Modern Bygrave Slide Rule

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I have developed a very compact and convenient method for doing the computations needed for traditional celestial navigation. This method relies on my adaptation of the long extinct Bygrave Slide Rule which is elegant in its simplicity and which produces altitudes and azimuths within one or two minutes of arc and takes less than two minutes to do the computation. The original Bygrave Slide Rule was invented by RAF Captain Bygrave in 1920 and consisted of three concentric tubes carrying a cotangent and a cosine scale which formed a cylindrical slide rule. Because the scales spiral around the tubes the scales were forty four feet long which provided the accuracy necessary to calculate altitude and azimuth for celestial navigation. A modified version was used by Germany in world war two for navigation of their U-boats. But these cylindrical slide rules were expensive to produce and they fell into disuse when precomputed tables became available.

I have developed a simpler implementation of the Bygrave, one that is very easy to make since it doesn't require concentric tubes. I made this by (in effect) unwinding the two scales and printing out the cotangent scale twice on a piece of paper. This avoids the problem encountered with normal slide rules of running off the ends of the scale. I then printed the cosine scale in red on a transparent sheet so that the cosine scale can be placed directly on top of the cotangent scale and aligned much like a normal slide rule. These two simple sheets replace the multi-volume sets of tables normally used for celestial computations. You can combine this slide rule with eight photocopied pages from H.O. 249 (four pieces of paper printed on both sides) containing the long term almanac, list of stars, and the precession and nutation correction tables and you have a very compact, self contained celestial navigation system usable for ten years!

I will illustrate how easy it is to use with an example. I have included pictures of the scales and of the computation form showing this computation.

See also: Bygrave instructional form, Bygrave instructional form example , Bygrave Nautical Almanac Form. and Nautical Almanac Example.

illustration 1 , cotangent scale

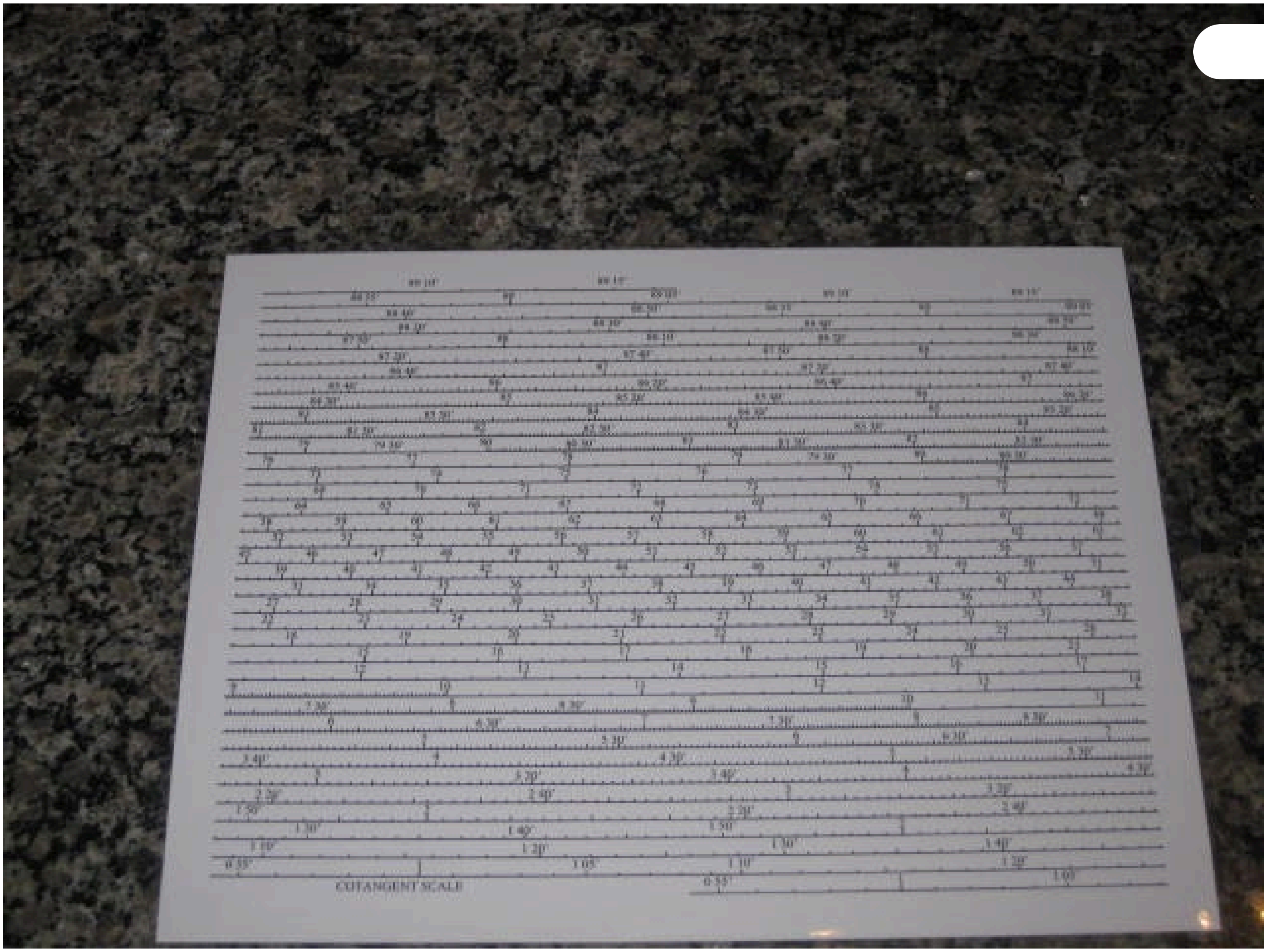


illustration 2 , cosine scale

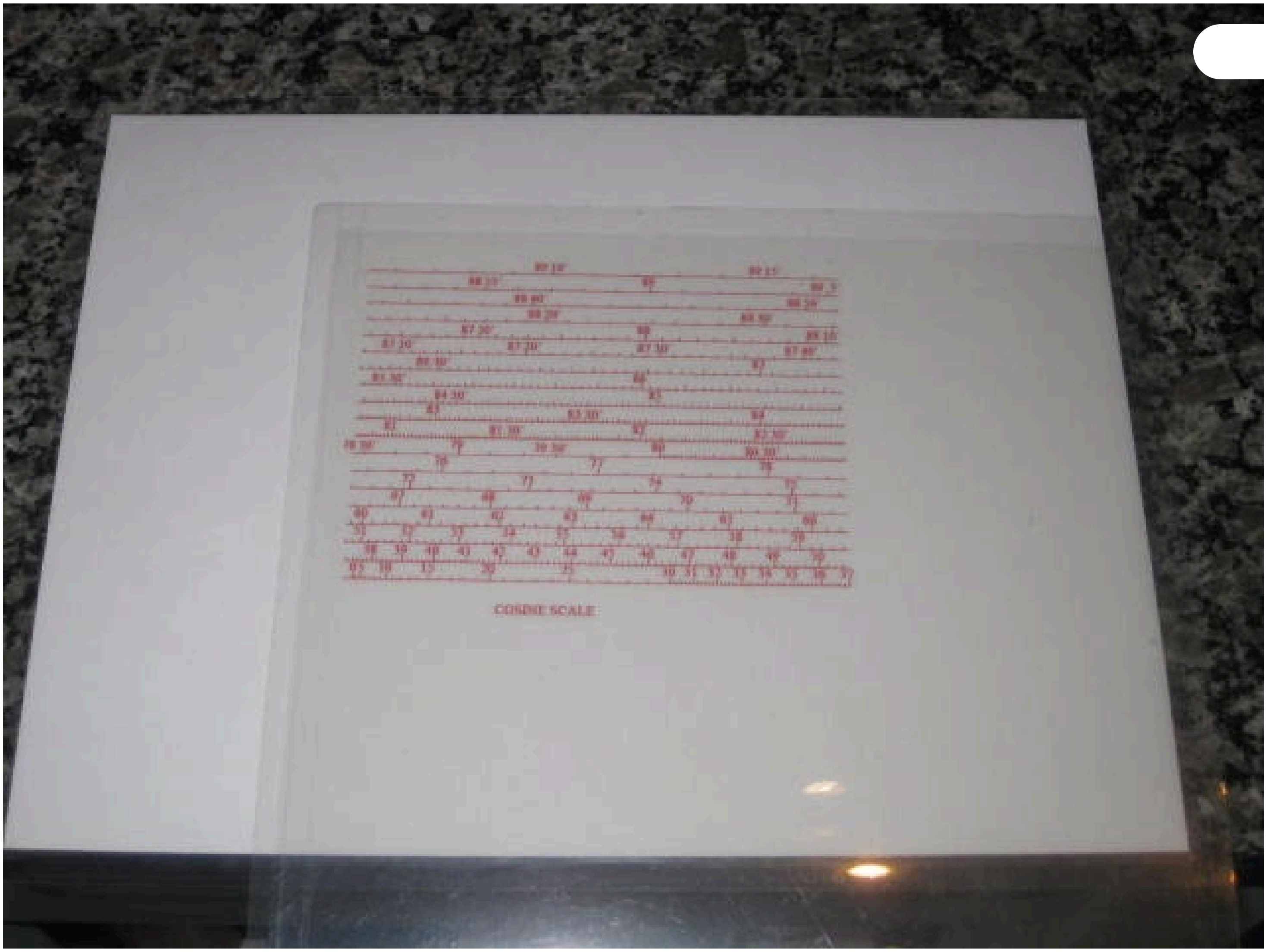


illustration 3 , computation form

scales.

If declination is less than 1° omit the first step and set W equal to declination. If latitude is also less than 1° then assume a latitude of 1°. Compute Az and Zn. Interchange declination and latitude then start over again computing Hc using those values and disregard the Az derived during this second computation.

If Y is less than 1° or greater than 89° choose a different assumed latitude to bring Y within the range of the scales.

Compute Az. If Az is greater than 85° use this Az for computing Zn and for plotting the LOP. Interchange declination and latitude then start over again computing Hc using those values and disregard the Az derived during this second computation.

0 < LHA < 90	= LHA < 180	< LHA = 270	< LHA < 360
	180	LHA ______	360
	- LHA ______	-180	- LHA ______
H = LHA ______	H = ______	H = ______	H = ______

(If H < 1° or if H > 89° see special rules)

X = Co-Lat + or - W:

Declination same name:	+ W	-W	- W	+ W
Declination contrary name:	- W	- W	-W	- W

D ____________________
(If declination is less than 1° see special rules)

H ____________________

(90) (89:60)

Lat. – ____________________
Co-Lat. ____________________
W (+/-) ____________________

(179:60)

X ____________________(Ignore sign of X)
(If X < 90 then Y=X ; If X > 90 then Y = 180-X)

Y ____________________(Ignore sign of Y)
(If 1° >Y > 89° choose a different assumed latitude)

(180) (179:60) (360) (359:60)

Az ____________________
(If Az > 85° see special rules)

Zn ____________________

Hc ____________________

Ho ____________________

INT ____________________T/A

Cos	Co-Tan
Set 0	Set D
Set H	Read W
Set W	Set H
Set Y	Read Az
Set Az	Set Y
Set 0	Read Hc

Azimuth Rules

North Latitude

	0 < LHA < 180	< LHA < 360
If X > 90°	Zn = 360 - Az	Zn = Az
If X < 90°	Zn = 180 + Az	Zn = 180 - Az

South latitude

If X > 90°	Zn = 180 + Az	Zn = 180 - Az
If X < 90°	Zn = 360 - Az	Zn = Az

The Bygrave doesn't require that the latitude or the LHA to be whole degrees so you can compute altitude (Hc) for your De Reckoned position but for convenience in this example we will use whole degrees, 34° North for latitude, 20° North for declination and a Local Hour Angle (LHA) of 346°.

