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**REPORT No. 198**

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**ASTRONOMICAL METHODS IN AERIAL  
NAVIGATION**

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TABLE II

| Altitude of star | Correction—add algebraically to altitude of Polaris |      |           |      |          |      |        |      |         |      |        |      |  |
|------------------|---|------|-----------|------|----------|------|--------|------|---------|------|--------|------|--|
|                  | Arcturus  |      | Betelgeux |      | Denebola |      | Pollux |      | Regulus |      | Sirius |      |  |
|                  | East  | West | East      | West | East     | West | East   | West | East    | West | East   | West |  |
| 6                |   |      |           |      |          |      |        |      |         |      |        |      |  |
| 8                |   |      |           |      |          |      |        |      |         |      |        |      |  |
| 10               |   |      | -65       | +61  | -36      | +24  |        |      |         | -53  | +50    |      |  |
| 12               |   |      | -67       | +59  | -32      | +28  |        |      |         | -56  | +53    |      |  |
| 14               | +12   |      | -67       | +58  | -29      | +31  |        |      |         | -53  | +55    |      |  |
| 16               |   |      | -68       | +56  | -25      | +35  |        |      |         | -51  | +57    |      |  |
| 18               | +16   | -11  | -69       | +54  | -22      | +38  | -69    | +69  | -45     | +61  |        |      |  |
| 20               | +19   | -8   | -69       | +51  | -18      | +41  | -69    | +69  | -43     | +62  |        |      |  |
| 22               | +23   | -4   | -69       | +48  | -15      | +44  | -69    | +69  | -39     | +64  |        |      |  |
| 24               | +26   | 0    | -69       | +45  | -11      | +46  | -69    | +69  | -36     | +65  |        |      |  |
| 26               | +30   | +4   | -69       | +42  | -7       | +49  | -68    | +69  | -33     | +66  |        |      |  |
| 28               | +33   | +7   | -69       | +39  | -3       | +52  | -68    | +69  | -30     | +67  | +16    | -33  |  |
| 30               | +37   | +11  | -68       |      | 0        | +54  | -67    | +69  | -26     | +68  | +12    | -37  |  |
| 32               | +40   | +15  | -67       |      | +4       | +57  | -66    | +68  | -22     | +69  | +8     | -40  |  |
| 34               | +43   | +19  |           |      | +8       | +59  | -65    | +67  |         | +69  | +4     | -43  |  |
| 36               | +45   | +23  |           |      |          |      | -64    | +67  |         |      | +1     | -45  |  |
| 38               |   |      |           |      |          |      |        |      |         |      |        |      |  |
| 40               |   |      |           |      |          |      |        |      |         |      |        |      |  |
| 42               |   |      |           |      |          |      |        |      |         |      |        |      |  |
| 44               |   |      |           |      |          |      |        |      |         |      |        |      |  |
| 46               |   |      |           |      |          |      |        |      |         |      |        |      |  |
| 48               |   |      |           |      |          |      |        |      |         |      |        |      |  |
| 50               |   |      |           |      |          |      |        |      |         |      |        |      |  |
| 52               |   |      |           |      |          |      |        |      |         |      |        |      |  |
| 54               |   |      |           |      |          |      |        |      |         |      |        |      |  |

Example:  
 Altitude of Polaris.....  $h=49^{\circ} 16'$   
 Altitude of Regulus (in west)=15°.....  $Dh= +60'$   
 Latitude.....  $L=50^{\circ} 16'$

3. SOLUTIONS BY MEANS OF NOMOGRAMS AND DIAGRAMS

The haversine formulas for zenith distance and azimuth may be written in the form of the straight line equation

$$y = b + mx$$

as follows:

$$\begin{aligned} \text{hav } z &= \text{hav } (L - D) + \{ \text{hav } (180^{\circ} - (L + D)) - \text{hav } (L - D) \} \text{hav } t \\ \text{hav } (90^{\circ} \pm D) &= \text{hav } (L - h) + \{ \text{hav } (180^{\circ} - (L + h)) - \text{hav } (L - h) \} \text{hav } A \end{aligned}$$

These formulas may be solved on a nomogram which was first published in 1899 by D'Ocagne in his "Traité de Nomographie."

Let the sides of a square (fig. 20) be divided according to the haversines of angles from 0° to 180° and the corresponding points on opposite sides be connected by straight lines. The left-hand side is numbered from bottom to top in degrees and the right-hand side in the reverse order, likewise in degrees. The upper and lower edges are numbered from left to right in degrees and in time units, respectively.

To find zenith distance, enter the diagram at the left with  $L - D$  and at the right with  $L + D$ , and join these points by a straight line. Find the hour angle ( $t$ ) at top or bottom and follow along the vertical to the straight line already determined. Then pass horizontally to the left-hand scale and read off the zenith distance.

To find the azimuth, the left and right hand scales are entered with  $L - h$  and  $L + h$ , respectively, and the straight line drawn as before. The left-hand scale is then entered with  $90^{\circ} - D$  and the horizontal followed to the straight line. The vertical from this point is followed to the top scale from which the azimuth is read.

If the altitude, or zenith distance, be known, the hour angle may be determined by reversing the last two steps in the procedure for finding the zenith distance.

