Coastal Navigation by Station Pointer & the Snellius Resection[1]

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Introduction

When navigating at sea the coast is friend as well as foe: foe by danger of running aground, but friend by the possibility of position-finding using visible landmarks – such as lighthouses or church towers.

One of the instruments used in coastal navigation is the *station pointer*, or 3-way protractor. This is a *goniometer* with three long arms for setting out on a sea chart two adjacent horizontal angles observed between three landmarks.

The Station Pointer

The station pointer depicted in Figures 1 to 4, was produced by H. Hughes & Son[2] in the second quarter of the 20th century. Each of the three protractor arms consists of two parts so that the containing box (Figure 1) can have a manageable size. The 0º - 360º scale on the silver-plated ring is divided in full degrees (Figure 3), but more expensive models do exist with micrometers and magnifying glasses to allow divisions up to one minute of arc.

The print on the box (Figure 2) reveals that this instrument

Figure 1. Station Pointer in its Box

Figure 2. Station Pointer on its Box

was part of the navigation equipment of the 75 meter Dutch Indies Navy gunboat *Soemba*, of the *Flores* class. The *Soemba* was commissioned in 1926 and saw active service during World War II under allied command, in the East Indies and later in the Mediterranean. The *Soemba* was decommissioned in 1985 [1].

Principle of the station pointer

Any coastline of interest will have some visible landmarks, clearly indicated on large-scale sea charts to allow position finding by observation from sea. The simplest method is by taking bearings on two landmarks through the *pelorus* of the bridge compass so that the position can be constructed on the sea chart by drawing the intersection of the bearing lines. The station pointer method requires the observer O to take horizontal sextant angles between three landmarks A, B, and C: the result is in the measured angles AOB and BOC.

Figure 4 shows an example on the sea chart of Puget Sound on the west coast of the USA, just north of Seattle. The landmarks are:

A = Point Jefferson, B = Point Monroe, C = Skiff Point

The two measured angles AOB and BOC are set up between the respective outer arms of the station pointer and the middle arm, which is connected to the ring, fixed at the value of 0º. The outer arms are then fixated by their respective clamping screws near the ring scale.

Now the bevelled sides of the arms form an *analogon* of the two measured triangles on the chart. Note that the bevelled "radial" sides must be used because only those line up to the centre of the ring scale. This radial is on opposite sides of the outer arms!

The thus adjusted station pointer can be put on the chart, in such a way that all three bevelled sides of the instrument touch the respective landmarks A, B, and C; in that position the centre of the station pointer's ring (cross hair or notch) will represent the location of the observer O.

Use of the station pointer

This seemingly simple guideline described above is often used in documents about the station pointer, but the exact procedure to accomplish this is missing in most descriptions.

Shifting and rotating the station pointer until all arms touch their respective landmarks A, B, and C, seems a random trial and error exercise.

The best method to reach a solution in a systematic way is the following:

1. Position the station pointer anywhere on the chart with the lower arm touching point C.
2. Rotate the station pointer around C until the middle arm touches B; now the lower triangle has the correct angle BOC at O.
3. Shift the station pointer in such a way that the lower arm keeps touching C and the middle arm keeps touching B, until the upper arm

