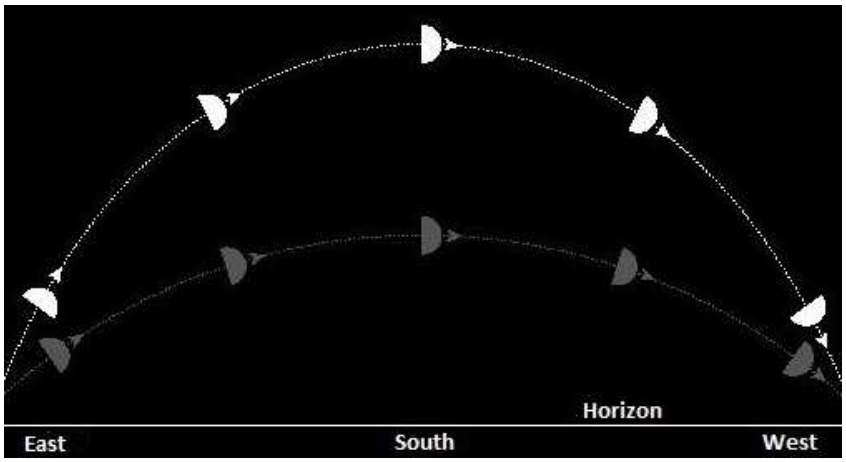
Tilt of the Moon during Meridian Passage

Introduction

There is an idea that during moon meridian passage the line connecting horns of the moon is always vertical (see schematic drawing below), in other words moon's tilt is zero.

To test this idea we will calculate the moon tilt angle at the moment of its meridian passage. Sake of simplicity we will choose Greenwich Meridian (LON = 0° 0.0′) as the meridian of observer. Year 2010 is chosen since I have a hard copy of *Nautical Almanac 2010*. Moon phases through the year 2010 can be traced either from Nautical Almanac or from different online sources[1]. We will consider the last quarter phase.



Theory

The formula for the moon tilt (angle with a vertical) is quite rare and can be found in a few references [1-3] along with its derivation [3]:

$$\tan(Tilt) = \frac{\cos(h_m)\sin(h_s) - sin(h_m)\cos(h_s)\cos(\Delta AZ)}{\cos(h_s)\sin(\Delta AZ)} \quad (1)$$

where[2]

$$\Delta AZ = AZ_m - AZ_s \quad (2)$$

[1] https://www.calendar-12.com/moon_phases/2010

[2] In reference [2] they specified the absolute value: $\Delta AZ = |AZ_m - AZ_s|$

Both the height (h) and azimuth (AZ) of the Sun and Moon can be calculated as standard:

$$h = sin^{-1}(\sin(LAT)\sin(\delta) + \cos(LAT)\cos(\delta)\cos(P)) \quad (3)$$

and

$$T^* = tan^{-1}\left(\frac{\sin(P)}{\cos(LAT)\tan(\delta) - \sin(LAT)\cos(P)}\right) \quad (4)$$

where P is local hour angle of celestial body (which is either E or W):

$$P = \begin{cases} LHA\ (if\ 0 < LHA < 180) & \boldsymbol{W} \\ 360 - LHA\ (if 180 < LHA < 360) & \boldsymbol{E} \end{cases} \quad (5)$$

and finally true azimuth is:

AZ =

T* / P	+	−
+	T*	T*+180°
−	T*+180°	T*+360°

Pay attention on signs entering the formulas (3-4):

LAT, δ		LON, P	
N	S	E	W
+	−	+	−

Calculations & Results

During Mer. Pass. of the moon it's azimuth (AZ_m) is either 180° (South) or 0° (North) depending on your *LAT* in relation to moon's declination δ_m.

The GP of the Sun (δ_m and GHA_m) and moon (δ_s and GHA_s) at the time of Meridian passage were accurately predicted by computer program *Navigator*[3] (see Table 1). The results of calculation for h_s AZ_s and h_m AZ_m are given in Appendix A in corresponding tables (Table 3 - Table 6) and were verified with the same computer program. These results we can plug into (1) to find moon's *Tilt*-angle. The final results for the calculated tilt angle we can find in Table 2. We can clearly see that the tilt angle is not zero.

