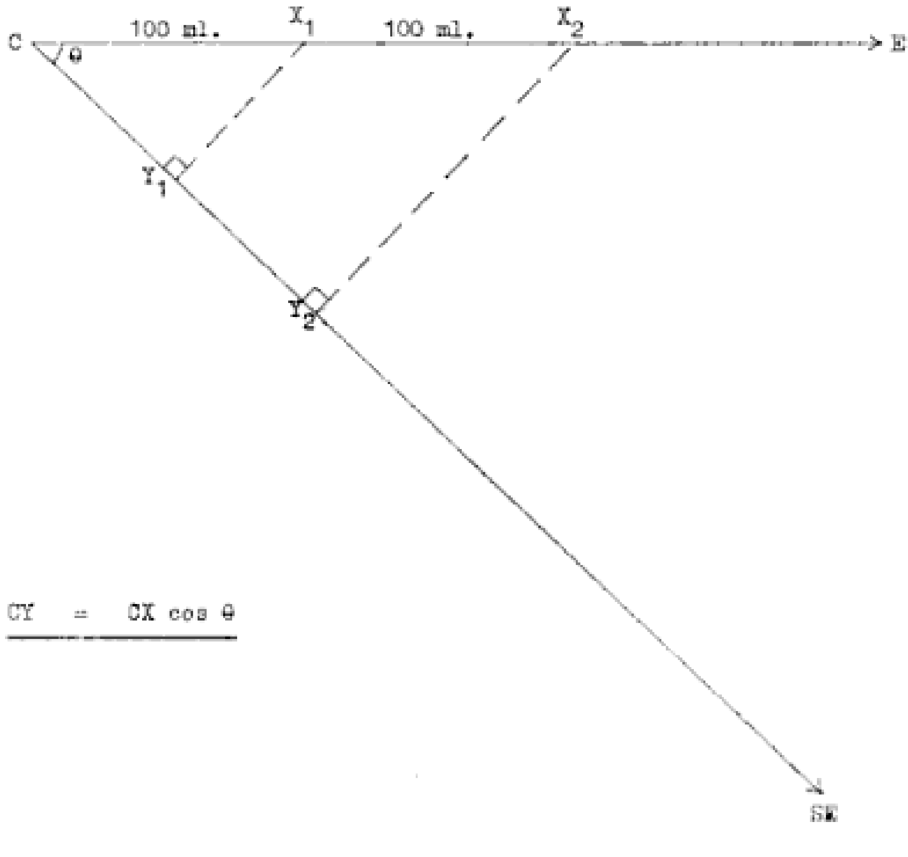

FIGURE 1

(1) 'The seamen consider four general winds and another four derived from them, viz. NE (grec), SE (exaloc), SW (lebeg) and NW (maestre), and they consider the centre of the circle at which the winds (rhumbs) make angles: and they consider for the E wind, the ship sailing a distance of 100 miles from the centre, how many miles she makes on the SE wind, and they double the number of miles to 200, and they know how many miles are multiplied, which are 200 from the E wind to the SE, by multiplication by the miles from the end point of each 100 miles E to the end point of the SE. And for this they have instrument, chart, compass (drawing compass), needle and north.'

(2) Taylor, E. G. R. (1956). *op. cit.* p. 118.

(3) Cortesão, A. (1969). *op. cit.* p. 209.

It follows from Ramon's explanation that a mariner in desiring to make good a course of SE but, because of the wind direction, sails on a course of E, makes good a distance equal to the distance sailed multiplied by the cosine of the angle between SE and E. Referring to figure 1, in which C denotes the 'centre of the circle at which the wind makes angle', it is seen that by sailing 100 miles due E from C to X_1, the ship makes good a distance CY_1 along the SE rhumb towards her destination. Had she sailed 200 miles from C to X_2 she would have made good a distance CY_1 along the SE rhumb; and, clearly, this is double the distance CY_2.

The procedure described by Ramon Lull might appear strange to the present-day navigator who is accustomed to resolving the distance travelled on a given leg of a traverse into a northing (or southing) and an easting (or westing), these being, respectively, 'd.lat' and 'departure'. But in mediaeval times the terms latitude and longitude were only beginning to have significance. Up to that time what was important in the sailing problem was the angle between the course to make good and the course actually steered.

In his *Ars magna generalis et ultima* written in c. 1305, Ramon Lull gives a geometrical explanation of his earlier remarks given in his *Arbre de sciencia*. The text of part of this explanation, in English translation by Cortesão (1), is illuminating in relation to Anderson's recent work on the philosophy of navigation (2).

> 'Navigation is the science by means of which sailors know how to sail through the seas. Indeed, navigation derives and springs from geometry and movement, in the relativity of time and space, that is, the ship is moveable and moves in time and space... Since geometry and arithmetic derive from this science (of time and space), it is evident that the science of navigation derives firstly from this knowledge and secondly from geometry and arithmetic...'

Ramon, in explaining the geometrical principle of the 'instrument' — a table for solving the sailing problem — supposed a navigator to be obliged to sail SE when he requires to make good a course of E. He explained that for each 4 miles covered on SE the ship makes good only 3 miles (2.83 more accurately) on E. This means that he 'loses' one mile in four or 25 (29 more accurately) in 100. Moreover, as Ramon explained, for each 4 miles sailed on a course of SE, the ship would be 3 miles from the desired path, as illustrated in figure 2.

The method of plotting a ship's position on a chart, a process known as 'pricking a chart', required the use of two pairs of dividers. One was opened up to an amount corresponding to the distance sailed, and the other, as in finding a course to steer, was employed to trace the appropriate rhumb-line on the chart. The aim was to determine the position of a point on

(1) *Ibid.* p. 207.

(2) Anderson, E. W. (1972). *The Education Potential of Navigation.* Unpublished M. Sc. thesis, University of Wales Institute of Science and Technology, Cardiff.

the chart lying on two loci; one corresponding to the distance sailed, and the other to the course steered. The name given by the Portuguese to the position thus obtained was 'ponto de fantasia', rendered by Edward Wright, in his *Certaine Errors in Navigation* of 1599, as the 'point of imagination'.