Figure 3. Station Pointer's Scale

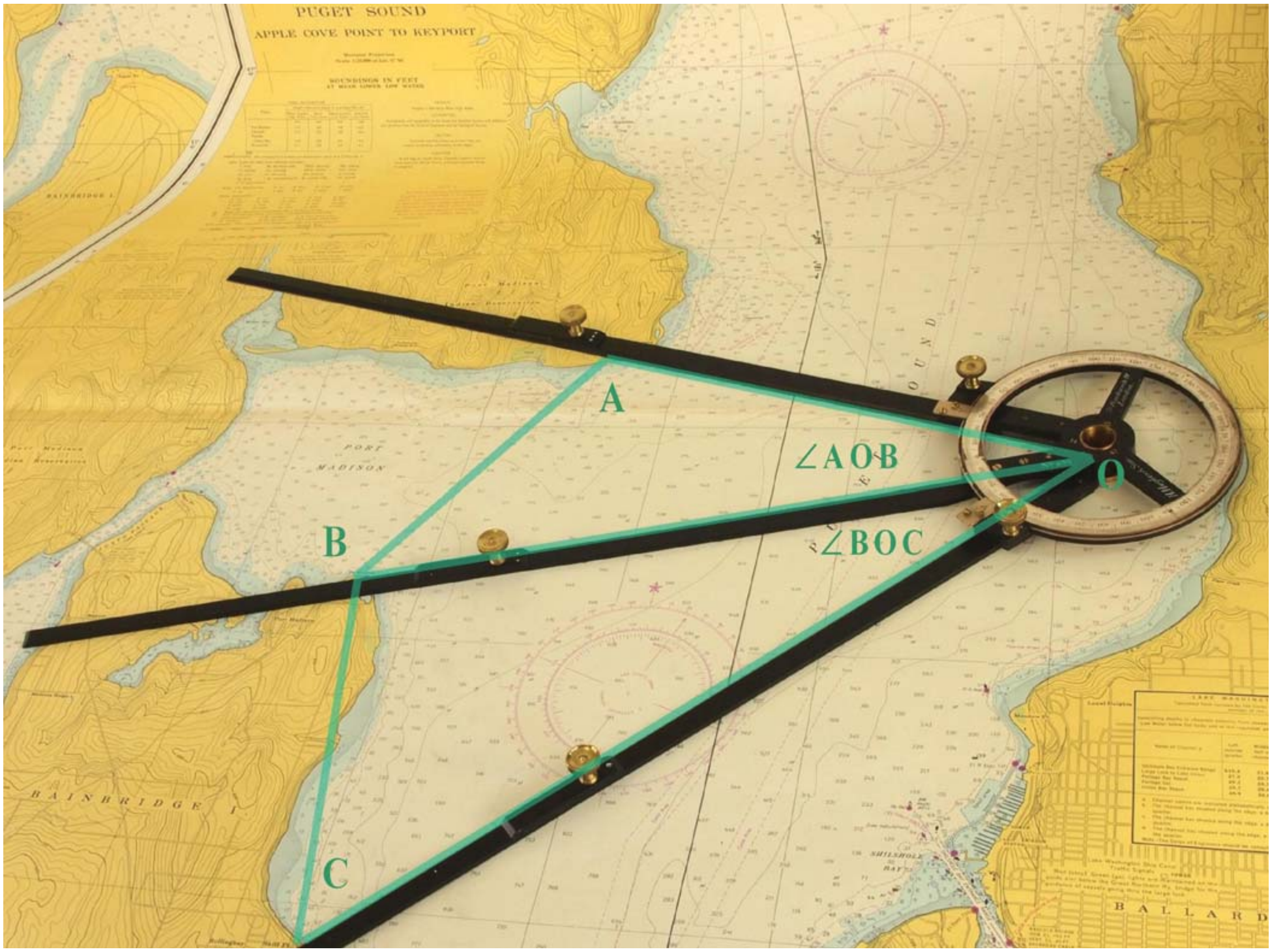


Figure 4. Using the station pointer

touches its landmark A; now the upper triangle also has the correct angle AOB at O.

The curve followed by the station pointer's centre in step 3, is actually the arc of a circle through B, C, and O. This nonlinear movement can be assisted by guiding the lower and middle arm over pins or thumb tacks in the chart at C and B respectively.

The Line of Position formed by equal top angles

From basic trigonometry we know that the radius of the circle circumscribing a triangle OAB can be calculated from the rectangular triangle AML (ML being the perpendicular bisector of AB):

$$R_{AOB} = \frac{\tfrac{1}{2}\,\mathrm{AB}}{\sin(\angle \mathrm{AOB})} \qquad (1)$$

Therefore, the circumscribed circle only depends on the length of base AB and the top angle AOB, but not on the top's position (the top angle AOB is named ω in Figure 5). The top's position can be anywhere on the so-called Line of Position, along the circumscribing circle (see the various light green triangles in Fig. 5). The centre of this Line of Position circle is determined by M in the rectangular triangle AML.

The second Line of Position is determined likewise by triangle OBC. The two points of intersection between the two Lines of Position represent the known point B and the required Observer's position O respectively.

Accuracy of the station pointer

The accuracy of the angles to be set out on the chart depends on many factors. In the first place the scale of the chart is very influential; large-scale charts for coastal navigation will provide the best precision in locating the ship's position. Then we have to consider the precision of the readings from the sextant and the setting of the position pointer. Although an instrument's circle scale may be divided up to arc-minutes or even less, the final accuracy of the pointer arms on the chart may come out no better than ¼ degree.

There are other factors that may influence the accuracy. The handling of the sextant during measurements may have some influence; time differences between measuring the angles AOB and BOC from a moving ship will decrease accuracy (correction is possible, but complicated). Even a deviation from the horizontal plane of the sextant during measurements will result theoretically in a too small horizontal angle, although this effect will be negligible for observed points that are neither too close nor too high.