To verify the calculated tilt angle we can use program Stellarium (see Appendix B). As we can see the magnitude of calculated lunar tilt agrees well with that shown by Stellarium.

Conclusion: Moon's tilt is not necessarily zero at meridian passage.

[3] http://www.tecepe.com.br/nav/default.htm

Table 1: Sun and Moon GP during Mer. Pass. of the last quarter Moon Phase through the year 2010.

Last Quarter UTC date ◑	Mer. Pass. UTC time ◑	δ_s	GHA_s	δ_m	GHA_m
Jan. 7, Th	05:37	22° 23.0′ S	262° 43.1′	10° 25.9′ S	0° 0.0′
Feb. 6, Sa	06:02	15° 38.4′ S	266° 59.6′	22° 15.3′ S	0° 0.0′
Mar. 7, Su	05:40	05° 18.8′ S	262° 13.7′	25° 17.7′S	0° 0.0′
Apr. 6, Tu	06:06	06° 25.4′ N	270° 52.6′	22° 47.1′ S	0° 0.0′
May 6, Th	06:17	16° 31.1′ N	275° 05.6′	13° 13.7′ S	0° 0.0′
Jun 5, Sa	06:15	22° 32.3′ N	274° 08.6′	0° 16.6′ S	0° 0.0′
Jul 4, Su	05:32	22° 52.9′ N	261° 54.8′	07° 59.5′ N	0° 0.0′
Aug. 3, Tu	05:42	17° 31.4′ N	263° 56.8′	19° 29.4′ N	0° 0.0′
Sep. 1, We	05:19	08° 18.9′ N	259° 43.3′	23° 43.8′ N	0° 0.0′
Oct. 1, Fr	06:00	03° 09.5′ S	272° 33.5′	22° 57.8′ N	0° 0.0′
Oct. 30, Sa	05:45	13° 45.0′ S	270° 19.8′	17° 07.5′ N	0° 0.0′
Nov. 28, Su	05:25	21° 17.0′ S	264° 17.6′	08° 18.3′ N	0° 0.0′
Dec. 28, Tu	05:52	23° 17.1′ S	267° 39.5′	07° 42.4′ S	0° 0.0′

Table 2: Lunar tilt (°) calculated for last moon quarter (◑) for several latitudes during the year 2010. NaN means that moon is below horizon ($h_m < 0$). Tilt angle is rounded to full degree.

UTC Date ↓ & Time / LAT→		−90° S	−60° S	−30° S	0°	+30° N	+60° N	+90° N
Jan. 7, Th	05:37	-23	-23	-23	-23	-23	-23	NaN
Feb. 6, Sa	06:02	-16	-16	-16	-16	-16	-16	NaN
Mar. 7, Su	05:40	-8	-8	-8	-8	-8	-8	NaN
Apr. 6, Tu	06:06	6	6	6	6	6	6	NaN
May 6, Th	06:17	17	17	17	17	17	17	NaN
Jun 5, Sa	06:15	23	23	23	23	23	23	NaN
Jul 4, Su	05:32	NaN	24	24	24	24	24	24
Aug. 3, Tu	05:42	NaN	19	19	19	19	19	19
Sep. 1, We	05:19	NaN	12	12	12	12	12	12
Oct. 1, Fr	06:00	NaN	-4	-4	-4	-4	-4	-4
Oct. 30, Sa	05:45	NaN	-13	-13	-13	-13	-13	-13
Nov. 28, Su	05:25	NaN	-20	-20	-20	-20	-20	-20
Dec. 28, Tu	05:52	-23	-23	-23	-23	-23	-23	NaN

References:

[1] S. van der Werf, Nederlands tijdschrift voor Natuurkunde *april 2000*

https://sites.google.com/site/siebrenvanderwerf/files-dutch/2000_NZeffect_NTvN.pdf?attredirects=0&d=1

[2] A. K. Mayers-Beaghton et.a. “The Moon Tilt Illusion”.

https://hrcak.srce.hr/file/197562

[3] S. van der Werf, Cornelis Douwes, 162 “Waarom de maan scheef en waarom zien wij steeds dezelfde kant”.

https://sites.google.com/site/siebrenvanderwerf/files-dutch/2005_ScheveMaan_CD162.pdf?attredirects=0&d=1

Appendix A

Table 3: h_s (°) during moon meridian passage (2010).