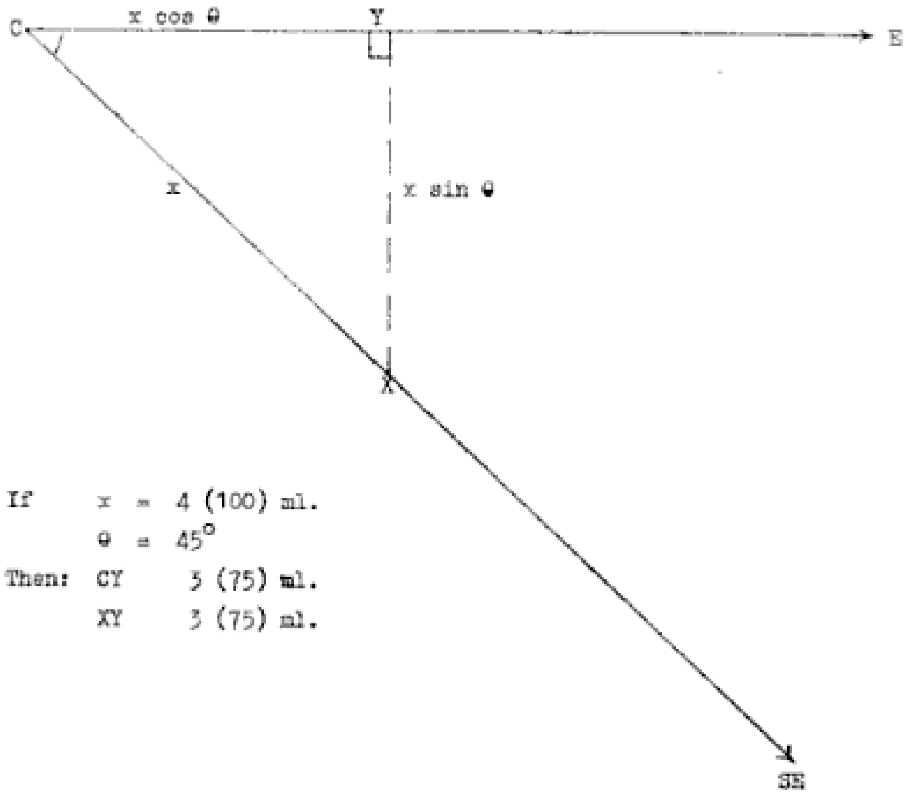

FIGURE 2

Ramon Lull's form of traverse table was given the name 'toleta de marteloio', and the rules for using it the 'raxon de marteloio'. The explanation of the word 'marteloio' has puzzled scholars, and Professor Taylor declared as recently as 1956 (1) that it had not been explained. Nordenskiöld, however, in his *Periplus* (2), argues that the expression is connected with the custom, still practised, of marking the time by striking the bell. The Spanish word 'marteloio' means 'hammer' (used for striking a bell), and 'raxon' means 'counting', so that 'raxon de marteloio' might signify dead reckoning by the hour or watch. Nordenskiold adds that the name is Spanish or Catalan in origin.

'The toleta de marteloio' described by Nordenskiöld in his *Periplus*, is that which appears on one of the sheets of an atlas by Andrea Bianco produced in 1436. Bianco's 'toleta' and 'raxon' are the earliest known

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(1) Taylor, E. G. R. (1956). *op. cit.* p. 117.

(2) Nordenskiöld, A. E. (1897). *Periplus: An Essay on the Early History of Charts and Sailing Directions.* Stockholm.

descriptions of these devices. In the first and fourth columns of the 'toleta', illustrated in figure 3, are given the angle (θ) in 'quarter-winds' (= compass

· largar ·

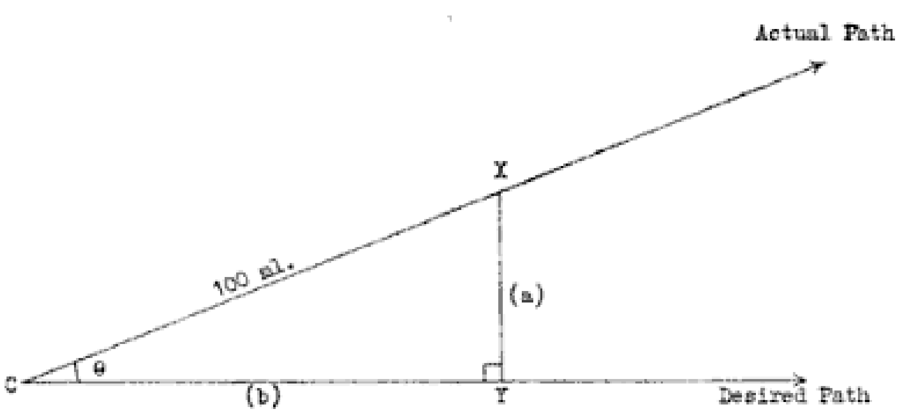
p una quarta	·20·	·98·	p 1 quarta	·51·	·5·
p 2 quarte	·38·	·92·	p 2 q°	·26·	·24·
p tre quarte	·55·	·83·	p 3 q°	·18·	·15·
p quatro q°	·71·	·71·	p 4 q°	·14·	·10·
p cinque q°	·83·	·55·	p 5 q°	·12·	[illegible]
p sie quarte	·92·	·38·	p 6 q°	·11·	·4·
p sete quarte	·98·	·20·	p 7 q°	[illegible]	[illegible]
p oto quarte	·100·	000	p 8 q°	[illegible]	·000·

FIGURE 3 — Andrea Bianco's Toleta de Marteloio (1436)

points of $11^1/_4°$ each) between the direction steered and the direction to make good. For each of the eight quarter-winds five quantities are given, as follows:

In the second column, 100 sin θ. (a)
In the third column, 100 cos θ. (b)
In the fifth column, 10/sin θ. (c)
In the sixth column, 10/tan θ. (d)