A substantial contribution to loss of accuracy may be caused by ill-chosen positions of the observation points A, B, and C. Where the intersection of two lines of position (circles) approach the perpendicular, the accuracy will be optimal. Smaller intersecting angles will diminish the accuracy, and in the extreme case of coinciding circles (one circle through A, B, C, and O, called the *danger circle*) there is no solution at all.

When the observation points A, B, and C can be chosen from a larger number, some base rules should be used, such as: use angles larger than 30 degrees, or choose the mid-point B closer to the observer than A and C, see [2] and [3].
A check on accuracy may be done by measuring a fourth observation point, to construct a third circle of position; the intersection of this additional one with the other two should not deviate too much from the Observer's position O as determined earlier.

Alternatives of the Position Pointer

Drawing a schematic station pointer: if an actual station pointer is not available, a construction of the station pointer's arm settings can be drawn on a piece of transparent paper. Three lines, intersecting each other by the measured angles between A, B, and C respectively, are sufficient as this piece of paper can be manoeuvred around the chart like a real station pointer.

Graphical construction: the two Lines of Position (circles) may be constructed with ruler and compasses on the chart, directly or on a transparency to protect the chart. The lines AB and BC are drawn first through A, B, and C respectively. Then a protractor is used to draw a line from A by 90º - ω away from AB; likewise a line from B by 90º - ω away from BA. These two new lines intersect at the centre M of the Line of Position around triangle OAB; this circle can now be drawn with the compasses. Note that for checking pur-