When using a Bygrave slide rule the azimuth and altitude are calculated in three steps using the same manipulations of the slide rule for each step. We first calculate an intermediate value, "W," in the first step and then use it to compute azimuth in the second step. Then using the azimuth we finally compute altitude in the third step. Before we begin we need to bring the LHA into a reduced range of 0° to 90° and we also figure our co-latitude. After we compute "W" we will be combining it with co-latitude, either adding or subtracting depending on the LHA and whether the latitude and the declination have the same or different names, to form the intermediate value "X" which is then brought into the range of 0° to 90° to form "Y." The computation form leads you through all of these steps.

Illustration number 3 shows the form to be used with this simplified model of the Bygrave. The top of the form is used to compute hour angle, "H", which is LHA converted into the range of 0° through 90°. Enter the LHA in the proper column and follow the form to make the computation. You can see in our example we have placed the LHA, 346°, in the column for LHAs in the range of 270° to 360°. The form shows that in this case we subtract LHA from 360° to find "H", hour angle, in this example, 14°. We have carried this 14° down to the "H" blank on the form and we have entered declination and latitude in the appropriate blanks (illustration 4.)

illustration 3

scales.

If declination is less than 1° omit the first step and set W equal to declination. If latitude is also less than 1° then assume a latitude of 1°. Compute Az and Zn. Interchange declination and latitude then start over again computing Hc using those values and disregard the Az derived during this second computation.

If Y is less than 1° or greater than 89° choose a different assumed latitude to bring Y within the range of the scales.

Compute Az. If Az is greater than 85° use this Az for computing Zn and for plotting the LOP. Interchange declination and latitude then start over again computing Hc using those values and disregard the Az derived during this second computation.

0 < LHA < 90	< LHA < 180	< LHA < 270	< LHA < 360
	180	LHA ______	360
	- LHA ______	-180	- LHA ______
H = LHA ______	H = ______	H = ______	H = ______

(If H < 1° or if H > 89° see special rules)

X = Co-Lat + or - W:

Declination same name:	+ W	-W	- W	+ W
Declination contrary name:	- W	- W	-W	- W

D ______________

(If declination is less than 1° see special rules)

H ______________

(90) (89:60)

Lat. – ______________

Co-Lat. ______________

W (+/-) ______________

(179:60)

X ______________ (Ignore sign of X)

(If X < 90 then Y=X ; If X > 90 then Y = 180-X)

Y ______________ (Ignore sign of Y)

(If 1° >Y > 89° choose a different assumed latitude)

(180) (179:60) (360) (359:60)

Az ______________

(If Az > 85° see special rules)

Zn ______________

Hc ______________

Ho ______________

INT ______________ T/A

Cos	Co-Tan
Set 0 →	Set D
Set H ←	→ Read W
Set W →	Set H
Set Y ←	→ Read Az
Set Az →	Set Y
Set 0 ←	→ Read Hc

Azimuth Rules

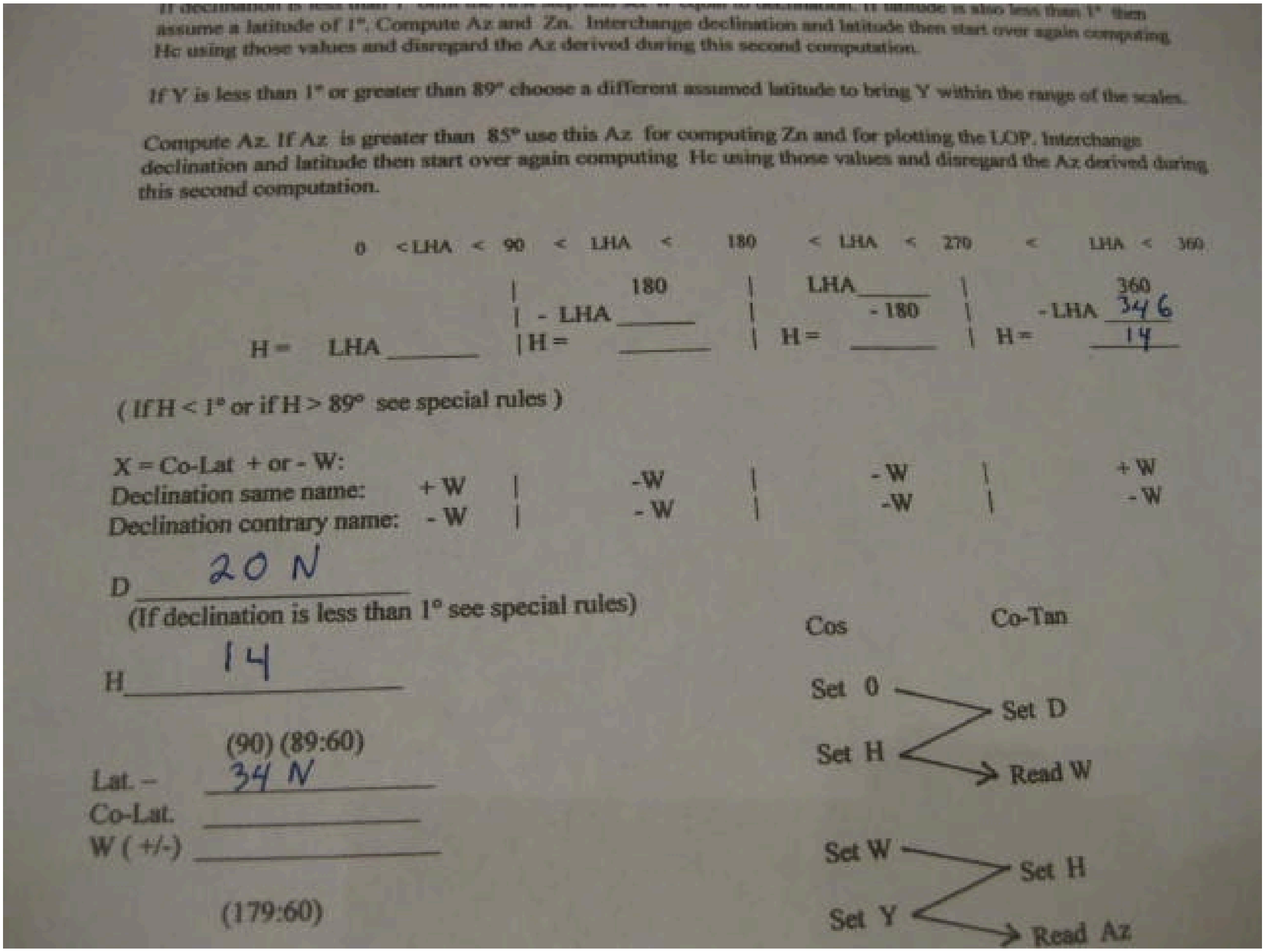
North Latitude

	0 < LHA < 180	< LHA < 360
If X > 90°	Zn = 360 - Az	Zn = Az
If X < 90°	Zn = 180 + Az	Zn = 180 - Az

South latitude

If X > 90°	Zn = 180 + Az	Zn = 180 - Az
If X < 90°	Zn = 360 - Az	Zn = Az

illustration 4



If declination is less than 1° ... If latitude is also less than 1° then assume a latitude of 1°. Compute Az and Zn. Interchange declination and latitude then start over again computing Hc using those values and disregard the Az derived during this second computation.

If Y is less than 1° or greater than 89° choose a different assumed latitude to bring Y within the range of the scales.

Compute Az. If Az is greater than 85° use this Az for computing Zn and for plotting the LOP. Interchange declination and latitude then start over again computing Hc using those values and disregard the Az derived during this second computation.

0 < LHA < 90	< LHA < 180	< LHA < 270	< LHA < 360
	180	LHA ____	360
	- LHA ____	- 180	- LHA 346
H = LHA ____	H = ____	H = ____	H = 14

(If H < 1° or if H > 89° see special rules)

X = Co-Lat + or - W:				
Declination same name:	+ W	-W	- W	+ W
Declination contrary name:	- W	- W	-W	- W

D 20 N
(If declination is less than 1° see special rules)

H 14

(90) (89:60)

Lat. – 34 N
Co-Lat. ________
W (+/-) ________

(179:60)

Cos	Co-Tan
Set 0	Set D
Set H	Read W
Set W	Set H
Set Y	Read Az

We also use the top of the form to determine if we will be adding or subtracting the intermediate value of "W" to co-latitud This is determined by the column of the LHA and by the names of the latitude and declination. In our example we can see that we will be adding "W" since LHA is in the last column and the names of the latitude and the declination are the same. Just circle the "+" mark at the bottom of this column and also place a "+" mark on the "W" line under the co-latitude line. (Illustrations 5 and 6) Shown on the form, but not happening in practice, are the two cells for LHA between 90° and 180° and 270° and contrary declination because such combinations can never happen with the celestial body above the horizon. .So the rule simplifies to:

IF

270° < LHA < 90°

and Latitude and Declination have the same name then ADD W

Otherwise SUBTRACT

illustration 5

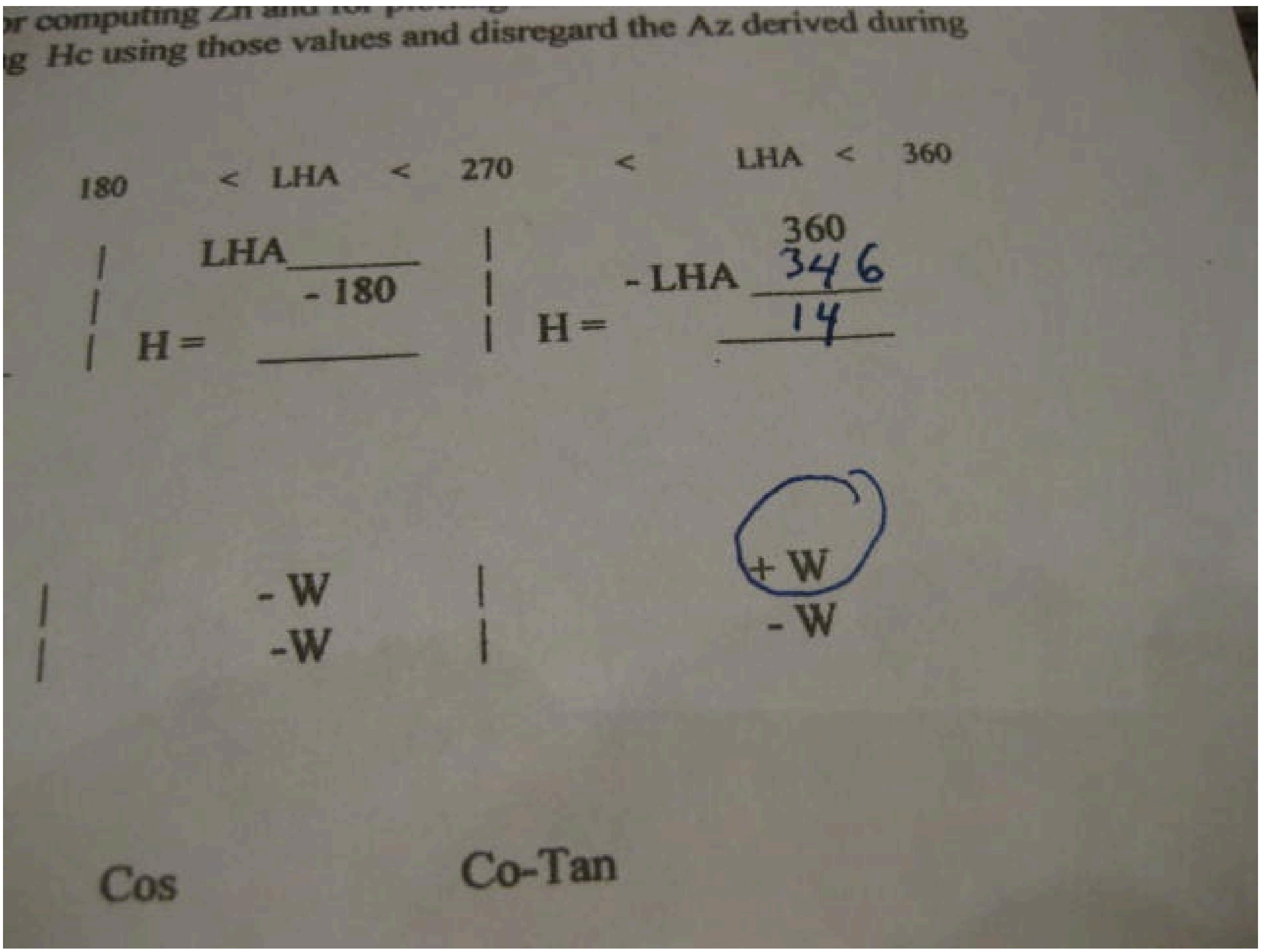
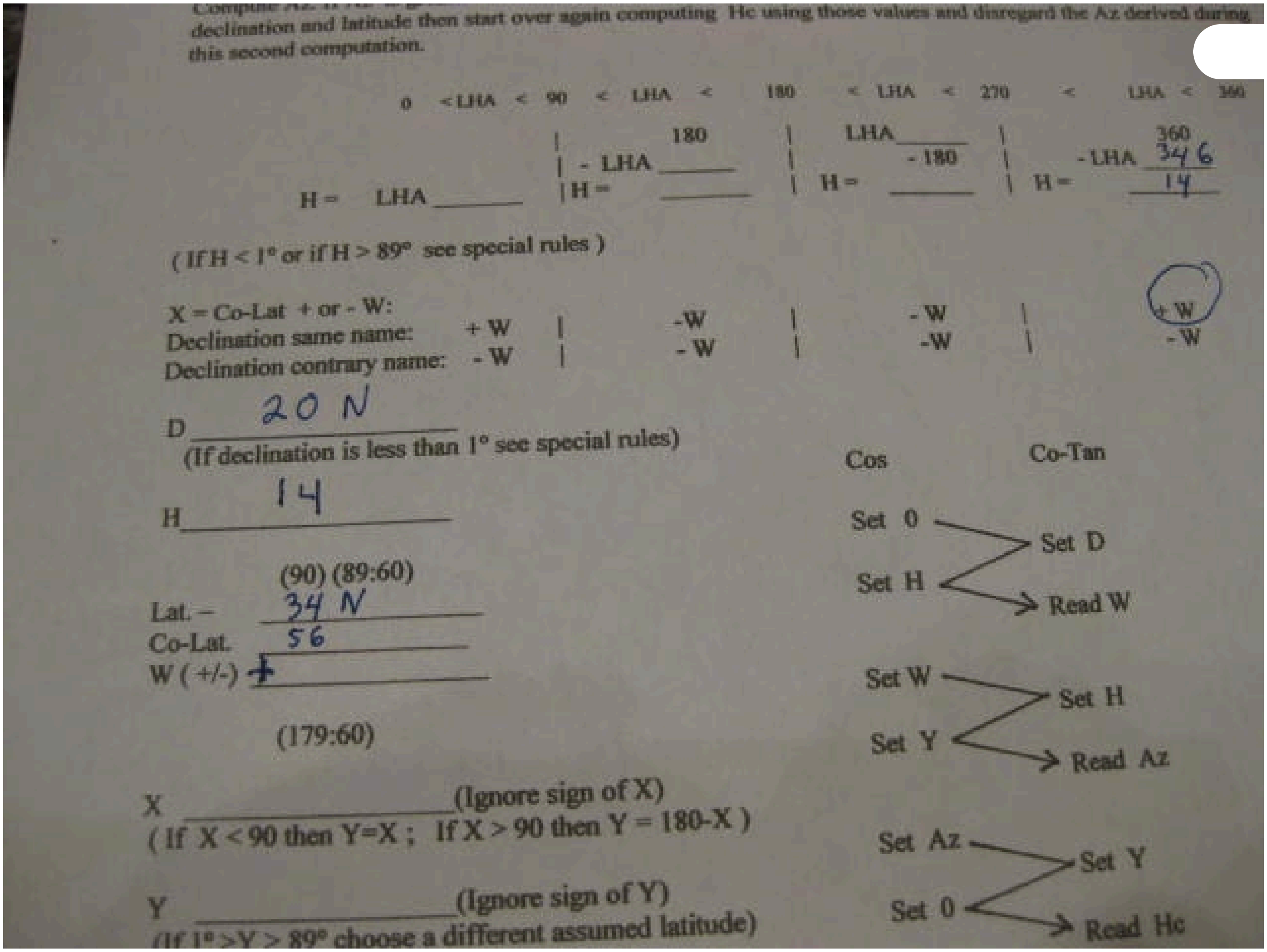


illustration 6


Compute Az. ... declination and latitude then start over again computing Hc using those values and disregard the Az derived during this second computation.

0 < LHA < 90 < LHA < 180 < LHA < 270 < LHA < 360

H = LHA ______ | 180 - LHA ______ H = ______ | LHA ______ - 180 H = ______ | 360 - LHA 346 H = 14

(If H < 1° or if H > 89° see special rules)

X = Co-Lat + or - W:
Declination same name: + W | -W | - W | + W
Declination contrary name: - W | - W | -W | - W

D 20 N
(If declination is less than 1° see special rules)

H 14

(90) (89:60)
Lat. – 34 N
Co-Lat. 56
W (+/-) +

(179:60)

X ______ (Ignore sign of X)
(If X < 90 then Y=X ; If X > 90 then Y = 180-X)

Y ______ (Ignore sign of Y)
(If 1° > Y > 89° choose a different assumed latitude)

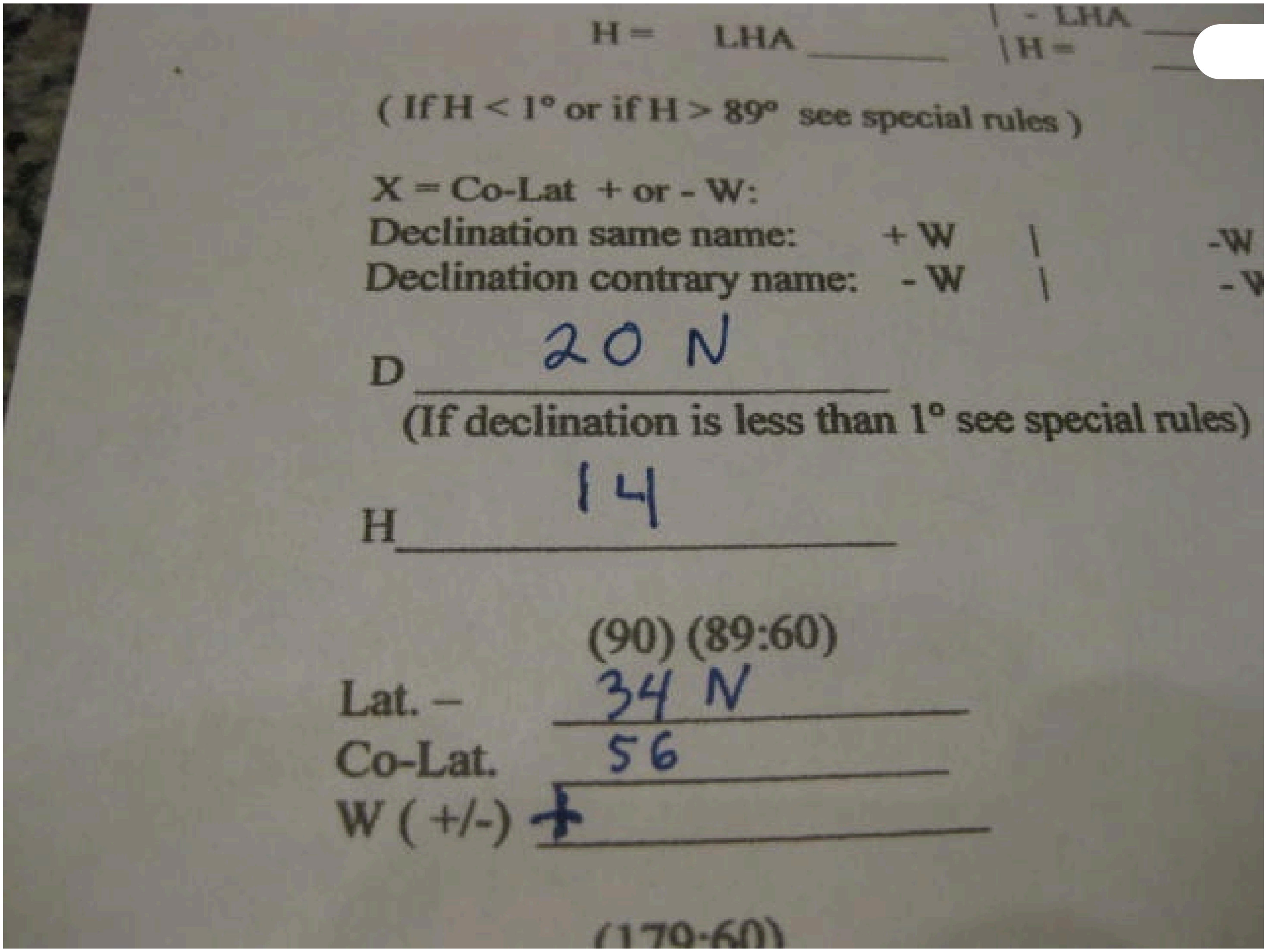
Cos Co-Tan
Set 0 → Set D
Set H → Read W

Set W → Set H
Set Y → Read Az

Set Az → Set Y
Set 0 → Read Hc

Next we subtract the latitude from 90° to form co-latitude. If we were using our D.R. latitude we would subtract it from 89° 60' since this notation makes it easy to subtract degrees and minutes. In our example the co-latitude is 56°. (Illustration 7)

illustration 7


H = LHA ______ | - LHA ___
| H = ___

(If H < 1° or if H > 89° see special rules)