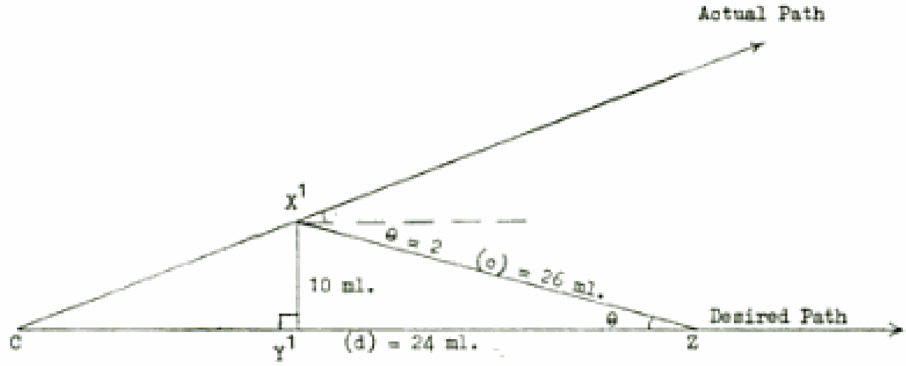
Figures 4 and 5 serve to explain the significance of these quantities. Referring to figure 4, the quantities (a) and (b) are 'alargar' and 'avançar'



If CX=100 ml. and θ=3 (a)=XY=55 ml. (alargar) (b)=CY=83 ml. (avançar)

FIGURE 4

respectively. In figure 5, the quantities (c) and (d) are 'retorno' and 'avanço' respectively.



If $X^1Y^1 = 10$ ml. and $\theta = 2$ (c) $= X^1Z = 26$ ml. (retorno) (d) $= Y^1Z = 24$ ml. (avanço)

FIGURE 5

3. THE CIRCLE AND SQUARE DIAGRAM

The Circle and Square Diagram, an example of which, like the toleta described above, was given in Bianco's atlas of 1436, is illustrated in figure 6.

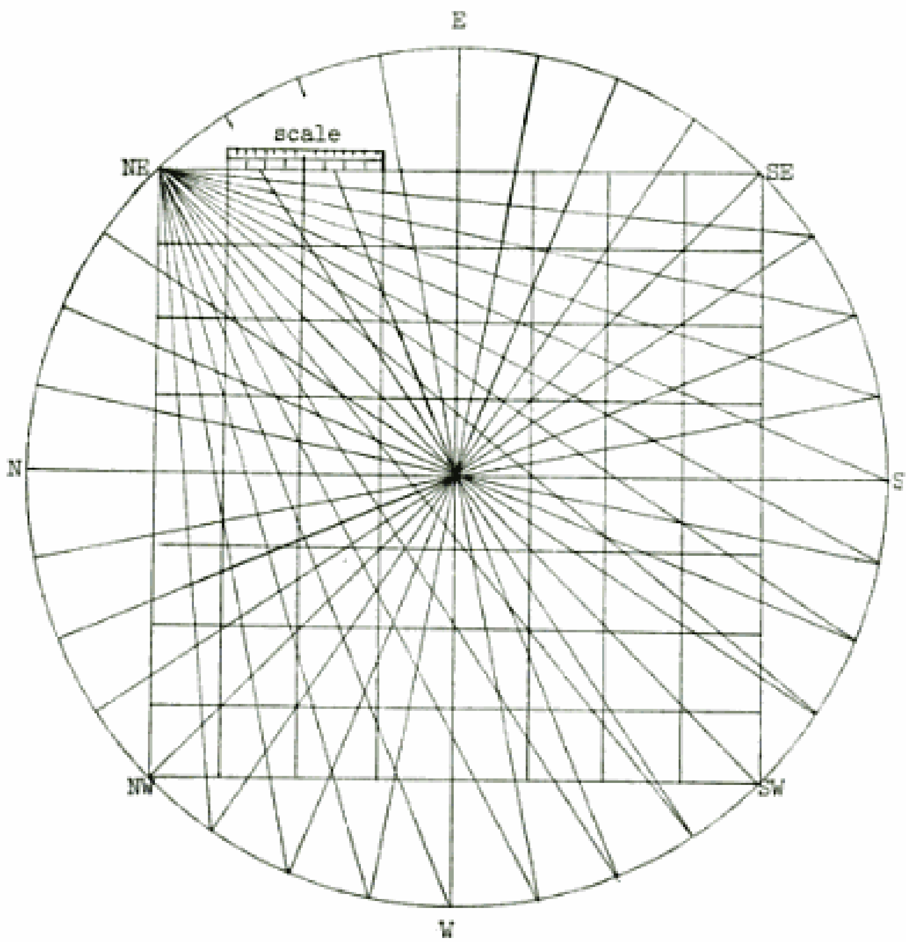


FIGURE 6 — The 'Ciecle and Square' Diagram for Solving Traverse Sailing Problems (Andrea Bianco, 1436)

It consists of a circle with the 32 winds or rhumbs radiating from the centre. Inscribed within the circle is a square the opposite sides of which are parallel to the North and South, and East and West rhumbs, respectively. The sides of the square are subdivided and a uniform grid is constructed within it. From one corner of the square 16 rhumbs are drawn within the right angle formed by adjacent sides of the square. Each of these meets one of the rhumbs drawn from the centre of the circle at the circumference of the circle.

In using the 'toleta de marteloio' the navigator was able to find 'alargar' and 'avançar' corresponding to a distance of 100 miles. For other distances the corresponding 'alargars' and 'avançars' would have to be found by the 'Rule of Three'. But this would have involved tedious arithmetic for which the early seaman was not generally equipped. The Circle and Square Diagram provided a simple graphical solution to the sailing problem. In particular the diagram afforded a rapid solution for finding 'd.lat.' and 'departure', although when 'latitude sailing' *(vide infra)* was introduced to the sailor in the Golden Age of Discovery the 'Regiment of the Leagues' (1) was invented by Portuguese cosmographers for the use of seamen.