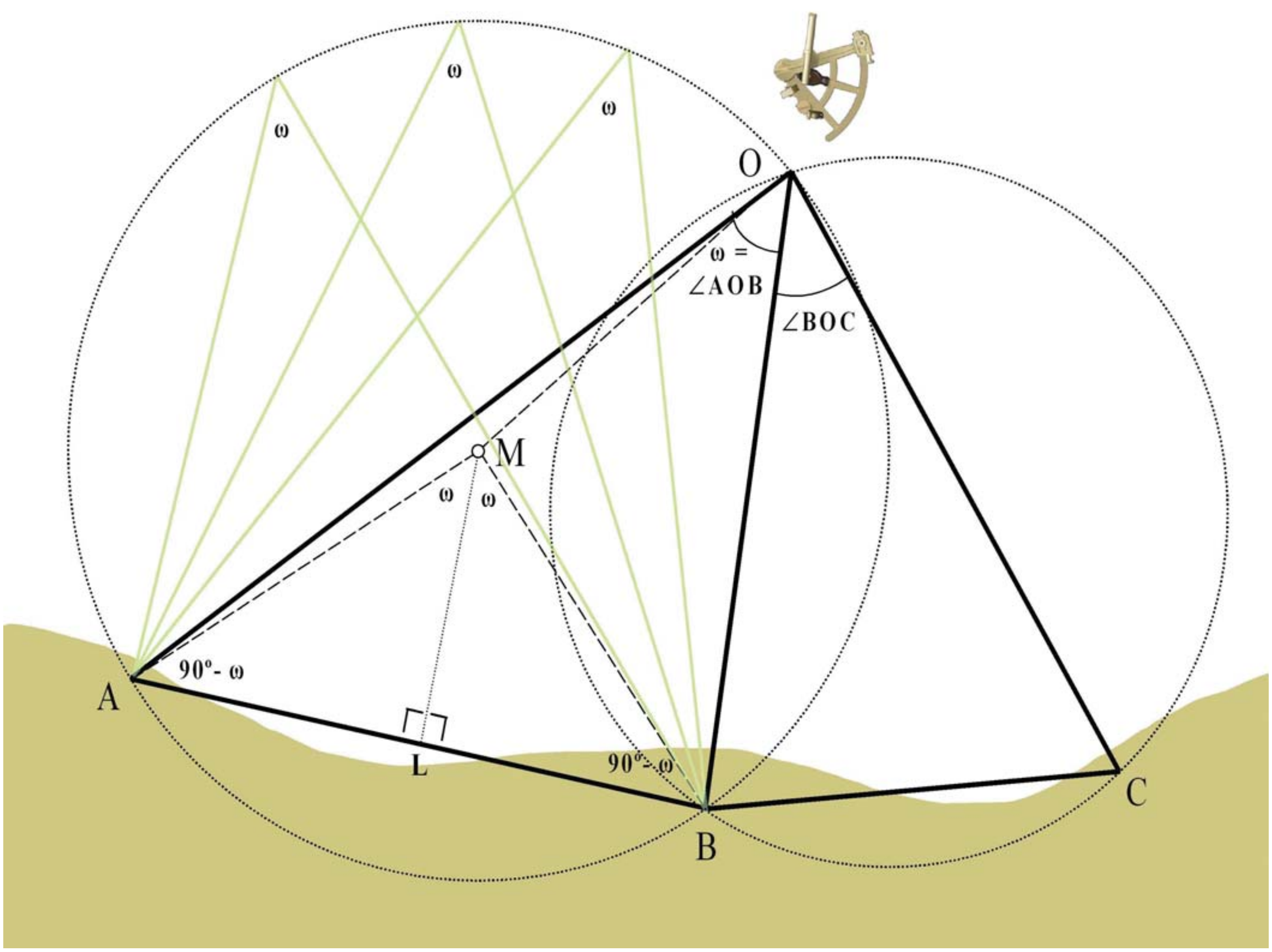


Figure 5. Line of Position Circles

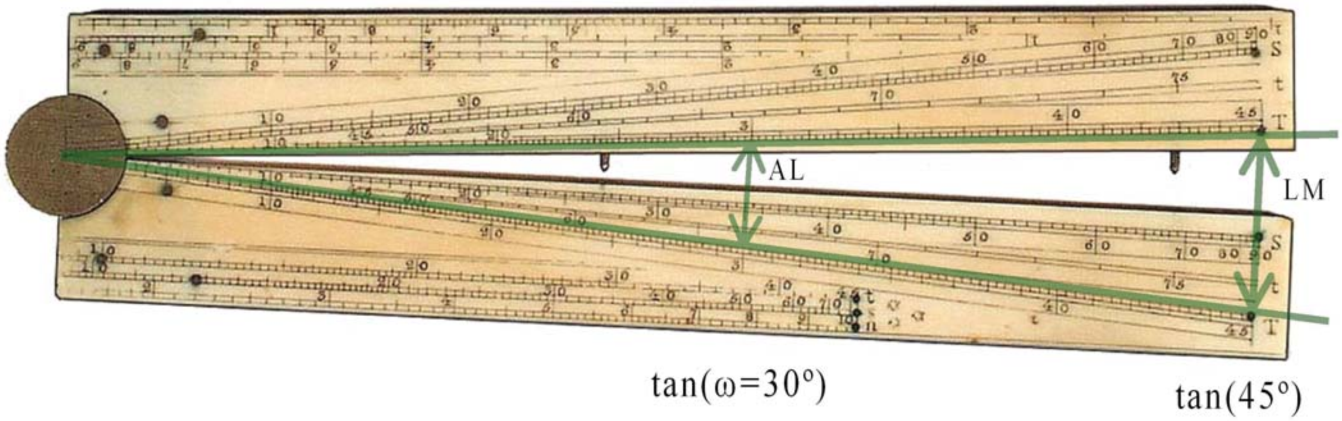


Figure 6. Using the proportional dividers

poses the perpendicular bisector of AB with the compasses between A and B can be drawn, and should also intersect with the circle's centre M.
The Line of Position circle around the triangle OBC can be drawn in the same manner, and the intersection between the two circles (not point B of course!) will be the Observer's position O.

In early times - before the 19th century - protractors did not yet exist, so angles had to be constructed on a chart by moving the required angles with a "parallel ruler" between the compass rose and the required position. As an alternative, angles could be constructed by means of "Chord" scales which were available already before the 17th centuries on "plain scale" rulers, and later on Gunter rules too.

Numerical Solutions

The graphical constructions described above need to set out angles on the chart, but do not use numerical calculations. When calculations are introduced, there is no need to set out angles.

In the rectangular triangle ALM the lengths of the unknown sides AM and LM can be calculated from the side AL = ½AB and the top angle ω:

$$AM = \frac{\mathrm{AL}}{\sin(\mathrm{w})} \quad \text{and} \quad LM = \frac{\mathrm{AL}}{\tan(\mathrm{w})} \qquad (2)$$

If one of these sides, for example AM, is calculated then the circle's centre M can be constructed directly on the chart by circling the length AM from A until intersecting with the perpendicular bisector of AB. If LM is calculated then the circle's centre M can be constructed directly on the chart by setting off from L the length LM on the perpendicular bisector of AB.

These calculations can be done on an electronic calculator (today), or on a logarithmic slide rule (long ago), or even on a Gunter rule (a very long time ago).

