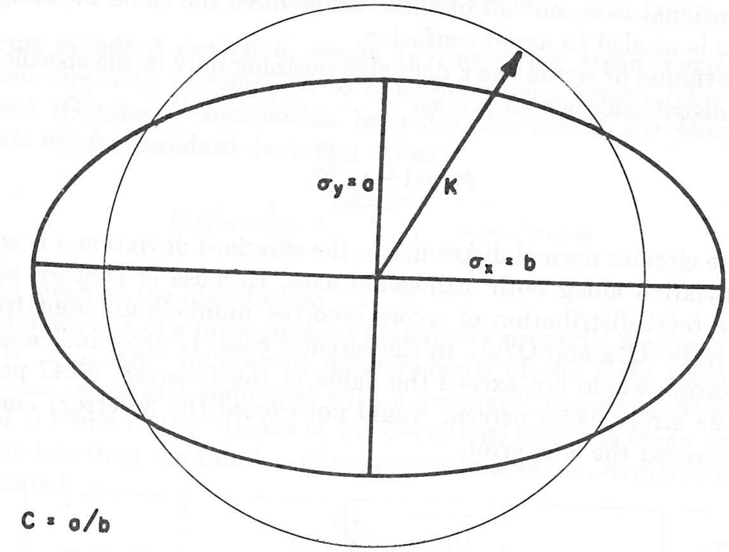

FIGURE Q7a.—Error ellipse and circle of equivalent probability.

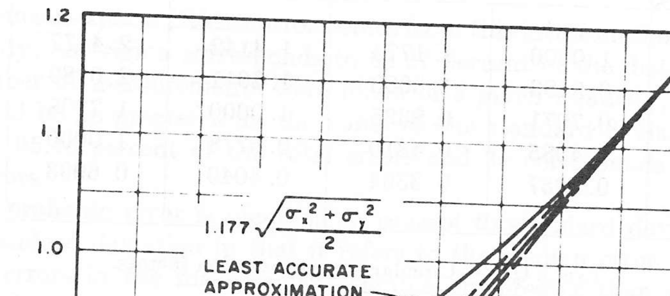

FIGURE Q7b.—CEP for elliptical error distribution approximations.

culation. Despite the availabilty of these curves and tables, approximations nade for this calculation of a CEP when the actual error distribution is ellip- ral of these approximations are indicated and plotted for comparison with urve in figure Q7b. Of the various approximations shown, the top curve, the diverges the most rapidly, appears to be the most commonly used.

er factor of interest concerning the relationship of the CEP to various hat the area of the CEP circle is always greater than the basic ellipse. Table tes that the divergence between the actual area of the ellipse of interest and of equivalent probability increases as the ellipse becomes thinner and more

$C=a/b$	Area of 50% ellipse	Area of equivalent circle
0.0	0	1.43
0.1	0.437	1.46
0.2	0.874	1.56
0.3	1.31	1.76
0.4	1.75	2.06
0.5	2.08	2.37
0.6	2.62	2.74
0.7	3.06	3.12
0.8	3.49	3.52
0.9	3.93	3.94
1.0	4.37	4.37

TABLE Q7c.—Comparison of areas of 50 percent ellipses of varying eccentricities with areas of circles of equivalent probability.

value of the CEP may be related to the radius of other values of probability lytically for the case of the circular normal distribution by solving the basic or various values of probability. For this special case of the circular normal n, these relationships are shown drawn to scale in figure Q7c with the associ- s tabulated in table Q7d.

lerivation of these values is shown in the following analysis. First, the factor e CEP to the circular sigma is derived, then, as a second example, the rela- etween the 75 percent probability circle and the circular sigma is derived. of these two values is then the value shown in table Q7d for the 75 percent

ircular normal distribution equation is:

$$P_R=1-e^{-\frac{R^2}{2\sigma^2}},$$

$$\text{CEP}\equiv P(R)=0.5$$

$$1-e^{-\frac{R^2}{2\sigma^2}}=0.5$$

$$e^{-\frac{R^2}{2\sigma^2}}=0.5.$$

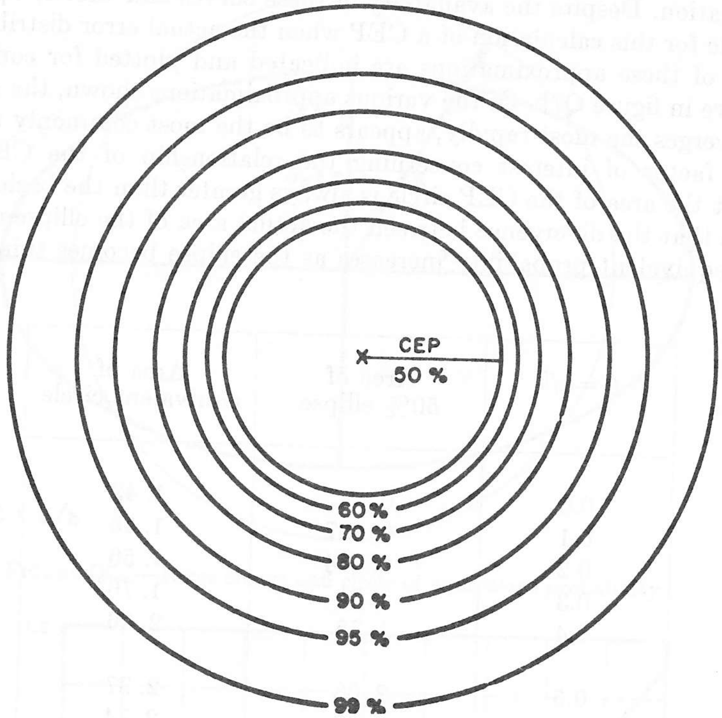

FIGURE Q7c.—Relationship between CEP and other probability circles.

Take natural logarithm of both sides

$$\ln\left(e^{-\frac{R^2}{2\sigma^2}}\right)=\ln 0.5$$

$$\frac{R^2}{2\sigma^2}=\ln 2 \qquad (\ln 0.5=-\ln 2)$$

$$R=1.1774\sigma.$$

For the 75 percent probability circle,

$$1-e^{-\frac{R^2}{2\sigma^2}}=0.75$$

$$e^{-\frac{R^2}{2\sigma^2}}=0.25$$

$$\ln\left(e^{-\frac{R^2}{2\sigma^2}}\right)=\ln 0.25$$

$$\frac{R^2}{2\sigma^2}=\ln 4$$

$$R=1.665\sigma$$

$$\frac{R(75\%)}{R(50\%)}=\frac{1.665\sigma}{1.177\sigma}=1.414.$$

Multiply value of CEP by	To obtain radius of circle of probability
1.150	60%
1.318	70%
1.414	75%
1.524	80%
1.655	85%
1.823	90%
2.079	95%
2.578	99%

TABLE Q7d.—Relationship between CEP and radii of other probability circles of the circular normal distribution.

The factors tabulated in table Q7d are sometimes used to relate varying probability circles when the basic distribution is not circular, but elliptical. That such a procedure is inaccurate may be seen by the curves of figure Q7d. It can be seen that the errors involved are small when the eccentricities are small. But the errors increase significantly when both high values of probability are desired and when the ellipticity increases in the direction of long, narrow distributions.

The terms **radial error, root mean square error,** and $\boldsymbol{d_{rms}}$ are identical in meaning when applied to two-dimensional errors. Figure Q7e illustrates the definition of d_{rms}. It is seen to be the square root of the sum of the square of the 1 sigma error components along the major and minor axes of a probability ellipse. The figure details the definition of 1 d_{rms}. Similarly, other values of d_{rms} can be derived by using the corresponding values of sigma. The measure d_{rms} is not equal to the square root of the sum of the squares of σ_1 and σ_2 that are the basic errors associated with the lines of position of a particular measuring system. The procedures described in art. Q6 must first be utilized to obtain the values shown as σ_x and ${}_y\sigma$.

The three terms (radial error, root-sum-square error, and d_{rms}) used as a measure of error are somewhat confusing because they do not correspond to a fixed value of probability for a given value of the error measure. The terms can be conveniently related to other error measures only when $\sigma_x=\sigma_y$, and the probability figure is a circle. In the more common elliptical cases, the probability associated with a fixed value of d_{rms} varies as a function of the eccentricity of the ellipse. One d_{rms} is defined as the radius of the circle obtained when $\sigma_x=1$, in figure Q7e, and σ_y varies from 0 to 1. Likewise, 2 d_{rms} is the radius of the circle obtained when $\sigma_x=2$, and σ_y varies from 0 to 2. Values of the length of the radius d_{rms} can be calculated as shown in table Q7e. From these values the associated probabilities can be determined from the tables of article Q6. The variations of probability associated with the values of 1 d_{rms} and 2 d_{rms} are shown in the curves of figures Q7f and Q7g. Figure Q7h shows the lack of a constant relationship in a slighly different way. Here the ratio d_{rms}/CEP is plotted against the same measure of ellipticity. The three figures show graphically that there is not a constant value of probability associated with a single value of d_{rms}.

Figure Q7i shows the substitution of the circular form for elliptical error distributions. When σ_x and σ_y are equal, the probability represented by 1 d_{rms} is 63.21 percent. When σ_x and σ_y are unequal (σ_x being the greater value), the probability varies from 64 percent when $\sigma_y/\sigma_x=0.8$ to 68 percent when $\sigma_y/\sigma_x=0.3$.