4. THE REGIMENT OF THE LEAGUES

The first method used by Portuguese seamen for finding latitude at sea employed the Pole Star, the altitude of which approximates to the observer's latitude. The earliest instrument used for measuring an 'altura' of the Pole Star was the seaman's quadrant (2). At first the arc of the quadrant was graduated, not in degrees but relative to the 'altura' of the Pole Star at Lisbon. As exploration southward along the African coast proceeded seamen were required to mark on the arcs of their quadrants the positions of the plumb-line, when the Pole Star was observed, corresponding to newly-discovered capes, rivers and other geographical features. On their return to Lisbon the cosmographers, using this information, were able to map the progress of the discoveries. So that, at first, the use of 'alturas' served, in effect, to indicate the distance reached south of Lisbon. But in the course of time the technique developed into a system of navigation known as 'latitude sailing'. By this the seaman on an ocean voyage sailed northwards (or southwards) until he reached the latitude of his destination as determined from an 'altura'. He then sailed along the parallel, maintaining the altura as accurately as possible until a landfall was made.

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(1) *Vide* Albuquerque, Luis de, 'Astronomical Navigation' in Cortesão, A. (1971). *History of Portuguese Cartography*. Vol. 2. Junta de Investigações do Ultramar. p. 438.

(2) *Vide* Cotter, C. H. (1968). *A History of Nautical Astronomy*. Hollis and Carter, London. p. 58 et. seq.

In using the new method of 'running down the latitude' (as English seamen were to call it) the navigator was provided with means by which he could tell the distance sailed, on any given rhumb, as a consequence of changing his latitude one degree. The simple device by which he could get this information was the 'Regiment of the Leagues'.

When using the 'toleta de marteloio' the seaman used an *estimated* distance sailed: when using the 'Regiment of the Leagues' he used a *measured* change of latitude. So that, provided that altitudes were measured accurately, the new technique marked a decided improvement in the practice of navigation.

In the earliest regiments the meridianal distance between parallels of latitude one degree apart was taken as 17 $^{1}/_{2}$ leagues each of four miles. This reckoned 70 miles to the degree. In later examples a degree of a meridian was taken as 16 $^{2}/_{3}$ leagues or 67 miles per degree. But when the Earth's circumference was more accurately estimated in the seventeenth century, the minute-mile became firmly established and a degree of a meridian was taken as 20 leagues each of three miles.

The 'Regiment of the Leagues', or the 'Rule to raise or lay a degree', as the English called it, was sometimes given as a set of written instructions. William Bourne, for example, in his 'Rules of Navigation' appended to his *An Almanacke and Prognostication* (1) of 1571, after explaining that in sailing north or south the ship keeps on the meridian, pointed out that (in the northern hemisphere) when sailing north 'you rayse the pole and laie the equinoctial'. But if the course is southerly, 'you laye the pole and raise the equinoctial'. If, however, the course is due east or west, 'you doe not alter your pole or parrel but only your meridian':

> 'but in sayling on any other point, you doe alter your pole and parrel and also your meridian. Therefore I wil open unto you in the sayling vp on one of the quarters (points) of the cōpas, what euery point doth raise or laie one degree, in howe farre saylinge, and howe many myles you be departed from the place that you did departe from; & what you be departed from your meridian.'

Bourne cautioned his readers because on most charts of the time a degree of a meridian was taken as 17 $^{1}/_{2}$ leagues. But in England 60 miles per degree of a meridian was being allowed, and Bourne's rule for 'raising or laying a degree' was based on this figure:

> '...in sayling directly south or north, you doe raise or laie the pole in 60 miles going in the altering of one point from the southe or north 61 miles, and departed from the lyne of south or north, or the meridian 12 myles in the altering the second point,

(1) Bourne, W. (1571). *An Almanacke and Prognostication for three yeares, that is to saye for the yeare of oure Lord, 1571, and 1572 and 1573, now newlye added unto my late Rulles of Nauigation. London.* (*vide* Taylor, E. G. R. (1963). *A Regiment for the Sea and other writings on Navigation by William Bourne...* Hakluyt Society Publication No. 121. Cambridge.)

> you doe raise a degree in sailing of 65 miles and depart from your meridiã 25 miles in the altring of the third pointe you doe raise or laie one degree in the sailing of 72 myles and 9 part; and you doe depart from your meridiã 40 myles in ye altering of the 4 point you do raise or laie a degree in the going of 85 miles & departe from your meridian 60 myles... etc.'

Bourne added that for anyone who wished to know the 'raising or laiying of a degree by the legges (leagues) of the cardes that is of 17 leages and a halfe', reference should be made to the English translation of *Arte de Navegar* by Martin Cortes *(vide infra).* But he warned that the instructions given, both by himself and by Cortes, applied only for the equator, and that if used in any other latitude:

> 'you shall be deceiued. For as you goe to any of the twoo poles, so be youre degrees shorter and shorter, till that your meridians meete vnder the twoo poles...'

Figure 7 illustrates the diagram given in the English translation of *Arte de Navegar* by Cortes, first published in 1551. The quantities given along the winds, or rhumbs, are the great circle arcs in degrees and minutes, and the number of leagues, to raise or lay a degree corresponding to each

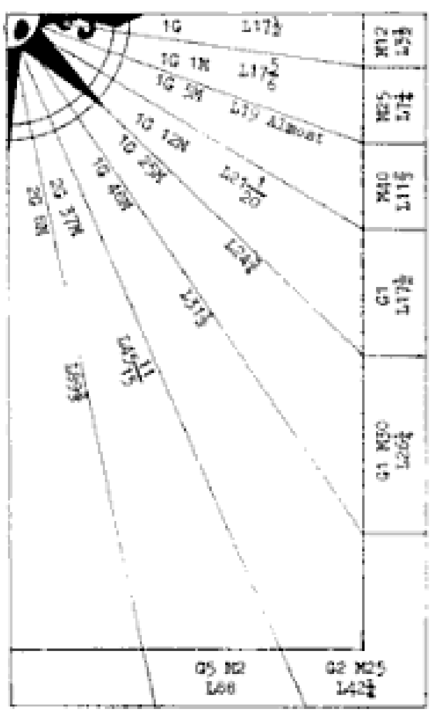

FIGURE 7 — Martin Cortes' Regiment for the Leagues (1551)

of the eight rhumbs. The quantities given at the side and bottom of the diagram are the great circle arcs in degrees and minutes, and the number o. leagues departure from the initial meridian, corresponding to the equator Thus, by sailing on the third rhumb (say NE x N) the great circle arc and distance to sail to raise or lay a degree are, respectively, $1^{\circ}\ 12'$ and $21\ ^{1}/_{20}$ leagues. The corresponding 'departure' quantities are $0^{\circ}\ 40'$ and $11\ ^{2}/_{3}$ leagues.