X = Co-Lat + or - W:
Declination same name: + W | -W
Declination contrary name: - W | - W

D 20 N
(If declination is less than 1° see special rules)

H 14

(90) (89:60)
Lat. – 34 N
Co-Lat. 56
W (+/-) +

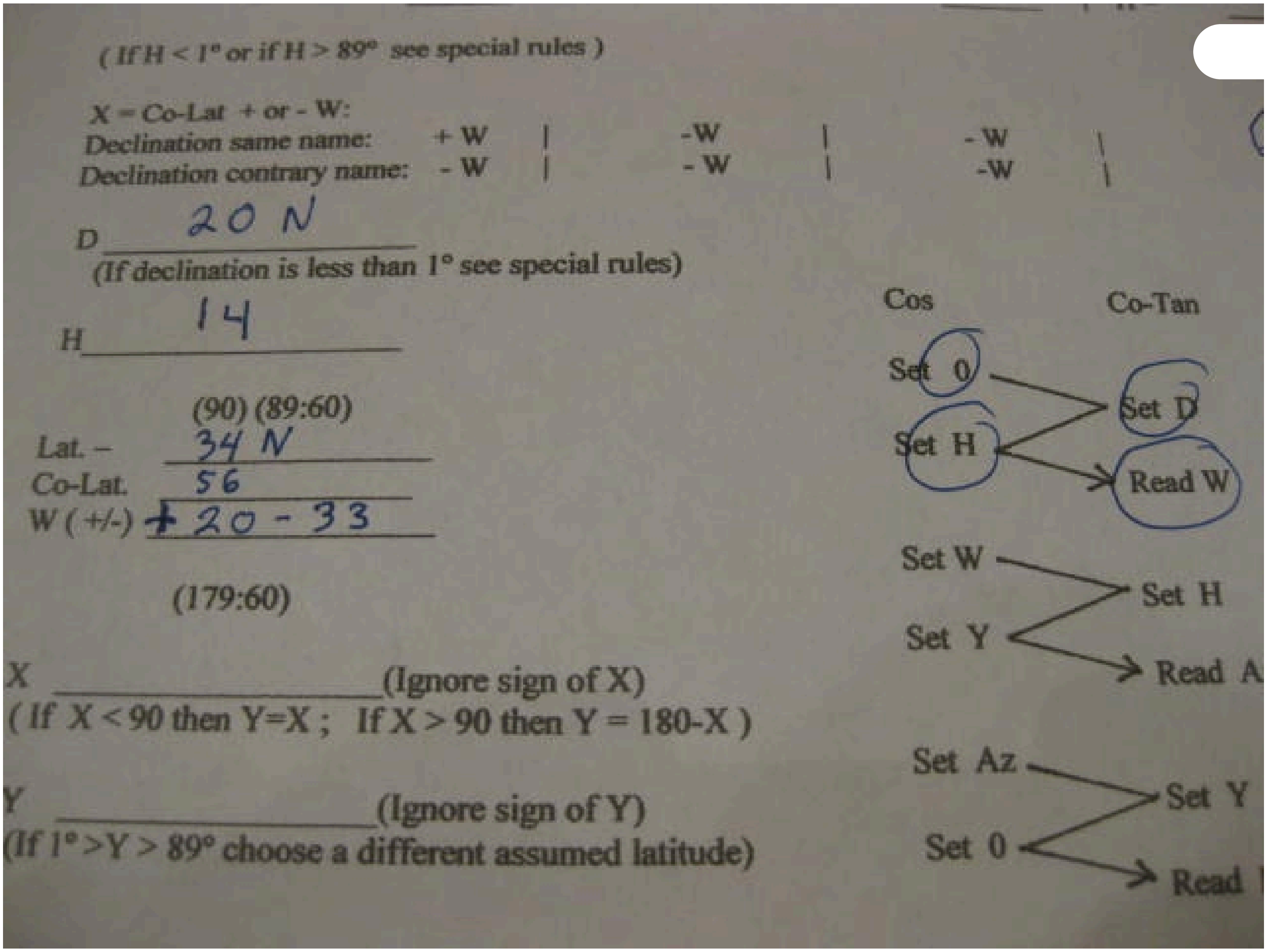
(179:60)

Now we calculate the intermediate value "W" which is found by the formula:

cotan W = cotan declination x cos H

We follow the zig-zag diagram along the right edge of the form to find the value of "W". See (Illustration 8) The left side of the zig-zag indicates the red cosine scale and the right side indicates the black cotangent scale. The same process is used three times and values are always taken off from the black cotangent scale, never from the red cosine scale. The zig-zag tells us to line up 0 (zero) on the red cosine scale with "D" (declination) on the black cotangent scale. We then look at "H" (hour angle) on the red scale and take out "W" from the adjacent black cotangent scale. (Illustration 9)

illustration 8



(If H < 1° or if H > 89° see special rules)

X = Co-Lat + or - W:

Declination same name:	+ W	-W	- W
Declination contrary name:	- W	- W	-W

D 20 N

(If declination is less than 1° see special rules)

H 14

(90) (89:60)

Lat. – 34 N

Co-Lat. 56

W (+/-) + 20 - 33

(179:60)

X ____________(Ignore sign of X)

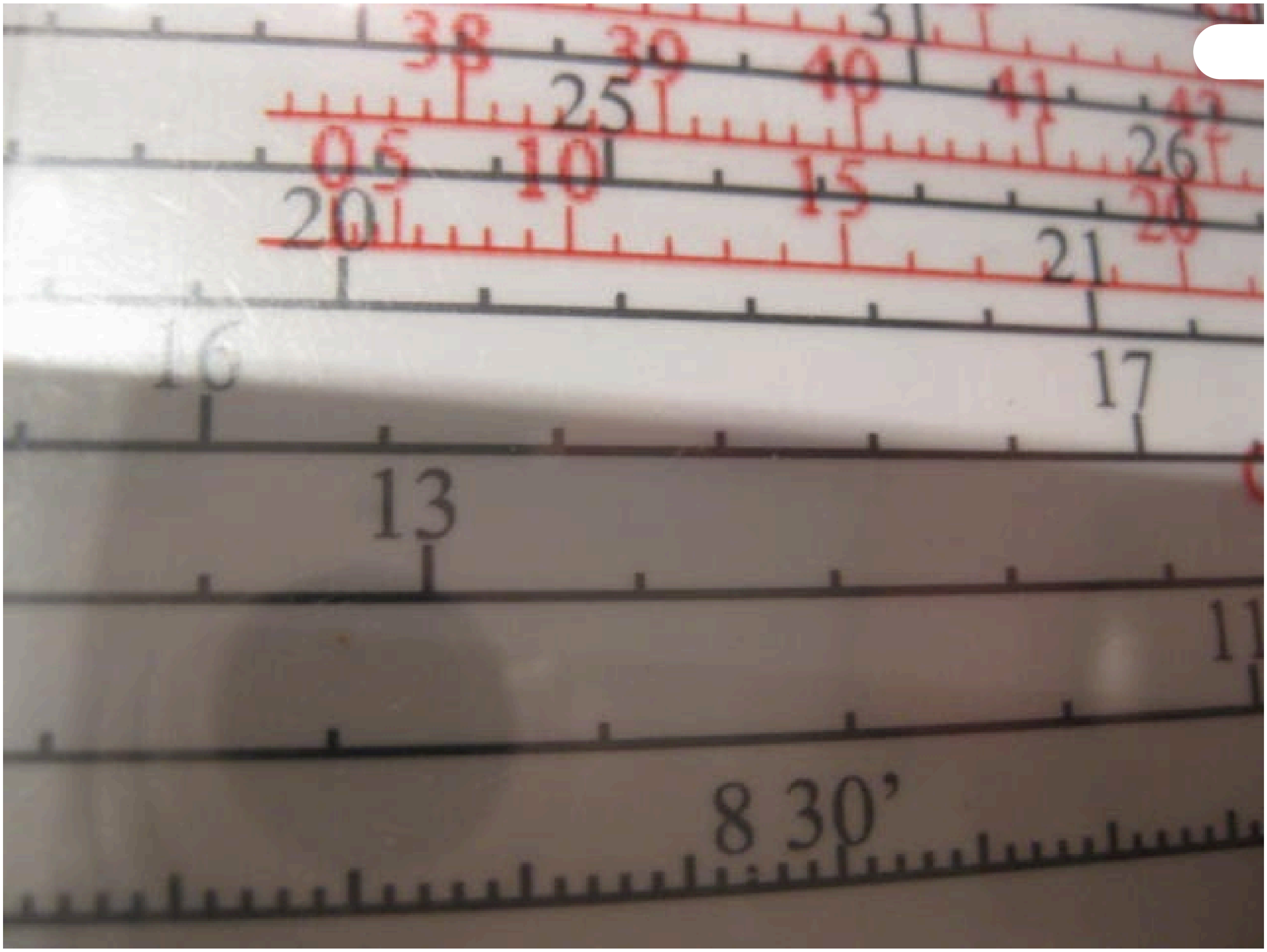
(If X < 90 then Y=X ; If X > 90 then Y = 180-X)

Y ____________(Ignore sign of Y)

(If 1° >Y > 89° choose a different assumed latitude)