Not only the angle ω but also a unit of length is needed for AL and the sides to be calculated; the most logical choice is to use the very scale of the map, so that with compasses or dividers a distance such as AL can be brought from the chart into the equation, in scaled miles or minutes of arc.

Proportional dividers: even before the 17th century, this computational problem could already be solved by means of the proportional dividers, without protractor or numerical calculations. The proportional dividers need to have sine and/or tangent scales to do this.

The method is simple. With chart dividers (or compasses) the perpendicular bisector of AB is constructed as described above; then the distance AL = ½AB is taken between the points of the chart dividers, and the proportional dividers are extended so far that the scale points for TAN (ω) on each arm have a distance AL between them (using the chart dividers that were just set to AL).

In that position of the proportional dividers, the chart dividers are now moved and extended until the points extend from the scale point TAN (90º) = 1 on one arm to the same scale point TAN (90º) = 1 on the other arm. The distance between the points of the chart dividers is now set to LM, and can be used to construct the circle's centre M by extending LM from L to M, along the perpendicular bisector of AB.

In fact two congruent equilateral triangles have been formed over the two tangent scales, representing the proportions:

tan (ω) : tan (45º) = AL : LM, see Figure 6

Note that the intermediate value of tan(30º) = 0.577 in this example need not even be known to the user!

If the angle ω > 45º then the method has to be adapted to use the complementary angle of ω:

$$\tan(90^\circ - \omega) : \tan(45^\circ) = LM : AL$$

The Resection Problem of Snellius

The use of the station pointer at sea has a striking resemblance with the resection problem in land surveying. The name "resection" is usually associated with the Dutch mathematician and scientist Willebrord Snel van Royen, best known as Snellius (1580 -1626) who designed and carried out the first large-scale surveying project in the Netherlands, the triangulation from Alkmaar to Bergen op Zoom, see [4] and Figure 7. His purpose was to determine the length of a part of the meridian through Alkmaar, in order to calculate accurately the circumference of the earth from this measured length and the difference in latitude of the arc's endpoints. This triangulation is described in his book "Eratosthenes Batavus" (1617), in translation *the Batavian Eratosthenes* comparing Snellius with the classical Greek mathematician, see [5] and Figure 8.

Some of his observations had been performed from the roof of his own house, which was not visible from prior triangulation points: the Hoogland church tower, the Leiden City Hall, and the Pieterskerk. Snellius solved that problem by measuring those points "backwards" from his roof, using the results to calculate the coordinates of his house, the so-called "Snellius-point". This was the first documented application of resection in the history of geodesy.

Snellius however is best known for his law on the refraction of light rays (not published in book or paper, but only written as manuscript).

Angle measurements in land surveying are possible with much greater precision than at sea, by using stationary theodolites. For example the Wild T2 theodolite of the 1930's could already distinguish angles within a few seconds. Graphical constructions and station pointers used at sea could not compete with that precision, and were therefore never used in land surveying. In land surveying the solution of the Snellius resection problem is mostly done by calculating schemes on fill-in forms, using National Grid rectangular coordinates (in contrast with the sexagesimal angle coordinates used at sea).

Surveying handbooks give various solutions for the resection problem, for example the construction of Cassini (with the two circles as described before), or the Collins construction with only one circle, or the method of *barycentric coordinates* (distances to the perpendiculars of a triangle). More than 500 solutions appear to exist for the resection problem.

On the Internet a striking animation can be found showing chosen (and variable) positions of the observation points A, B, and C, against the calculated observer's position O, see