An interesting diagram giving the 'Regiment of the Leagues' appeared in a work by Bartolomeu Velho in 1568 (1). Albuquerque suggests that Velho's diagram, which is illustrated in figure 8, provided the basis for finding graphically the data for the regiment of the leagues given in tabular form (2). To support this view, examination reveals that the quantities given in the early regiments were not consistent. This would not have been the case had they been computed.

The most common type of diagram giving the regiment of the leagues

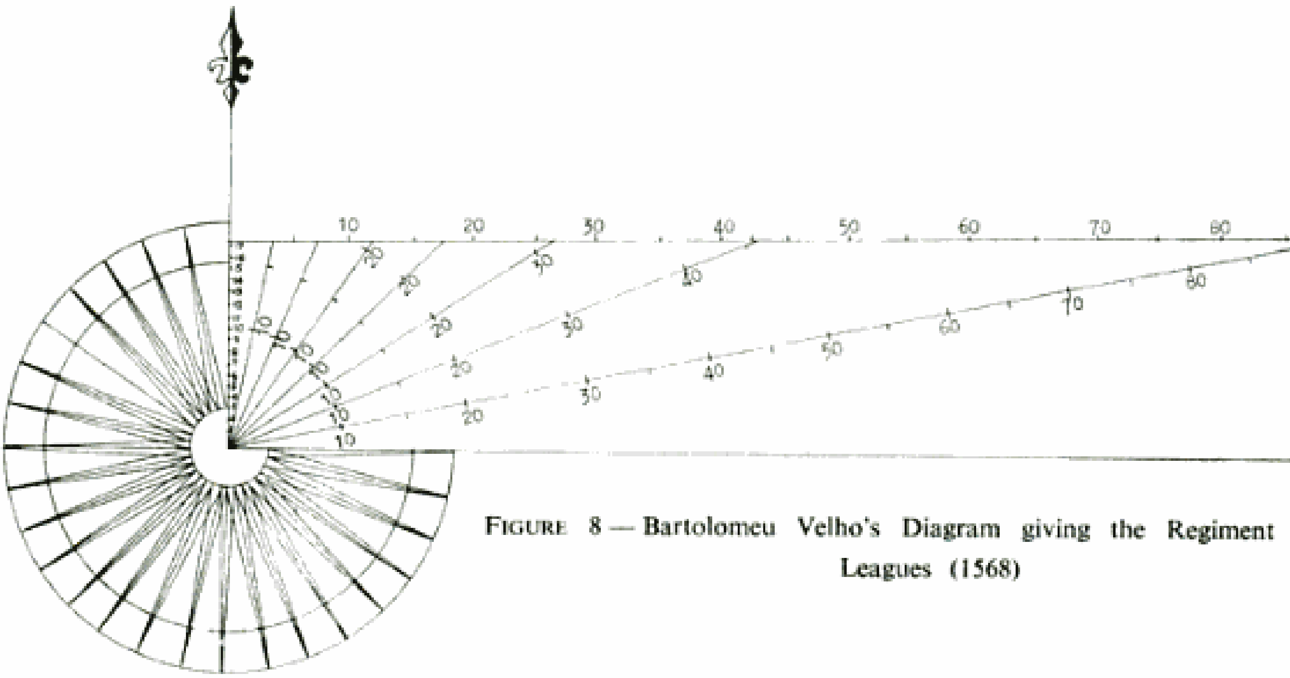

FIGURE 8 — Bartolomeu Velho's Diagram giving the Regiment Leagues (1568)

was a circular or 'wheel' diagram. The wheel diagram illustrated in figure 9, taken from Bourne's *Regiment for the Sea* (3), gives the number of English leagues (at 20 per degree) and the number of Spanish leagues (at 17 $^1/_2$ per degree), to sail to raise or lay a degree. Waghenaer, in his *Spieghel der Zeevaert* (4), gave a similar diagram from which the number of Dutch miles (15 per degree) to sail on each compass point to raise or lay a degree, could be found. Willem Blaeu, in his *Light of Navigation* of 1612 (5), gave a table (reproduced in figure 10) giving the number of leagues 'a man may sayle upon everie stroke (= compass point) before he winne or loose a degree...'.

Among the treatises forming Blundevil's *Exercises* (6), is *A New and*

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(1) Cortes, M. (1579). *Arte of Nauigation.* Englishd out of Spanyshe by Rich[d]. Eden. London.

(2) Albuquerque, L. de (1971). *op. cit.* p. 438.

(3) Bourne, W. (1574). *A Regiment for the Sea.* London.

(4) Waghenaer, W. J. (1584). *Spieghel der Zeevaert.* Leyden. p. 28.

(5) Blaeu, W. J. (1612). *The Light of Navigation.* Amsterdam.

(6) Blundevil, T. (1636). *Mr. Blundevil His Exercises Contayning Eight Treatises...* Seventh Edition by R. Hartwell. London.

Necessary Treatise on Navigation. This contains all the 'chiefe Principles of that Art' and Blundevil informs us that these principles were 'lately corrected out of the best Moderne Writers thereof'. Among the authors consulted was Michel Coignet whose *Instruction nouvelle des... l'art de nauiguer* appeared in 1581. Blundevil gave three tables for solving the sailing problems.