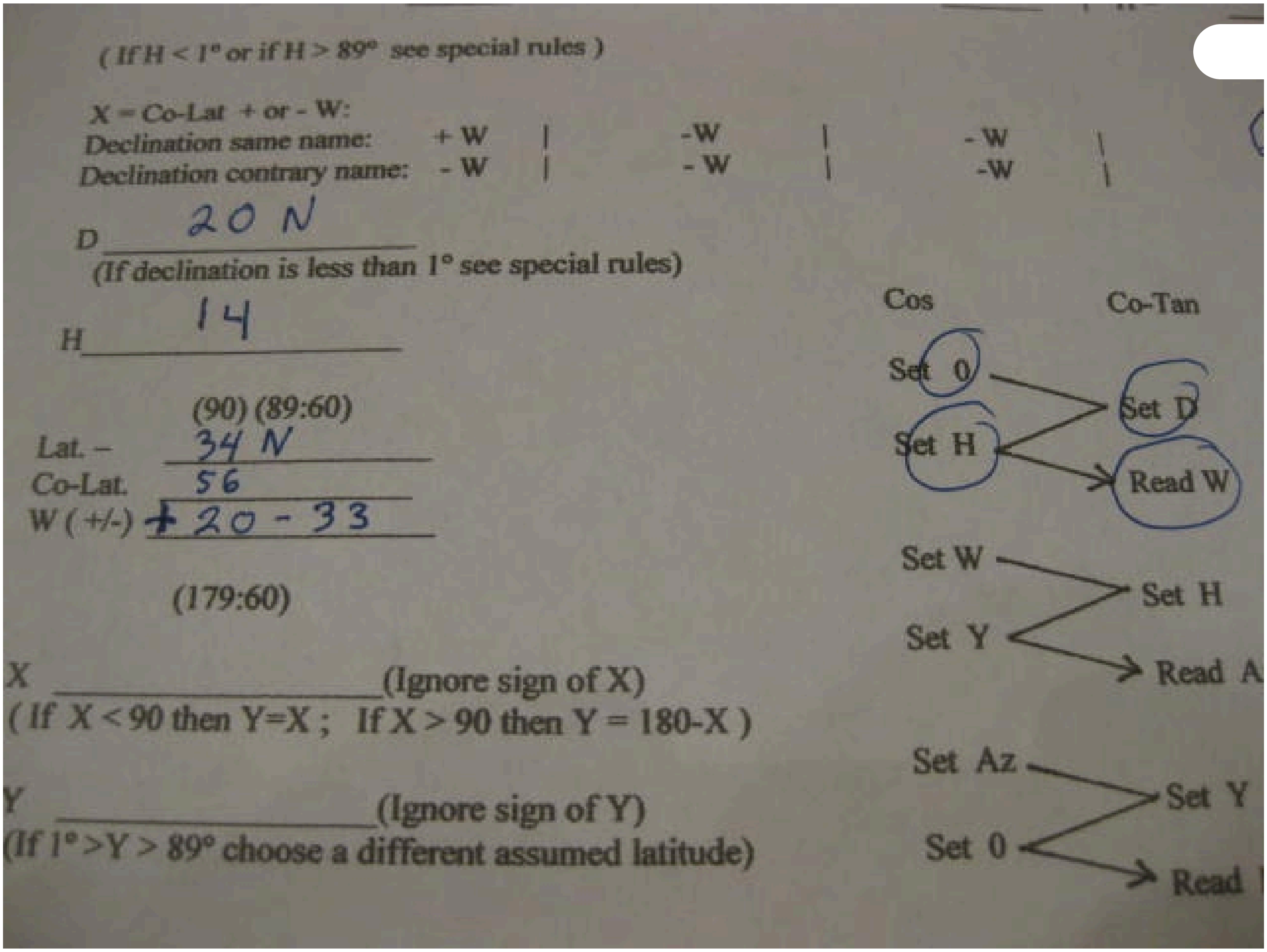
Cos	Co-Tan
Set 0	Set D
Set H	Read W
Set W	Set H
Set Y	Read A
Set Az	Set Y
Set 0	Read

illustration 9



On both scales the numbering is above the marks on the scales. I have aligned the red cosine scale slightly above the black cotangent scale for clarity. We can see that the red zero is lined up directly above the black tick mark for 20° (the declination). You use visual interpolation for the minutes of declination. Now, without allowing the scales to shift, we locate "H" (14°) on the red scale and read out "W" from the adjacent black scale, in this case 20° 33' by visual interpolation, and write this value on the "W" line below the co-latitude. (Illustration 8)

illustration 8


(If H < 1° or if H > 89° see special rules)

X = Co-Lat + or - W:
Declination same name: + W | -W | - W |
Declination contrary name: - W | - W | -W |

D 20 N
(If declination is less than 1° see special rules)

H 14

(90) (89:60)
Lat. – 34 N
Co-Lat. 56
W (+/-) + 20 - 33

(179:60)

X ________________(Ignore sign of X)
(If X < 90 then Y=X ; If X > 90 then Y = 180-X)

Y ________________(Ignore sign of Y)
(If 1° >Y > 89° choose a different assumed latitude)

Cos | Co-Tan
Set 0 → Set D
Set H → Read W
Set W → Set H
Set Y → Read A
Set Az → Set Y
Set 0 → Read

Next we determine "X" by adding "W" (20° 33') to co-latitude (56°), in this example "X" = 76° 33'. Then, following the rule on the form, since "X" < 90° , "Y" = "X" and we carry it down to the "Y" line. If "X" > 90° we would subtract "X" from 180° (or from 179° 60') to form "Y". (Illustrations 10 and 11)

illustration 10

Declination contrary name:

D 20 N
(If declination is less than 1° see special rules)

H 14

(90) (89:60)

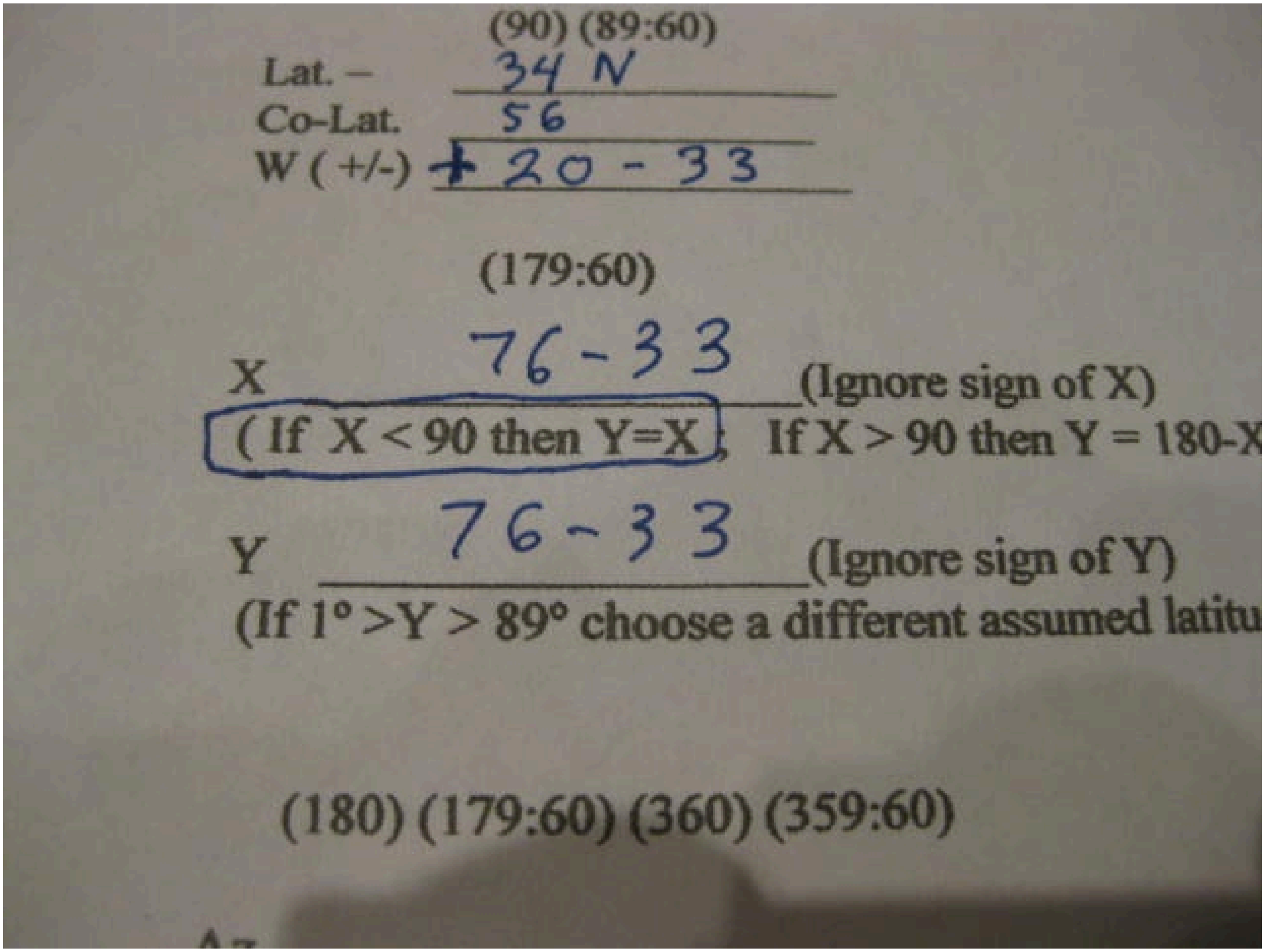
Lat. – 34 N
Co-Lat. 56
W (+/-) + 20 - 33

(179:60)

X 76 - 33 (Ignore sign of X)
(If X < 90 then Y=X ; If X > 90 then Y = 180-X)

Y (Ignore sign of Y)
(If 1° >Y > 89° choose a different assumed latitude)

illustration 11

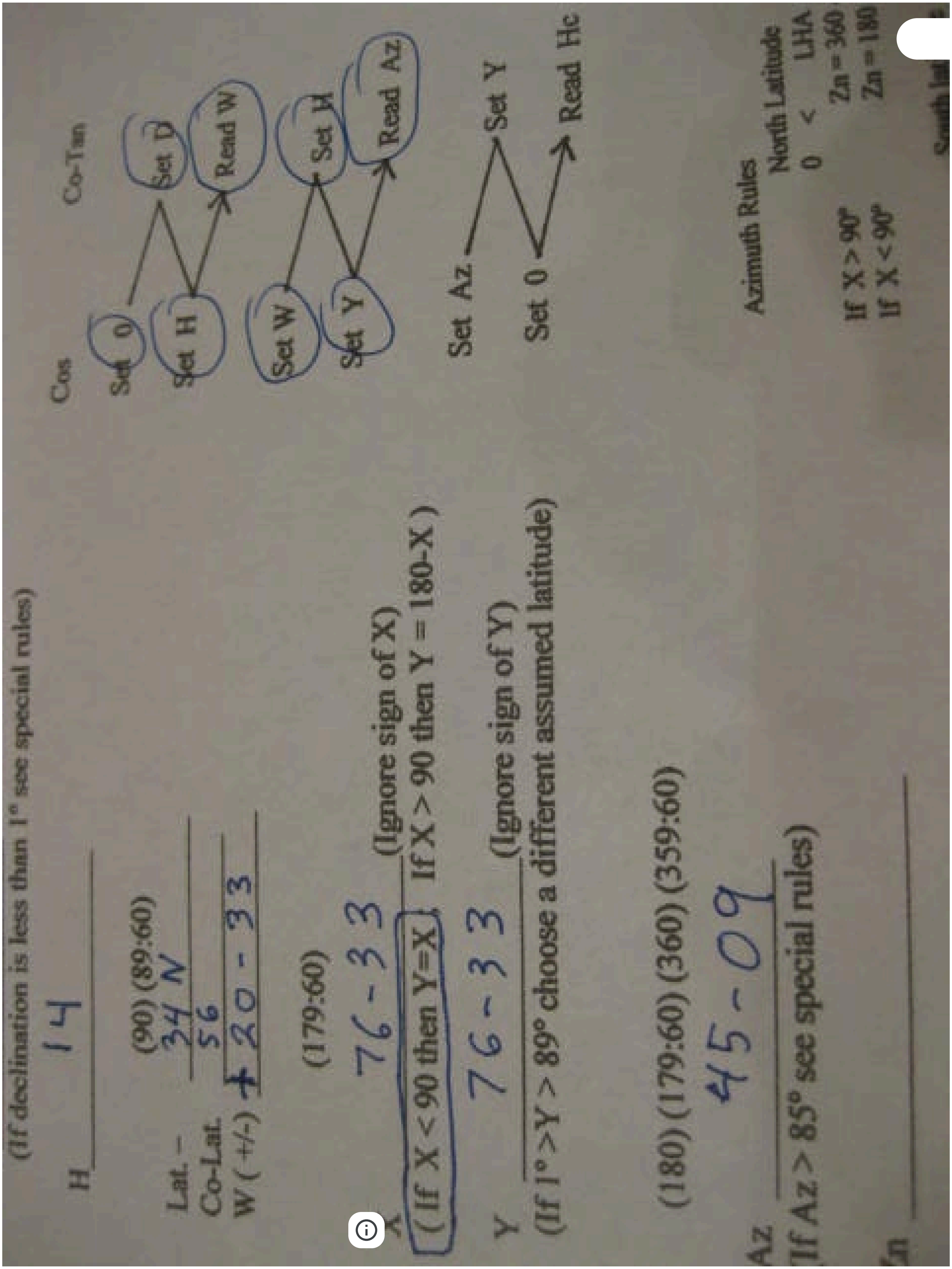


Next we find azimuth with the formula :