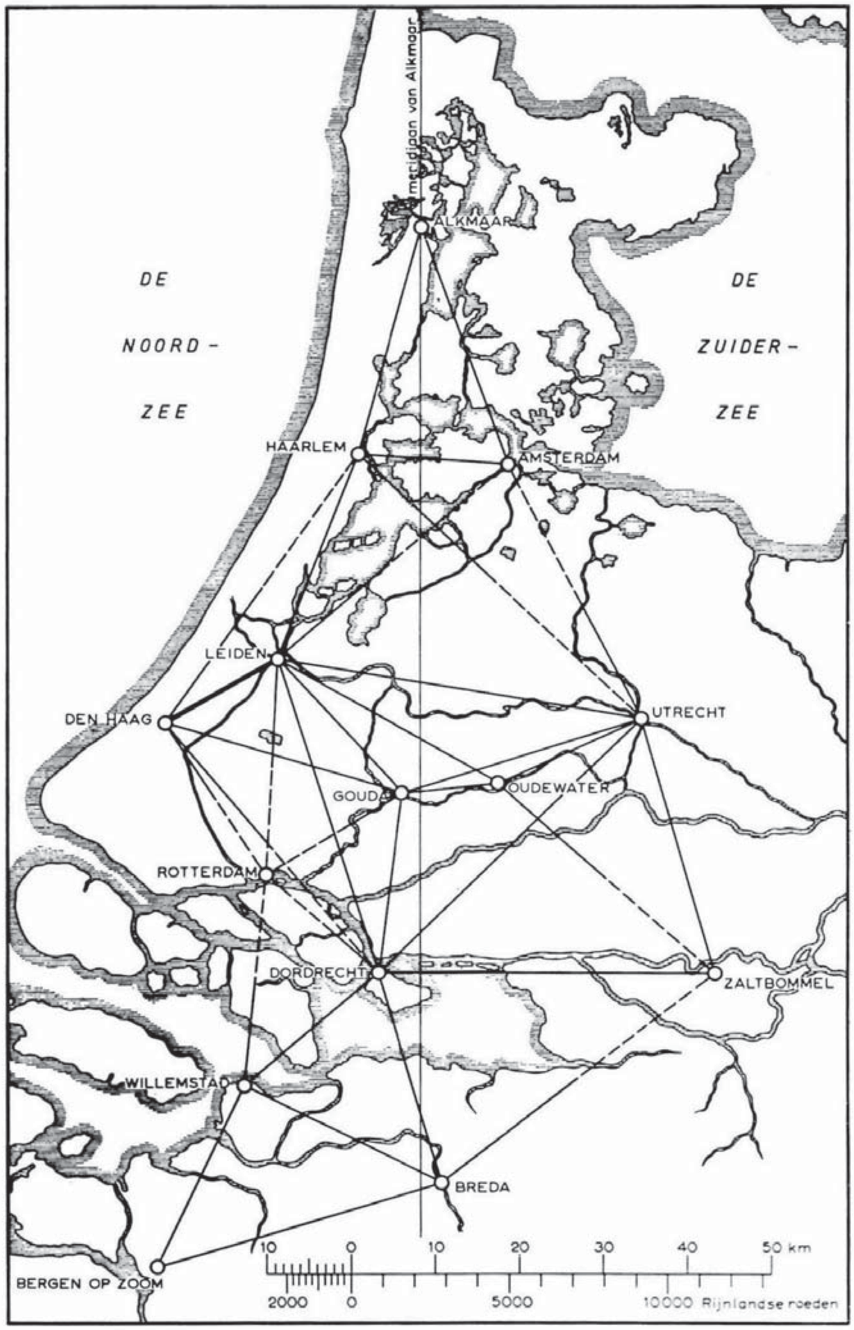
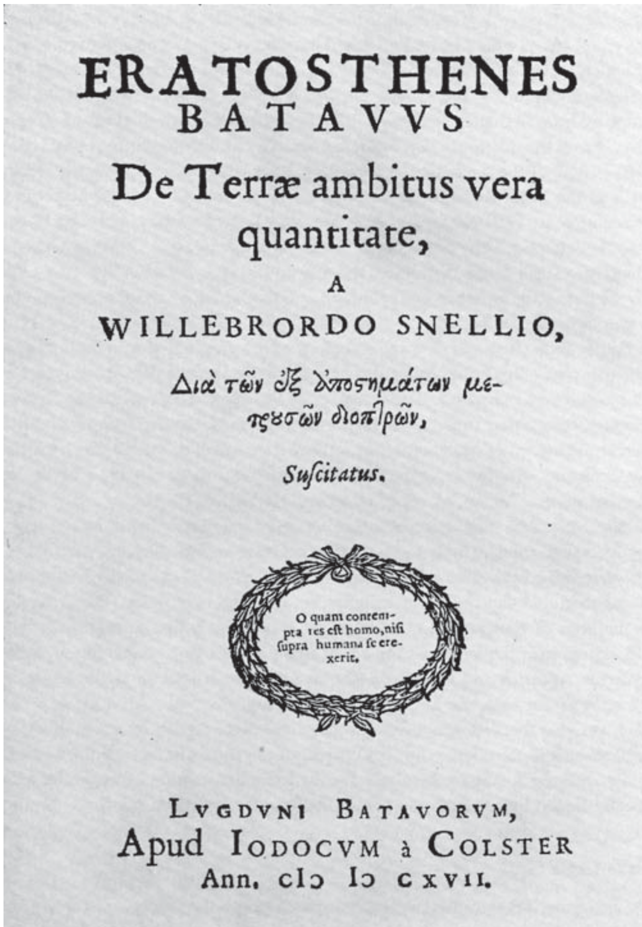


Figure 7. Snellius' Triangulation Net in the Netherlands


ERATOSTHENES
BATAVVS
De Terræ ambitus vera
quantitate,
A
WILLEBRORDO SNELLIO,
Διὰ τῶν ἐξ ἀποστημάτων με-
τρουσῶν διόπτρων,
Suſcitatus.