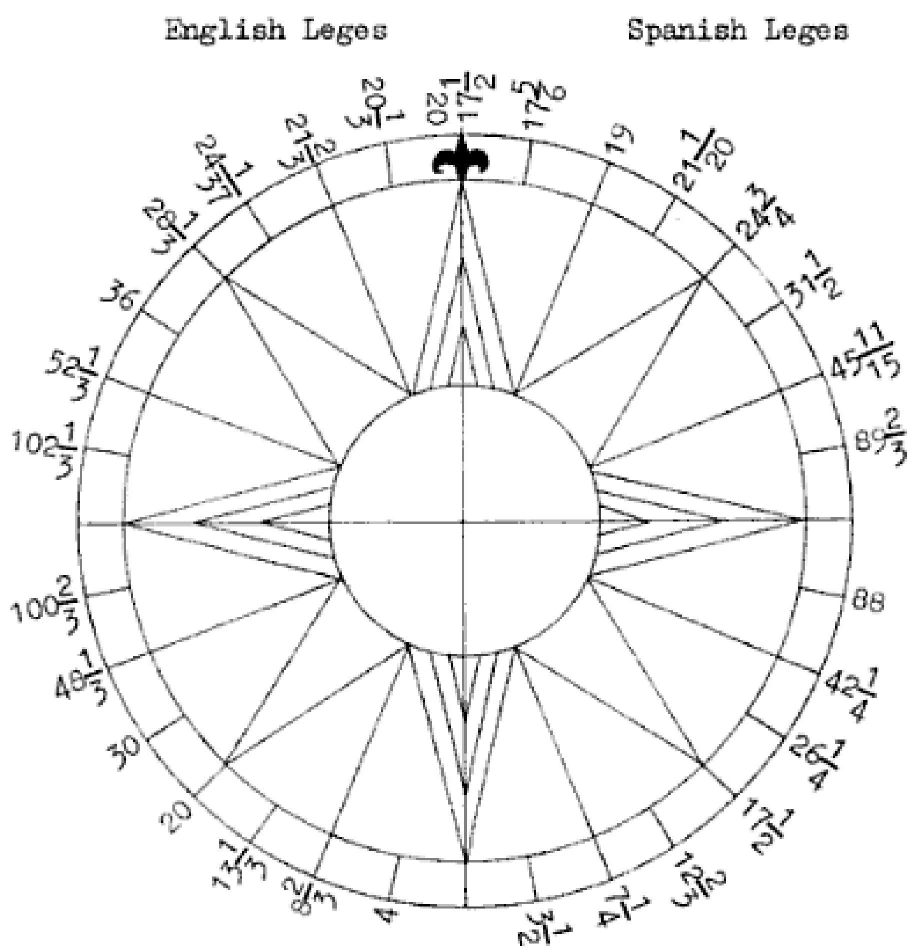


FIGURE 9 — William Bourne's Wheel Diagram for Raising or Laying a Degree (1574)

To handle the sailing problem for the convergency of the meridians Blundevil (and others) gave a table giving the number of miles per degree of longitude in different latitudes. This table was of great use in sailing due east or west, when the 'Regiment of the Leagues' was useless. Blundevil's 'Table to help you to know what way your ship hath made in sayling right East or West...' gave the same information as an interesting graphical method described by William Bourne in his *A Regiment for the Sea* (1). Referring

(1) Taylor, E. G. R. (1963). *A Regiment of the Sea and other writings on Navigation.* C.U.P. p. 241.

to figure 11, which illustrates Bourne's method, the arc of the semi--circle is graduated uniformly from 0° to 90°. To find the number of miles answering to a degree of longitude in any latitude $\theta°$, a thread was stretched from the zero mark on the diametrical scale (graduated uniformly from 0

How manie leagues a man may sayle upon everie stroke before he winne or loose a degree in height, & also how many leagues you are then without the Meridian or south and north line from which you sayled.

Vhen you sayle right south or north, then you sayle for a degree — — — — — 20 leagues.
And you stay under the same Meridian, that is, you goe neither more easterly nor westerly then at first you did.
Vpon the stroke of south and by west, or north and by west, you sayle for a degree — — — 20½ leagues
And then you have left the Meridian or the south and north Line, under which you were at the first — 4 leagues.
South-south-west, & n.n.e. you sayle for a degree 21 leagues.
And then you are out of the aforesaid Line — 8 leagues.
South w. by south, & n.e. by n. you sayle for a deg. 24 leagues.
And without the Meridian — — — 12½ leagues.
South-west & north-east for one degree — 28 leagues.
And without the Meridian — — — 20 leagues.
S.w. by w. & n.e. by n. you sayle for one degree 36 leagues.
Then you are without the Meridian — — 29 leagues.
W.s.w. & e.n.e. you sayle for a degree — 52½ leagues.
Then you are without the Meridian — — 48 leagues.
W. by south, & east by n. you sayle for a degree 103 leagues.
And then you are without the Meridian — 100 leagues.
Sayling east or west you neither winne nor loose, but you keepe alwaies in one height.

FIGURE 10 — Blaeu's Regiment of the Leagues from 'The Light of Navigation' (1612)

to 60 — the number of miles in a degree arc of the meridian) to the angle θ on the arc. This segment of thread was then stretched along the diametrical scale to give the required number of miles. As illustrated, the number of miles in a degree of longitude in latitude 60° is 30.

But Bourne's device had its origins as far back as 1537 the date of publication of *Tratado em defensam da carta de marear* by Pedro Nuñez.

Nuñez' instrument consists of a quadrant of brass or wood the arc of which is graduated in degrees from 0° to 90° as illustrated in figure 12. The lower radial edge of the quadrant is graduated in leagues from zero at centre to the circle to 17 $^1/_2$, the number of leagues reckoned to be in an equatorial degree. A semicircle is constructed on this edge as diameter. Fastened to the zero point on the scale of leagues is a thread on which a small bead may slide.

To find the number of leagues in a degree of longitude in any latitude θ, the thread is stretched so that it coincides with $\theta°$ on the graduated quadrant.