cot Az = (cotan H / cos W) x cos Y

We now follow the second zig-zag on the form to find Az. (Note we start with the value computed at the last step, "W.") (Illustration 12) Using the same manipulation as the first time, we line up "W" (20° 33') on the red scale with "H" (14°) on the black scale. We than look at "Y" (76°-33') on the red scale and take out Az from the adjacent black scale. Looking at illustration 13, we have lined up red 20° 33' with black 14° by aligning the 20° 30' mark slightly to the left of the "14" on the cotangent scale. (Note at this point on the red scale the tick marks show 1 degree.) Now, without allowing the scales to shift, we locate "Y" on the red scale (76° 33') and take out the Az from the black scale. (Illustration 14) The red 76° 30' mark is lightly to the left of the black 45° tick mark and the red 76° 40' is aligned with the black 45° 20' so by visual interpolation the red 76° 33' will align with the black 45° 09', the Az. (At this point on both scales each tick mark is 10'.)

illustration 12


(If declination is less than 1° see special rules)

H 14

(90) (89:60)

Lat. – 34 N

Co-Lat. 56

W (+/-) + 20 - 33

(179:60)

X 76-33 (Ignore sign of X)

(If X < 90 then Y=X ; If X > 90 then Y = 180-X)

Y 76-33 (Ignore sign of Y)

(If 1° >Y > 89° choose a different assumed latitude)

(180) (179:60) (360) (359:60)

Az 45-09

(If Az > 85° see special rules)

Zn

illustration 13

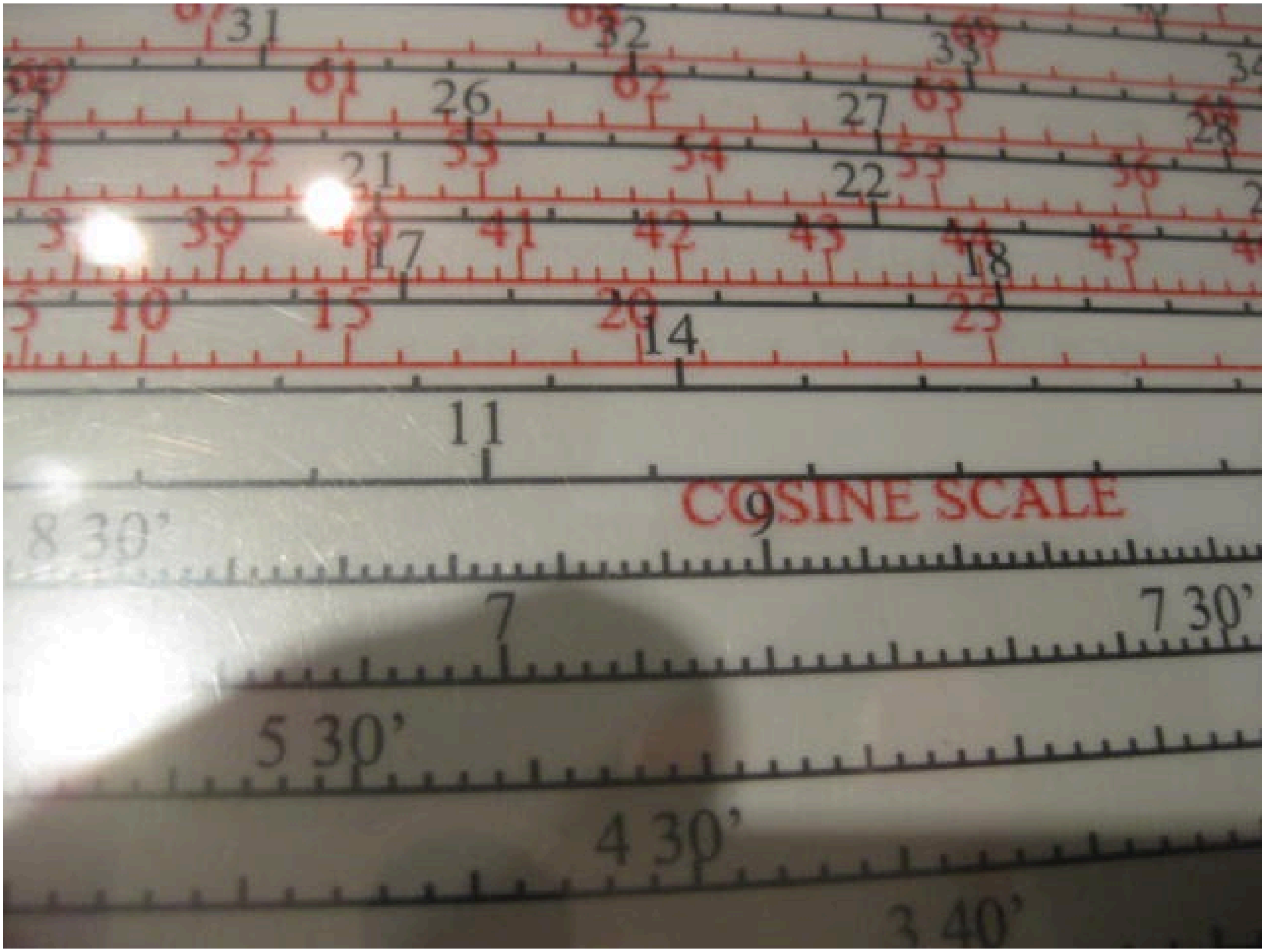
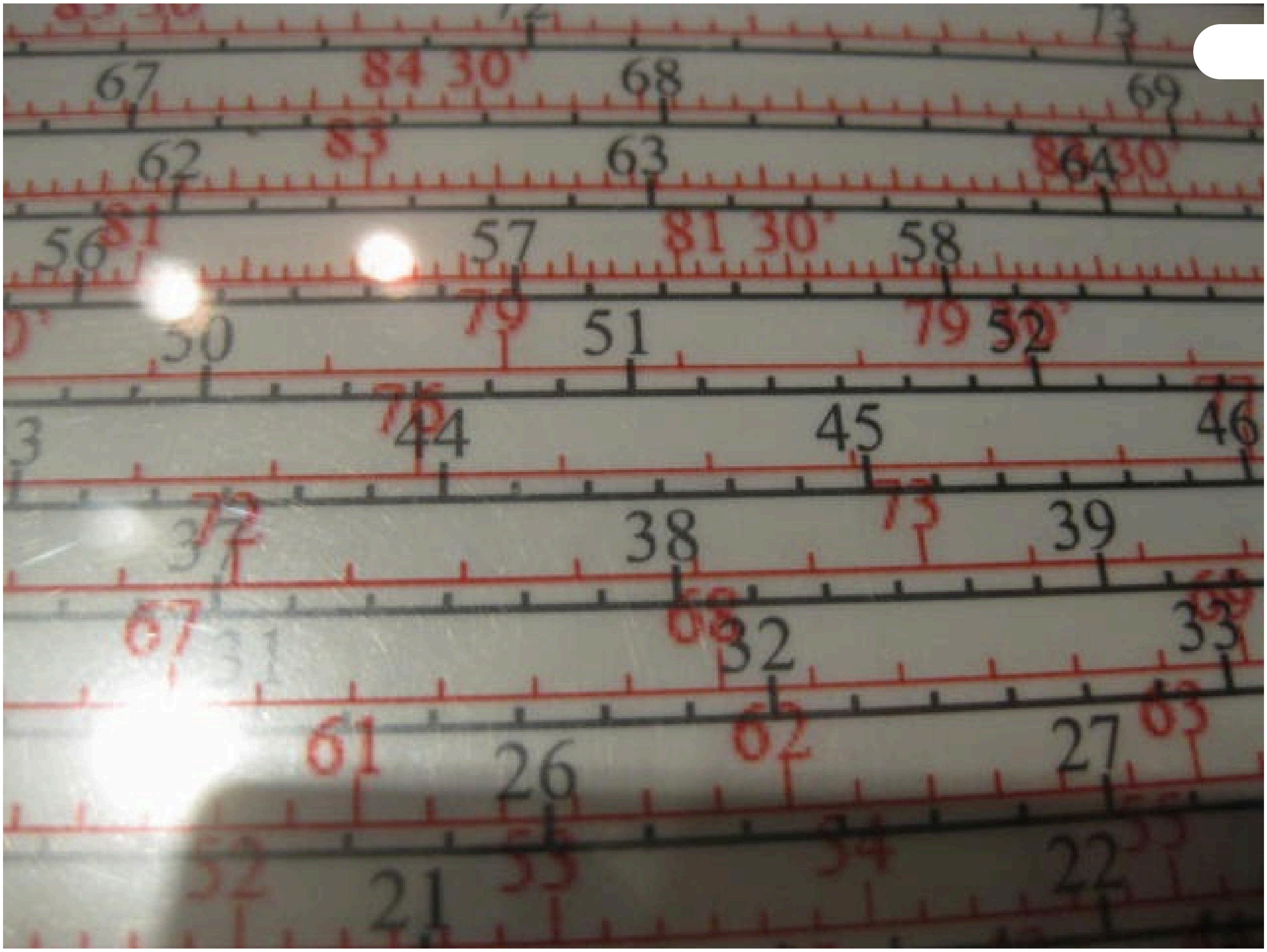


illustration 14



Next enter the Az on the form and compute Zn by applying the rules on the form so we subtract Az from 180° (179° 60') to determine Zn, 134° 51' (or 134.9°).(Illustration 15) , in this case the celestial body is to the south east. The determination of Zn is usually not a problem in real life since you know the approximate direction of the body when you take the sight. I have included rules on the form to resolve any ambiguity.

illustration 15

X 76-33 (Ignore sign of X)

$\boxed{\text{(If X < 90 then Y=X}}$; If X > 90 then Y = 180-X)

Y 76-33 (Ignore sign of Y)

(If 1° >Y > 89° choose a different assumed latitude)

~~(180)~~ (179:60) ~~(360) (359:60)~~

Az 45-09

(If Az > 85° see special rules)