UTC Date ↓ & Time / LAT→		−90° S	−60° S	−30° S	0°	+30° N	+60° N	+90° N
Jan. 7, Th	05:37	22.383	15.734	5.100	-6.730	-16.971	-22.853	-22.383
Feb. 6, Sa	06:02	15.640	12.017	5.224	-2.895	-10.284	-14.994	-15.640
Mar. 7, Su	05:40	5.313	0.737	-4.031	-7.738	-9.375	-8.483	-5.313
Apr. 6, Tu	06:06	-6.423	-5.122	-2.451	0.871	3.962	5.997	6.423
May 6, Th	06:17	-16.518	-11.751	-3.924	4.882	12.466	16.785	16.518
Jun 5, Sa	06:15	-22.538	-17.372	-7.692	3.826	14.444	21.427	22.538
Jul 4, Su	05:32	-22.881	-23.674	-17.857	-7.446	4.713	15.779	22.881
Aug. 3, Tu	05:42	-17.523	-18.121	-13.747	-5.771	3.638	12.150	17.523
Sep. 1, We	05:19	-8.315	-12.328	-13.015	-10.169	-4.622	2.118	8.315
Oct. 1, Fr	06:00	3.158	4.013	3.792	2.554	0.633	-1.457	-3.158
Oct. 30, Sa	05:45	13.750	12.042	7.104	0.320	-6.545	-11.715	-13.750
Nov. 28, Su	05:25	21.283	15.546	5.811	-5.316	-15.172	-21.141	-21.283
Dec. 28, Tu	05:52	23.285	18.879	9.505	-2.150	-13.306	-21.168	-23.285

Table 4: AZ_s (°) during moon meridian passage (2010).

UTC Date ↓ & Time / LAT→		−90° S	−60° S	−30° S	0°	+30° N	+60° N	+90° N
Jan. 7, Th	05:37	97.282	107.654	112.950	112.547	106.471	95.536	82.718
Feb. 6, Sa	06:02	93.007	100.518	105.059	105.661	102.217	95.409	86.993
Mar. 7, Su	05:40	97.772	99.376	98.504	95.362	90.748	85.924	82.228
Apr. 6, Tu	06:06	89.123	86.022	83.997	83.576	84.865	87.535	90.877
May 6, Th	06:17	84.907	77.262	73.174	73.420	77.960	85.900	95.093
Jun 5, Sa	06:15	85.857	74.849	68.369	67.409	72.041	81.732	94.143
Jul 4, Su	05:32	98.087	84.852	73.399	66.913	66.240	71.418	81.913
Aug. 3, Tu	05:42	96.053	86.171	77.486	72.384	71.840	75.932	83.947
Sep. 1, We	05:19	100.278	94.732	87.826	81.551	77.631	76.976	79.722
Oct. 1, Fr	06:00	87.442	89.365	91.460	93.161	94.014	93.794	92.558
Oct. 30, Sa	05:45	89.670	96.694	101.807	103.750	102.123	97.257	90.330
Nov. 28, Su	05:25	95.707	105.764	111.256	111.380	106.123	96.232	84.293
Dec. 28, Tu	05:52	92.342	104.078	111.478	113.302	109.421	100.201	87.658

Table 5: h_m (°) during moon meridian passage (2010).