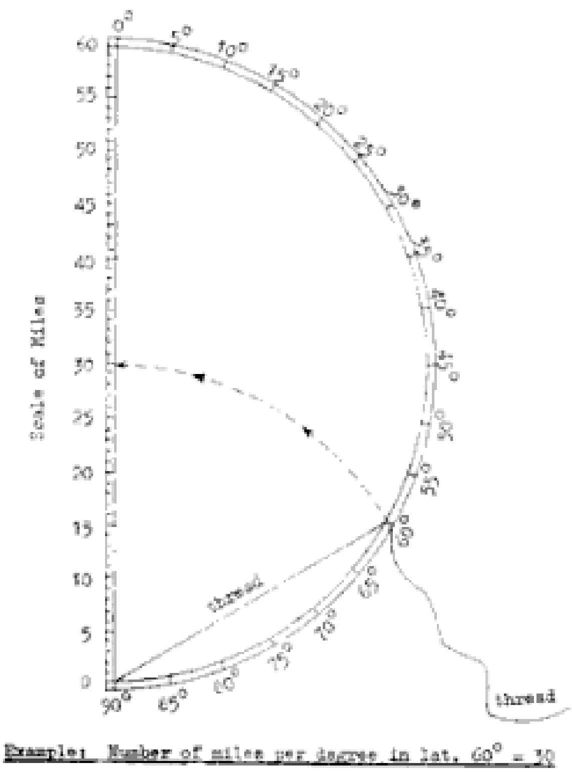


FIGURE 11 — Bourne's Diagram (1574)

The bead is then moved to the position at which the thread intersects the semicircle. Keeping the bead in this position, the thread is kept taut and swung down to the scale of leagues where the bead will indicate the required number of leagues per degree of latitude θ.

5. THE SHIPMAN'S QUADRANT

The shipman's quadrant is an instrument devised by Gemma Frisius, in c. 1545, for the purpose of finding, without a chart, the course to steer from one place to another given the latitudes and longitudes of the two places. The instrument, called by Gemma the *Quadratum Nauticum* or 'nautical square', suffered the same defects as the plane chart, on which the projection of a degree of longitude is constant in all latitudes.