LVGDVNI BATAVORVM,
Apud IODOCVM à COLSTER
Ann. cIɔ Iɔ CXVII.

Figure 8. Title page of Snellius' book

[6]. The accuracy of the determination of the Snellius point for a given configuration can be shown by a cloud of measurement points, simulated with a precision of 0.002 grad (about 6½ seconds). A denser cloud indicates a higher accuracy.

History of the Station Pointer

According to D. Baxandall [7], the station pointer was invented by the English hydrographer, cartographer, entrepreneur, and inventor J. Huddart, F.R.S.,1741-1816, known by, amongst other things, an extensive route description to the East Indies, ("The Oriental Navigator", 1801), and by his charts of Saint George's Channel, the Tigris, and West-Sumatra. In 1791 he became Fellow of the Royal Society. The station pointer of Huddart is only known from an article in Nicholson's Journal2, 1804 [8].

Actually the English hydrographer Murdoch Mackenzie, F.R.S., 1712-1791 (who had been appointed Maritime Surveyor in his Majesty's Service, and since 1774 also Fellow of the Royal Society), described the station pointer much earlier, in 1774. He became known by, amongst other things, his accurate sea charts of the Orkney islands in 1801. Mackenzie was the first to publish a book specifically addressing the science and techniques of Maritime Surveying; this book contained the description of the station pointer, see [9] and Figure 9. Having published the station pointer almost 30 years before Huddart, Murdoch Mackenzie must be considered its inventor.

The station pointer was used in the first place for accurately locating hydrographical data on sea charts, especially depth soundings. The angle measurements were not performed with sea sextants held horizontally, but with special-purpose "sounding sextants". Some versions were able to fix two adjacent angles simultaneously, very important on board moving vessels (the double sounding sextant).

The Point S may be readily laid down on a Draught, by drawing on a looſe tranſparent Paper indefinite Right-lines S A, S B, S C, at Angles equal to thoſe obſerved; which being placed on the Draught ſo as each Line may paſs over, or coincide with, its reſpective Object, the angular Point S will then coincide with the Place of Obſervation. Or,

Provide a graduated Semicircle of Braſs, about 6 Inches in Diameter, having three Radij with chamfered Edges, each about 20 Inches long, (or as long as it may be judged the Diſtance of the Stations from the three given Objects may require) one of which Radij to be a Continuation of the Diameter that paſſes through the Beginning of the Degrees on the Semicircle, but immoveably fixed to it, the other two moveable round the Center, ſo as to be ſet and ſcrewed faſt to the Semicircle at any Angle. In the Center let there be a ſmall Socket, or Hole, to admit a Pin for marking the central Point on the Draught. When the ſloped Edges of the two moveable Radij are ſet and ſcrewed faſt to the Semicircle, at the reſpective Degrees and Minutes of the two obſerved Angles, and the whole Inſtrument moved on the Draught until the Edges of the three Radij are made to lie along the three ſtaſimetric Points, each touching its reſpective Point, the Center of the Semicircle will then be in the Point of Station S; which may be marked on the Draught, through the Socket, with a Pin. Such an Inſtrument as this may be called a *Station-pointer*; and would be found convenient for finding the Point of Station readily and accurately, except when the given Objects were near; when the Breadth of the Arch, of the Radij, and of the Braſs about the Center of the Semicircle might hinder the Points to be ſeen, or the Radij to be placed ſo as to comprehend a very ſmall Angle between them.

Figure 9. Mackenzie on the StationPointer

Reading measured angles and adjusting the arms of the station pointer accordingly, was a time-consuming and error-prone activity. Combined instruments have been designed with the optical path of a double sounding sextant running along the arms of a protractor, see McCombie's "position finder" in Figure 10 from [10]. Also some regular (single) sextants are known to have been combined with a protractor, for example the English "reflecting protractor" by Douglas, or the Dutch "sextant-transporteur" by LaPorte.