Zn 134-51

Hc

Ho

The third step calculates computed altitude, Hc, using the formula:

cot Hc = cot Y / cos Az

We follow the last zig-zag to calculate Hc. (Note we again start with the value computed at the last step, Az). (Illustration 16) Using the same manipulation as before, we line up Az (45° 09') on the red scale with "Y" (76° 33') on the black scale. Next we locate zero on the red scale and take out Hc from the adjacent black scale. Looking at illustration 17 we see that we have aligned the red 45° tick mark slightly to the right of the black 76° 30' (each tick mark is 10' on both scales) and we visually interpolate red 45° 09' with black 76° 33'. Now, without allowing the scales to shift, we find zero on the red cosine scale and interpolate on the black scale to take out the Hc of 71° 17' (Illustration 18) and enter it on the form. (Illustration 19)

illustration 16

Set H → Read W

Set W → Set H

Set Y → Read Az

...gnore sign of X)

...> 90 then Y = 180-X)

Set Az → Set Y

Set 0 → Read Hc

...nore sign of Y)

...rent assumed latitude)

...0)

Azimuth Rules

	North Latitude	
	0 < LHA < 180	< LH...
If X > 90°	Zn = 360 - Az	Zn = ...
If X < 90°	Zn = 180 + Az	Zn = ...

illustration 17

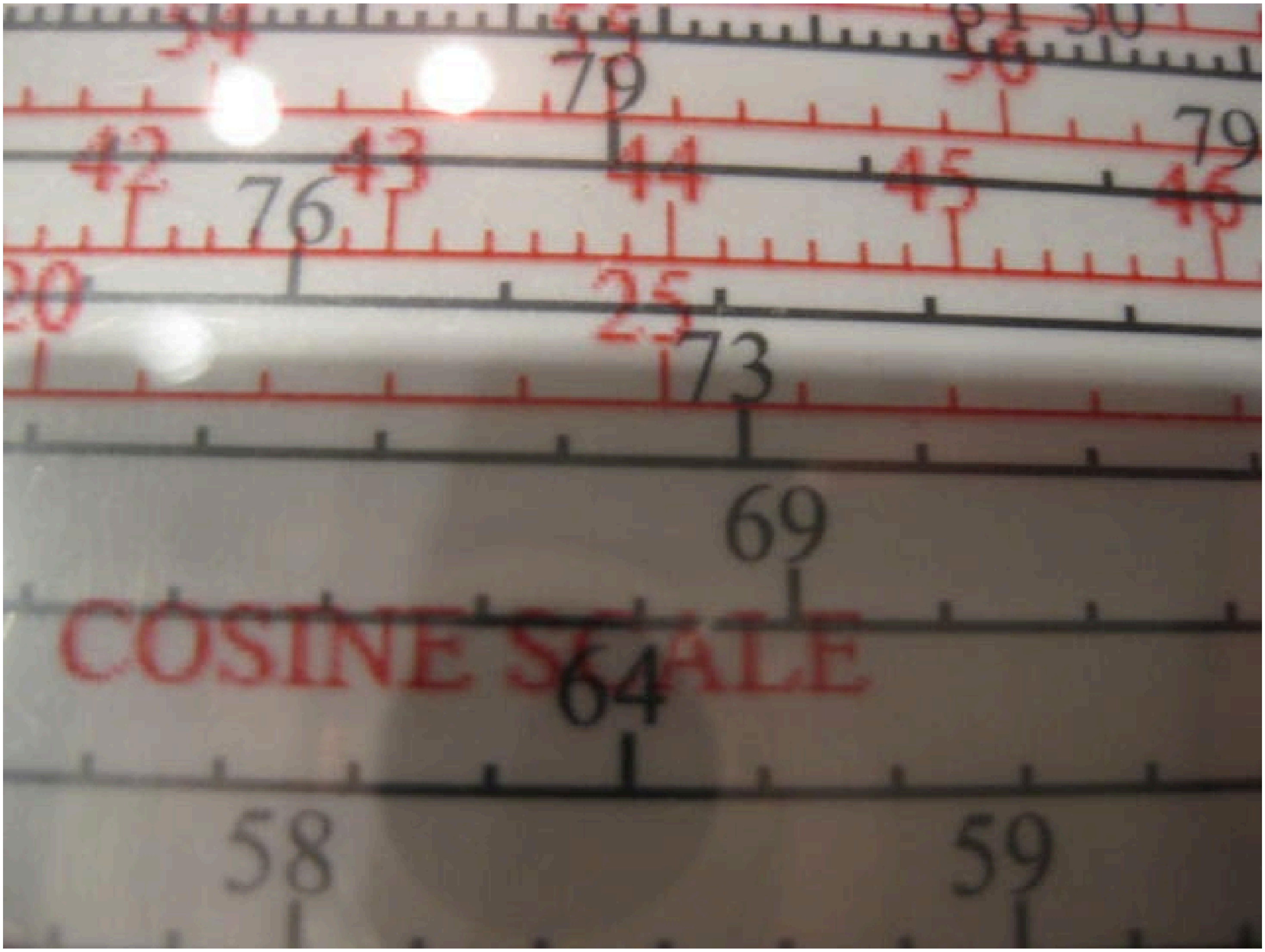


illustration 18

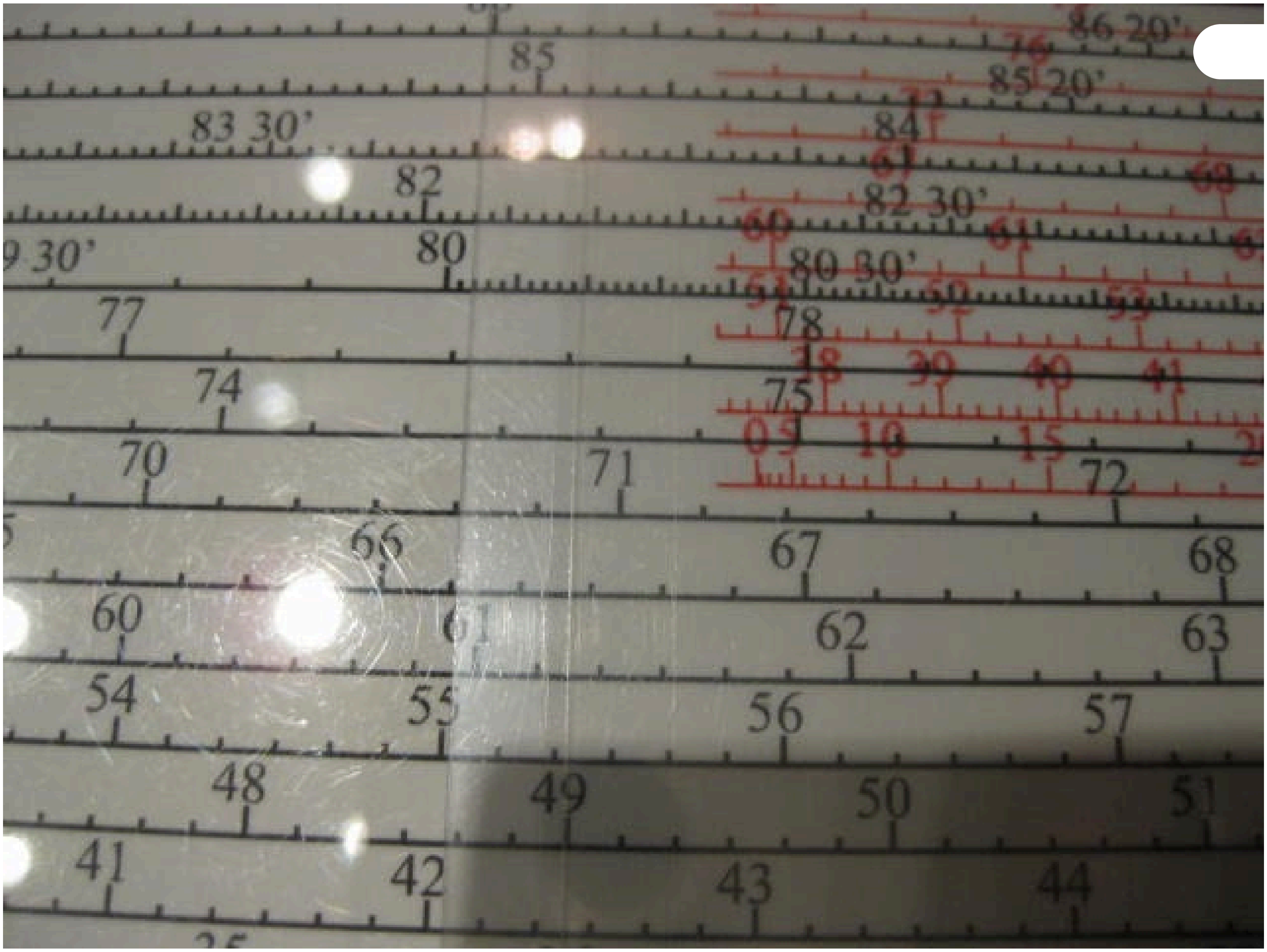
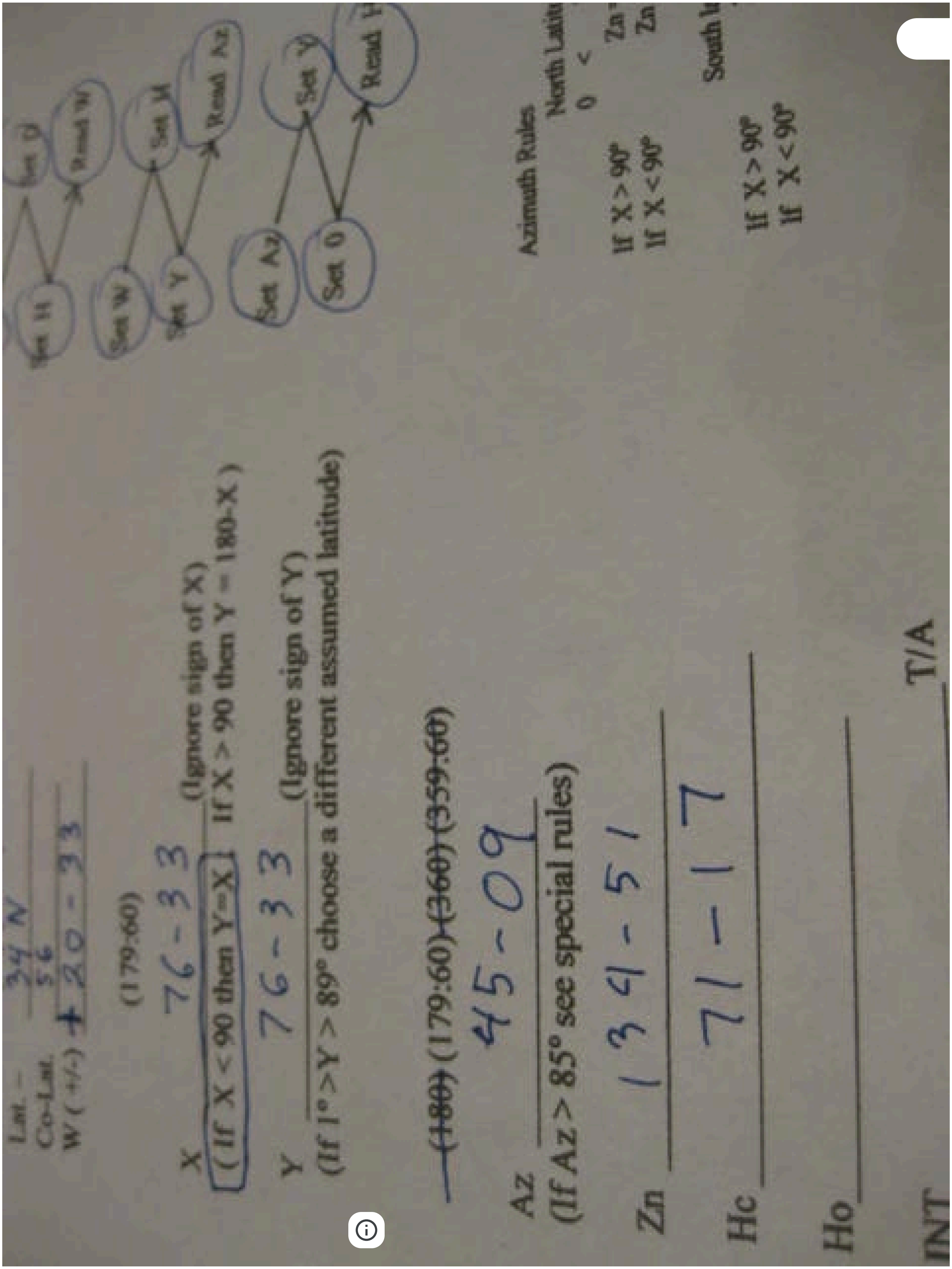


illustration 19



Lat. – 34 N
Co-Lat. 56
W (+/-) + 20 - 33

(179:60)