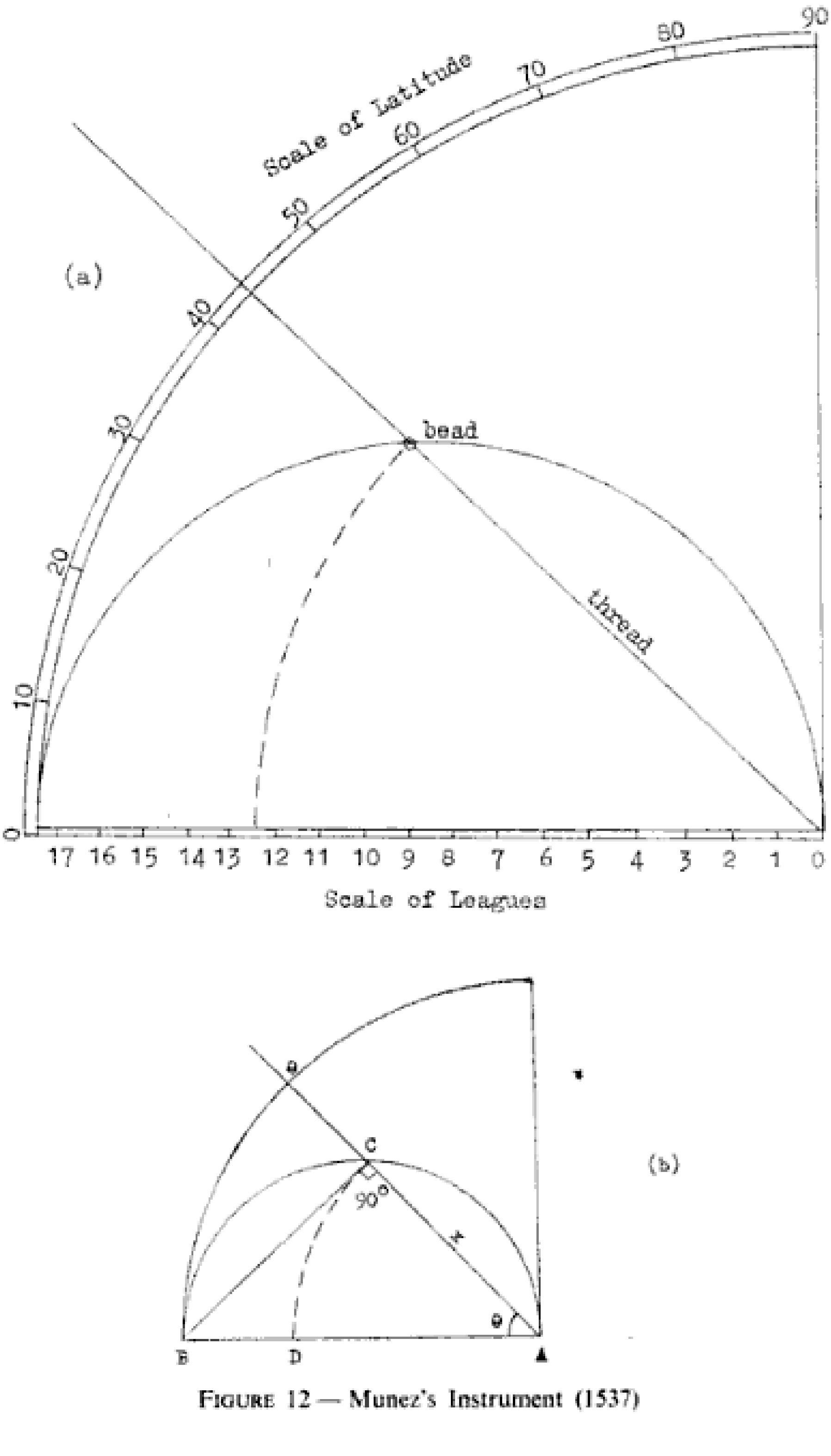


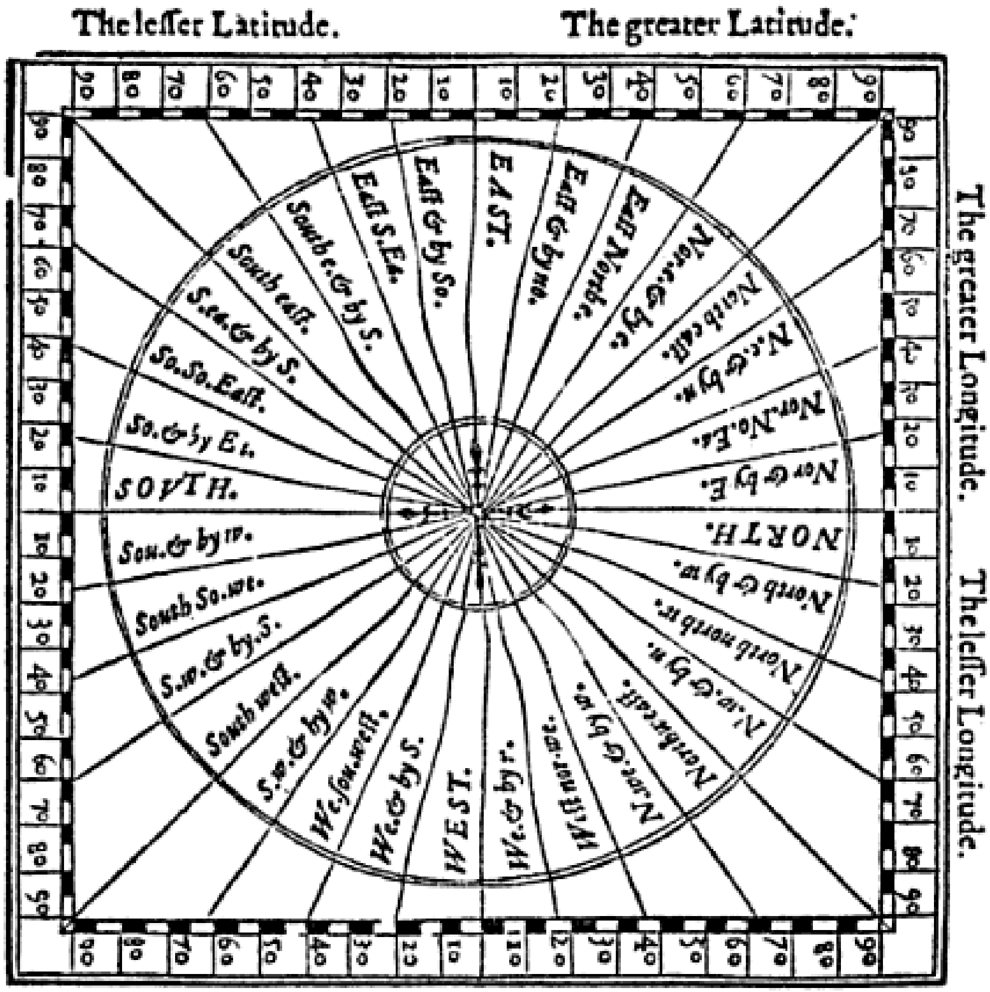
FIGURE 12 — Munez's Instrument (1537)

The shipman's quadrant consists of a square the upper and lower sides of which are graduated from zero at the centre to 90° of longitude E and W (right and left respectively) of the central zero. The right and left sides of the square are graduated similarly. Centrally placed within the square is a circle graduated with the 32 points of the compass with north uppermost.

Figure 13 illustrates the shipman's quadrant given in Cunningham's (1) *The Cosmographical Glasse* of 1559.

In reply to his pupils' desire to understand the use of the shipman's quadrant, the master instructed as follows:

'I will shew you, opening the whole Art of directing your shippe. First you must seke out the Longitude & Latitude both of the place from whence you saile, & also



FIGUR 13 — The Shipman's Quadrant from Cunningham's 'Cosmographical Glasse' (1559)

of that vnto which you intend to trauail. ...then substracte the smaller number of Longitude and Latitude oute of the greater, and with the difference of Logitude and Latitude, do in this manner. First if the Longitude of the place vnto whiche you trauell be greater than that from which you depart, entring into the hier part of the Quadrate (and towarde the left hande vnder thys title, the greater longitude) you shall seeke oute in degrees and minutes, thys difference. And do in like maner in the lower part of the table directly vnder it, & this difference so founde oute, apply

(1) Cunningham, W. (1559). *The Cosmographical Glasse, conteinyng the pleasant Principles of Cosmographie, Geographie, Hydrographie, or Nauigation.* London. Folio 162.

a thride (thread), or ruler, to the number found in the hier part of the Quadrat & also in the lower part... but now as touching the difference in Latitude of the two places, if the pole of the place (vnto which you direct your shippe) be greater, then the Pole of the place, from which you losen then accomptinge frō the middle line vpward, under this title, the greater Latitude, & in like case toward the right had, then draw and extend a thrid, or apply a rulers, vnto this nūber of Latitud̃, & wher̄ the ij thrides or rulers crosse one another, there make a marke for it is the place you desire... So that if you applie a ruler frō the Center of the Quadrante, vnto the intersectio of the two thrides or rulers, it shalbe manifest what point of winde, you must vse, vntill you haue finished your course...'

6. CONCLUSION

The devices described in this paper were designed to relieve the navigator of the tedium of computation. The basis of nearly all navigational problems is trigonometry — the mathematics of triangles; and the solutions described in the paper were based on what is now regarded as a trivial problem, viz. the solving of a plane right-angled triangle. It was not until relatively recently that navigators became accustomed to using strictly computational, rather than graphical or instrumental, methods for solving their trigonometrical problems. The instrumental successors to the devices covered in this paper include the Sinical Quadrant, the Quartier de Réduction (a favourite with French mariners); a variety of 'Sectors' (including those by Thomas Hood and Edmund Gunter), and 'Scales', and numerous 'slide rules'. These were largely replaced by 'figuring' techniques using logarithmic tables. We are now witnessing the advent of a new phase in navigational practice when the hand-held electronic calculator is fast becoming the stock-in-trade of almost every ship's officer, and even logarithmic tables are fast becoming redundant.

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