Conclusion

The classic station pointer, originally designed for maritime surveying, is not an essential instrument for navigation. The limited accuracy, the laborious copying of angles from sextant to station pointer, and the space needed on the chart table, are obstacles to its use. Still one can find station pointers for sale, for example the 30 dollar plastic instrument by *Starpath* [11], or more expensive ones, like a 450 dollar metal instrument by *Celestaire* [12].

However, the geometric background and the close relation to the resection point in land surveying makes the station pointer an interesting instrument in historical context.

Acknowledgments

Thanks to John Vossepoel and Nicolàs de Hilster for contributions to and review of this article from their respective fields of experience in surveying and hydrography. Also thanks to Durk Rienstra for giving me the opportunity to handle and photograph his Hughes station pointer.

Notes

1. Adapted and translated into English by O. E. van Poelje from his original Dutch article in the Dutch KRING's MIR 60, 2012, p 12 – 18.
2. Henry Hughes started his own company in 1838 in London as a maker of chronometers and scientific instruments.

Since 1859 the company, then "H. Hughes & Son", had the office located at 59 Fenchurch Street, London, where they stayed until 1941. The firm was incorporated as "Henry Hughes & Sons Ltd" in 1903. Since the 1920s the company also sold instruments under the brand name HUSUN, especially sextants.

Literature

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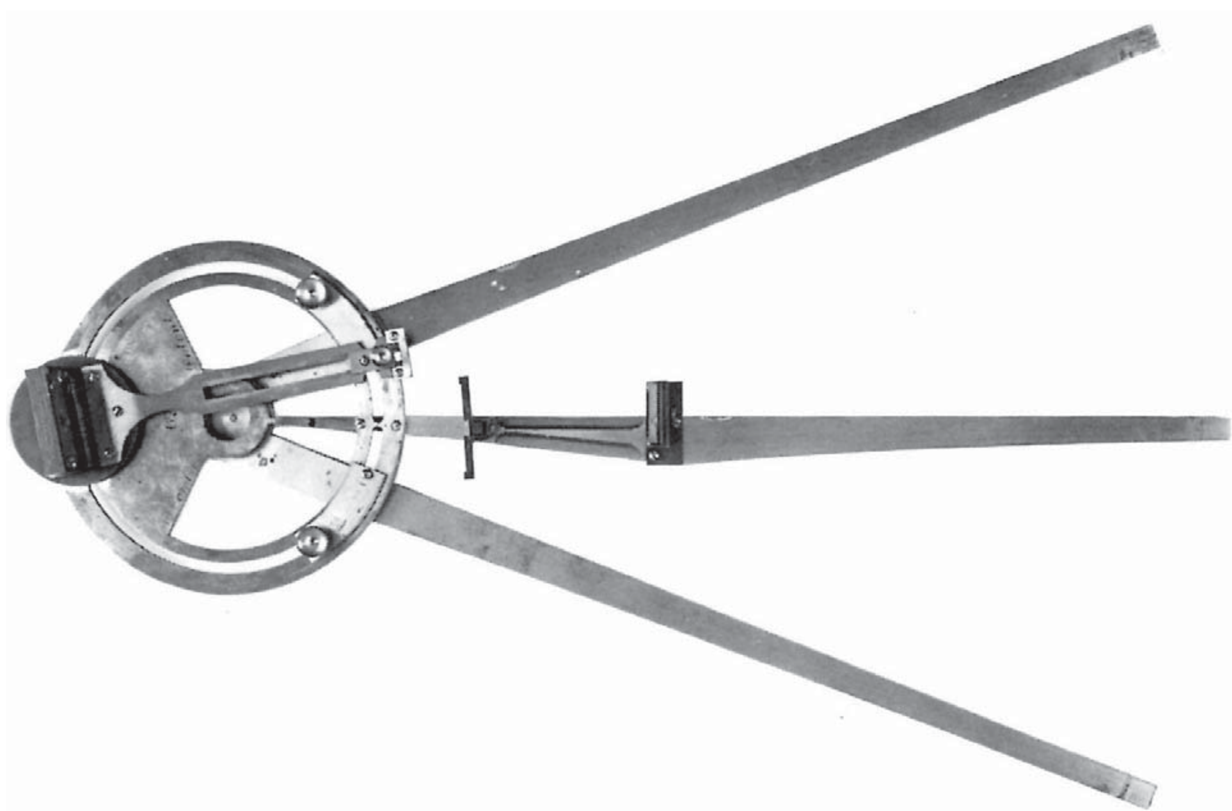

Figure 10. McCombie's Position Finder