Hints to Travellers, graphic method of clearing a lunar distance by the scale of chords, 1883, page 73-74

Symbols used:

H'	apparent altitude of moon
h'	apparent altitude of the other body
D'	apparent distance
D	true distance

The scale of chords referred to is a scale where you get the length of a chord with a pair of compasses from a given angle θ in degrees. The length of a chord c,

$c(\theta) = 2 \cdot r \cdot \sin(\theta/2)$, where r is the radius of the circle.

According to the text the radius should be taken from the scale for an angle of 60° and as $c(60°)$ equals r we can treat r as unity.

In the figure on page 74 call the crossing closest to the moon symbol M and the crossing closest to the planet symbol N. Angle BAS is the apparent distance D', angle MAy is $90°-H'$, and angle SAS'' is $90°-h'$.

The segment AM thus equals $\sin H'$ and segment AN equals $\sin h'$. Prolong the segment S''S' until it meets the prolonged segment AB. Call this point F. In the triangle AFN the angle at F equals $90°-D'$ and hence angle E in triangle MEF equals D'.

In triangle FAN segment AF equals $\sin h' / \cos D'$ and hence MF is $\sin h' / \cos D' - \sin H'$.

The correction ME is found from the triangle FEM as $\tan D' = (\sin h' / \cos D' - \sin H') / \text{ME}$, giving

$\text{ME} = \sin h' / \sin D' - \sin H' / \tan D'$.

The length of ME is measured with the compass and the scale of chords gives the result as an angle in degrees, instead of a fraction. Let's call that angle $\gamma°$. If we divide $\gamma°$ by $180°/\pi = 57.3°$ we get the angle $\gamma = \gamma° / 57.3°$ in radians, or as a fraction. Now

$\text{ME} = 2 \cdot \sin(\gamma/2) = 2 \cdot (\gamma/2 - (\gamma/2)^3/3! + \dots) = \gamma - \gamma^3/24 + \dots \approx \gamma$

This approximation seems acceptable, compared to all other uncertainties in this graphical solution.

"Multiply the line of correction by the horizontal parallax, expressed in minutes and decimals of a minute, and divide the product by 62 when the correction is subtractive; but when additive, divide by 53; …".

Now, $62 = 57.3 \cdot (1 + 0.08)$, and $53 = 57.3 \cdot (1 - 0.08)$ and this gives an explanation to the different constants. When the correction is subtractive, we have

$\text{correction} = \gamma° \cdot HP / 62 = \gamma \cdot HP / (1 + 0.08) \approx \gamma \cdot HP \cdot (1 - 0.08) = \gamma \cdot HP - 0.08 \cdot \gamma \cdot HP$

and thence $D = D' - \gamma \cdot HP + 0.08 \cdot \gamma \cdot HP$;

and when additive

$\text{correction} = \gamma° \cdot HP / 53 = \gamma \cdot HP / (1 - 0.08) \approx \gamma \cdot HP \cdot (1 + 0.08) = \gamma \cdot HP + 0.08 \cdot \gamma \cdot HP$

and thence $D = D' + \gamma \cdot HP + 0.08 \cdot \gamma \cdot HP$.

To summarize: $D = D' \pm \gamma \cdot HP + 0.08 \cdot \gamma \cdot HP$.

The last term, $0.08 \cdot \gamma \cdot HP$, is always positive and is an approximation of the "third" or "Q-" correction given in Bowditch (1851) table XLVIII.