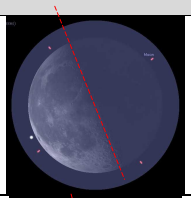
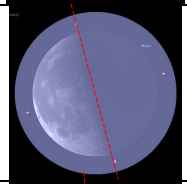
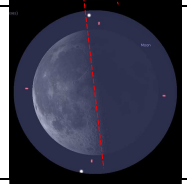
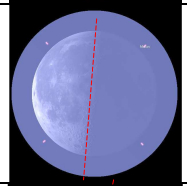
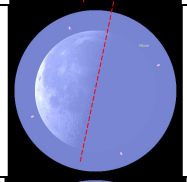
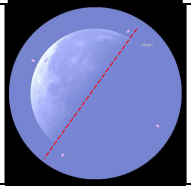
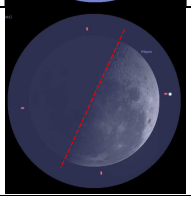
UTC Date ↓ & Time / LAT→		−90° S	−60° S	−30° S	0°	+30° N	+60° N	+90° N
Jan. 7, Th	05:37	10.431	40.431	70.431	79.568	49.568	19.568	-10.431
Feb. 6, Sa	06:02	22.255	52.255	82.255	67.745	37.745	7.745	-22.255
Mar. 7, Su	05:40	25.295	55.295	85.295	64.705	34.705	4.705	-25.295
Apr. 6, Tu	06:06	22.785	52.785	82.785	67.215	37.215	7.215	-22.785
May 6, Th	06:17	13.228	43.228	73.228	76.771	46.771	16.771	-13.228
Jun 5, Sa	06:15	0.276	30.276	60.276	89.723	59.723	29.723	-0.276
Jul 4, Su	05:32	-7.991	22.008	52.008	82.008	67.991	37.991	7.991
Aug. 3, Tu	05:42	-19.490	10.510	40.510	70.510	79.490	49.490	19.490
Sep. 1, We	05:19	-23.730	6.270	36.270	66.270	83.730	53.730	23.730
Oct. 1, Fr	06:00	-22.963	7.036	37.036	67.036	82.963	52.963	22.963
Oct. 30, Sa	05:45	-17.125	12.875	42.875	72.875	77.125	47.125	17.125
Nov. 28, Su	05:25	-8.305	21.695	51.695	81.695	68.305	38.305	8.305
Dec. 28, Tu	05:52	7.706	37.706	67.706	82.293	52.293	22.293	-7.706

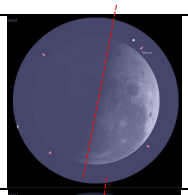
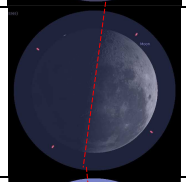
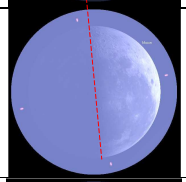
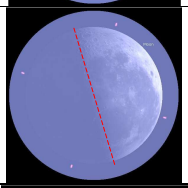
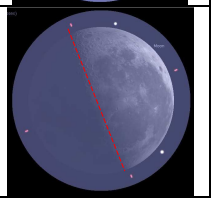
Table 6: AZ_m (°) during moon meridian passage (2010).

UTC Date ↓ & Time / LAT→		−90° S	−60° S	−30° S	0°	+30° N	+60° N	+90° N
Jan. 7, Th	05:37	0	0	0	180	180	180	180
Feb. 6, Sa	06:02	0	0	0	180	180	180	180
Mar. 7, Su	05:40	0	0	0	180	180	180	180
Apr. 6, Tu	06:06	0	0	0	180	180	180	180
May 6, Th	06:17	0	0	0	180	180	180	180
Jun 5, Sa	06:15	0	0	0	180	180	180	180
Jul 4, Su	05:32	0	0	0	0	180	180	180
Aug. 3, Tu	05:42	0	0	0	0	180	180	180
Sep. 1, We	05:19	0	0	0	0	180	180	180
Oct. 1, Fr	06:00	0	0	0	0	180	180	180
Oct. 30, Sa	05:45	0	0	0	0	180	180	180
Nov. 28, Su	05:25	0	0	0	0	180	180	180
Dec. 28, Tu	05:52	0	0	0	180	180	180	180

Appendix B

Comparison of calculated tilt with Stellarium visualization (screenshots). The chosen coordinates of observer are: LAT = 0° , LON = 0°. The red dotted line is added as a guidance for an eye.

Last Quarter UTC date ◑	Mer. Pass. UTC time ◑	Calculated Tilt	Stellarium view
Jan. 7, Th	05:37	-23	
Feb. 6, Sa	06:02	-16	
Mar. 7, Su	05:40	-8	
Apr. 6, Tu	06:06	6	
May 6, Th	06:17	17	
Jun 5, Sa	06:15	23	
Jul 4, Su	05:32	24	

Aug. 3, Tu	05:42	19	
Sep. 1, We	05:19	12	
Oct. 1, Fr	06:00	-4	
Oct. 30, Sa	05:45	-13	
Nov. 28, Su	5:25	-20	
Dec. 28, Tu	05:52	-23