Longitude by Equal Altitudes

Robin Stuart, July 2020

Define a timescale t = CT – CT_{LAN} where CT is the time read from the chronometer and CT_{LAN} is the time the chronometer would show when the Sun is on the meridian at local apparent noon (LAN). This is the Sun's local hour angle (LHA) expressed in time. Let t_{AM} and t_{PM} be times in the morning and afternoon when the altitude of the Sun is equal. Their average will be denoted by $\bar{t} = \tfrac{1}{2}\left(t_{PM} + t_{AM}\right)$ and their difference by $\Delta t = t_{PM} - t_{AM}$ and both are expressed in hours. Hence $t_{AM} = \bar{t} - \dfrac{\Delta t}{2}$ and $t_{PM} = \bar{t} + \dfrac{\Delta t}{2}$.

Let $\bar{\delta}$ be the Sun's declination at time $t = \bar{t}$ and δ' be the rate of change of declination in °/hour.
If h is the Sun's observed altitude at t_{AM} and t_{PM} then

$$\begin{aligned}\sin h &= \sin L \sin\left(\bar{\delta} - \delta' \frac{\Delta t}{2}\right) + \cos L \cos\left(\bar{\delta} - \delta' \frac{\Delta t}{2} \cdot\right) \cos\left(\bar{t} - \frac{\Delta t}{2}\right) \\ &= \sin L \sin\left(\bar{\delta} + \delta' \frac{\Delta t}{2}\right) + \cos L \cos\left(\bar{\delta} + \delta' \frac{\Delta t}{2}\right) \cos\left(\bar{t} + \frac{\Delta t}{2}\right)\end{aligned}$$

where the arguments of the trigonometric functions are converted to angular measure as necessary.

Performing a series expansion to first order in the small quantities $\bar{t}$ and $\delta'\Delta t$ gives

$$\bar{t} = \frac{\delta'\Delta t}{30}\left\{\frac{\tan L}{\sin\frac{\Delta t}{2}} - \frac{\tan\bar{\delta}}{\tan\frac{\Delta t}{2}}\right\}$$

From the definition of t it follows that at LAN the chronometer reads $CT_{LAN} = \overline{CT} - \bar{t}$ where $\overline{CT}$ is average of the AM and PM chronometer times. Up to a sign the quantity $\bar{t}$ is traditionally known as the *equation of equal altitudes* (Admiralty Manual of Navigation 1922, Chapter XIII, P. 204) or *noon correction* as it is added to the average of the chronometer times to obtain the CT of LAN.

Adding the *Equation of Time* (EqT) to CT_{LAN} gives CT_{LMN} the chronometer time at *local mean noon*. If the chronometer is set to GMT then the observer's longitude, λ, in time is

$$\lambda = 12 - CT_{LMN} = 12 - \left(CT_{LAN} + EqT\right)$$