X 76 - 33 (Ignore sign of X)
(If X < 90 then Y=X) If X > 90 then Y = 180-X)

Y 76 - 33 (Ignore sign of Y)
(If 1° > Y > 89° choose a different assumed latitude)

~~(180)~~ (179:60) ~~(360)~~ ~~(359:60)~~

Az 45 - 09
(If Az > 85° see special rules)

Zn 134 - 51

Hc 71 - 17

Ho

INT T/A

Azimuth Rules
North Latit
0 <
If X > 90°
If X < 90°
Zn
Zn
South la
If X > 90°
If X < 90°

Comparing our result with H.O. 249 (see HO 249.pdf) we find the Hc in H.O 249 of 71° 17' and Zn of 135°, the same as with Bygrave example.

Link to H.O. 249 Excerpt:

H.O. 249

SPECIAL RULES

There are some unusual cases that require slightly different procedures and all of these special cases are described on the form. If "H" is less than 1° or greater than 89° simply assume a longitude to bring "H" within the range of the scales. The intercept will be longer but perfectly usable for practical navigation.

If the computed azimuth angle is greater than 85° the computed altitude will lose accuracy even though the azimuth is accurate because with azimuth angles in this range, even rounding the azimuth angle up or down one half minute can change the Hc by ten minutes. So you use the azimuth you have calculated but you compute altitude by interchanging declination and latitude and then doing the normal computation a second time. You discard the azimuth derived during this second computation of altitude and use the original azimuth.

When declination is less than one degree you can't begin the computation the normal way to find "W" because you have to start the process with declination on the cotangent scale and this scale doesn't extend below 1°. So in this case you just skip the computation of "W" and simply set "W" equal to declination. Using this method you arrive at an azimuth that is not exact but is a close approximation and in the worst case I have found the azimuth is still within 0.9° of the true azimuth but most are much closer. If the declination is less than one degree and the latitude is also less than one degree, follow this procedure and also assume a latitude equal to one degree. After you have computed the Az you then follow the same procedure discussed above for azimuths exceeding 85° by interchanging the latitude and declination and then computing Hc which will produce an exact value of Hc.

Another rare possibility is that "Y" will be less than one degree or that it will exceed 89° after adding "W" to co-declination so it won't fit on the scale. The simple way to handle this situation is to assume a different latitude so the "Y" does fit on the scale even though the resulting intercept is longer but still usable.

An extremely unlikely case (I only mention it to be complete) is that "W" exceeds the range of the cotangent scale, 89°15', so cannot be computed in the first step of the process. This can only happen when shooting one star, Kochab, which has a declination of 74° 07' north and then only if "H" exceeds 87° 20', an extremely unlikely event.

The attached form steps you through the computation and contains the rules for the special cases. The special cases are likely to come up only very rarely in practice.

The first rule for H < 1° or H > 89° only involves LHAs covering 5 degrees out of 360° (LHA in the ranges of 0 -1, 89-91, 269-271, and 359-360) so only occurs by chance very rarely and these can be avoided if sights are preplanned as is the normal procedure for flight navigation. In the worst case you have to change the time of the observation by four minutes.

Rule 3 covers the case when Y is less than 1 or when Y exceeds 89° which covers a range of three degrees out of a possible 180°

so is also very rare. Co-lat is in the range of 0º-90º and W is also in the same range so X comes in the range of 0º -180º. If X is ' than 89º then Y is also less than 89º. If X is greater than 91º then Y is also less than 89º. Only in the case of X between 89º an. 91º will Y exceed 89º. This situation can't be avoided in advance because you can't predict what the value of W will be but if it occurs then just assuming a latitude that differs at most by one degree solves the problem which will result in a longer intercept but one that is still usable.

The fourth rule deals with cases of bodies bearing almost directly east or west and this situation can be avoided by choosing a different body to shoot or, if only the sun is available, by waiting a few minutes to allow the azimuth to change out of this range.

The remaining situation covered by rule two (declinations less than one degree) concerns only bodies in the solar system since none of the navigational stars have declinations less than one degree. Obviously the most important body is the sun and its declination is between 1º north and 1º south for five days in March and again in September so this situation can't be avoided and this is the most important special case. The special rule handles it nicely and the Hc is completely accurate. The computed azimuth is an approximation but is never more than one degree different than the actual azimuth and is usually much closer. Since you can use your D.R. for the A.P. for this situation the intercepts can be kept short and the slight inaccuracy in the azimuth will not make a noticeable difference in the LOP.

EXPLANATION OF AZIMUTH RULES

In most situations there is no ambiguity as to which

quarter the Zn lies since you know the approximate direction you are

looking when you take the sight. The problem arises because the azimuth

angle is limited to the range of zero to 90 degrees and when the Zn is near

east or west the correct Zn might fall either

side of the line so there is an ambiguity in converting from azimuth

angle to Zn.

One easy rule to apply first is that if the declination is greater than

the latitude then the azimuth can never be in the opposite semicircle.

To generalize this rule, if the declination has

the same name as the latitude and the declination is greater than the

latitude, then you start with the direction of the elevated pole (the

nearer pole) when converting from azimuth angle to azimuth (Zn.)

The second rule to apply is that if the declinati contrary then the Zn

must be in the opposite semicircle. To generalize this rule, if the declination and the latitude have contrary names then you start with the

direction of the depressed pole (the further pole) when converting from

azimuth angle to Zn.

These two rules take care of most of the cases, especially for

navigators in low latitudes.

The remaining ambiguity concerns situations in which the declination is

the same name as the latitude but is less than the latitude. In this

situation the azimuth of the body will be both north and south of the

east - west line during part of each day. The rules compare "W"

with latitude to resolve this remaining ambiguity.

So, combining all three rules:

If the declination or if "W" is greater than the latitude then combine

the azimuth angle with the direction of the elevated pole.

If the declination name is contrary to the name of the latitude or if

"W" is less than the latitude then combine azimuth angle with the

direction of the depressed pole.

CHECKING YOUR COMPUTATIONS

An easy way to check the computation on a Bygrave is to do the same computation on a calculator since this allows you to check the intermediate steps.

Just use the standard Bygrave formulas in the three step process following along on the form I have posted.

First calculate co-latitude and save it in a memory in the calculator. If you are using a value for hour angle that is not a whole number of degrees you might want to make the conversion to decimal degrees and save it in a memory since it will we used twice. If you are using whole degrees then this step is not necessary.

Then you calculate "W" using the formula:

tan W = tan D / cos H

and sum it to the memory where you have save -latitude which is then X and then make any adjustment necessary to convert X to Y. (If you are just making trials you can avoid this step by your choice of the trial values.) There is no reason to store

W itself since it is not used again. You can then convert W to degree and minute format to compare with the Bygrave derived value.

Then you compute azimuth angle using the formula:

tan Az = (cos W / cos Y) x tan H.

If you want you can also convert Az to degree and minute format to compare with the Bygrave.

The last step is to calculate altitude with the formula:

tan Hc = cos Az x tan Y.

Then convert to degree and minute format to compare with the Bygrave result.

(When entering values in the format of degrees minutes seconds, change decimal minutes to seconds, 6 seconds per tenth of a minute, in your head before punching in the assumed latitude, declination and hour angle if necessary.)

Using whole degrees for declination, assumed latitude and hour angle, using a TI-30 with only 3 memory locations the key strokes are:

(co-latitude = 90 - Assimed latitude)

90

-

Assumed Lat

=

STO 1 (co-latitude stored in memory 1)

(tan W = tan D / cos H)

Declination

tan

/

H

cos

=

inv

tan (computed W)

SUM 1 (X now stored in memory 1)(change X to Y if necessary)

(tan Az = (cos W / cos Y) x tan H)

cos (of W from prior step)

/

RCL 1 (recalls Y from memory 1)

cos

x

H

tan

=

inv

tan (computed Azimuth angle)

(tan Hc = cos Az x tan Y)

cos (of Az from prior step)

x

RCL 1 (recalls Y from memory 1)

tan

=

inv

tan (computed altitude, Hc)

2nd

D.D - DMS (changes Hc in decimal degrees to degrees, minutes and seconds)

DONE

HINTS FOR MAKING YOUR BYGRAVE

Print the cotan scale on photo paper. Print the cosine scale on ink jet transparency film such as 3M made"Transparency film for Ink Jet Printers," stock number CG3480.

To ensure that the two scales are printed at exactly the same scale remember that the cotan scale spans two cycles while the cosine scale covers only one cycle. To test the proper scale check that the cosine scale covers exactly one half of the cotan scale. An easy way to do this is to place the right hand end of the cosine scale on any value you choose on any line of the cotan scale then check that the left hand end of cosine scale, one line up, lines up with the same value one line higher on the cotan scale. If they don't, then adjust your printer until they do. You can use plain paper for making this check, instead of the more expensive transparency sheet, by holding them up to a light.

To learn how to use the Bygrave, work the example given on this page, using the instructional form. For actual navigation, use the abbreviated forms but keep a copy of the instructional form with you to refer to as it contains the rules for doing the computation. Download a PDF copy of this page to use as a training manual.

For increased durability you can seal each scale in 3M "Self-Sealing Laminating Pouches," stock number LS854-5G or -10G. When sealing the transparency sheet you must be careful to work slowly to smooth out any bubbles since the transparency sheet can trap bubbles.

Links to the two scales:

COTAN scale

COSINE scale

Link to original Bygrave manual:

Bygrave Manual

Link to paper on Bygrave type slide rules:

Position Line Slide Rules by van Riet

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