The nomogram is simple and convenient, but unfortunately has one great disadvantage. If it be made about 15 inches square, an accuracy in zenith distance of about 7' of arc is possible. An accuracy of at least 2' or 3' of arc is desirable for aerial navigation, and for this accuracy the diagram would be inconveniently large. Furthermore, it can not be subdivided, but must be used as a whole. This nomogram has also been published by Wimperis under the name of "Spherical triangle nomogram,"<sup>26</sup> and by Littlehales as the "Altitude, azimuth, and hour-angle diagram."<sup>27</sup>

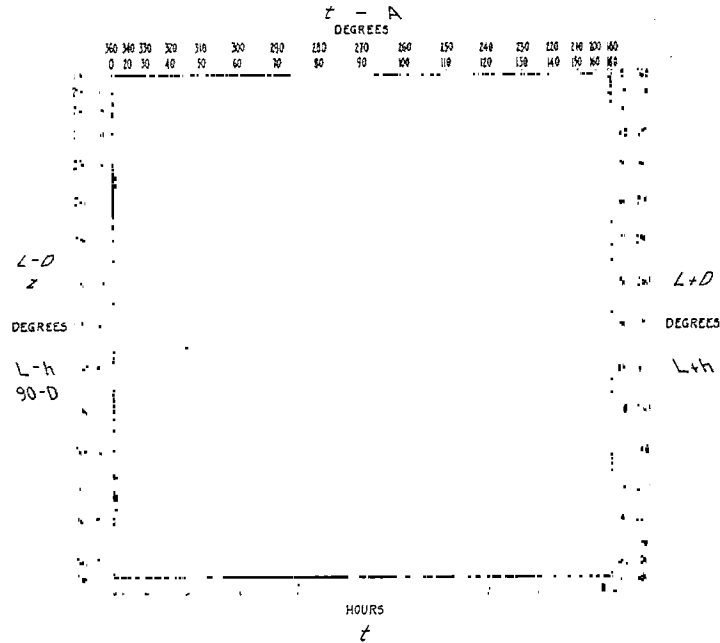


FIG. 20.—D'Ocagne nomogram

If the astronomical triangle be divided into two right triangles, as shown in Figure 19, the following formulas express the relations between the various parts:

$$\begin{aligned} \cos (90^\circ + D) &= \cos \beta \cos \alpha \\ \cot t &= \cot \alpha \sin \beta \\ \text{and } \cos A &= \cos \{(90^\circ - L) - \beta\} \cos \alpha \\ \cot z &= \cot \alpha \sin \{(90^\circ - L) - \beta\} \end{aligned}$$

It is evident that the two sets of equations are of the same form, and thus a graphical representation of the first set will also represent the second set. Taking  $\beta$  as abscissa and  $\alpha$  as ordinate and plotting values of  $D$  and  $t$ , the diagram in Figure 21 will be obtained, by means of which the astronomical triangle can be solved in two steps. The procedure is as follows: Find the intersection of the curves corresponding to the given declination ( $D$ ) and hour angle ( $t$ ), and read the coordinates  $\alpha$  and  $\beta$  of this point. Determine a new value of  $\beta$  according to the relation  $\beta' = \{(90^\circ - L) - \beta\}$ . Locate the point whose coordinates are  $\alpha$  and  $\beta'$ . Find the azimuth ( $A$ ) and zenith distance ( $z$ ) corresponding to this point by means of the declination and hour-angle curves, respectively, interpolating between the curves if necessary. Since the diagram represents only an octant of a spherical surface, provision must be made for the solution of all astronomical triangles which may be met with in practice. This may be done in either of two ways. The diagram may be enlarged to show three octants, or additional graduations may be applied to the curves and scales, with appropriate rules for each case. The diagram was published by Favé and Rollet de L'Isle in 1892.<sup>28</sup>

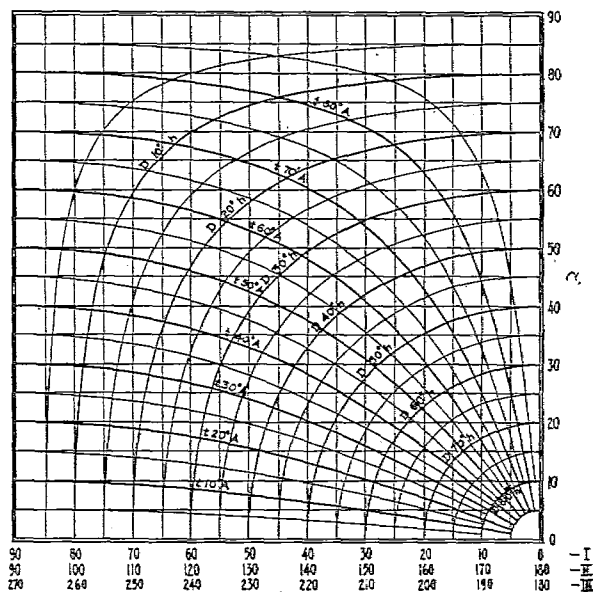
<sup>26</sup> H. F. Wimperis, loc. cit.

<sup>27</sup> Proceedings U. S. Naval Institute, November, 1917. "Altitude, azimuth, and hour-angle diagram," by G. W. Littlehales.

<sup>28</sup> Annales Hydrographiques, 1892. "Abaque pour la Détermination du Point à la Mer," by Favé and Rollet de L'Isle.

Numerical operations are reduced by the use of this diagram to a single addition or subtraction, and a complete solution can be obtained in two or three minutes' time. Deformations of the paper have no effect on the accuracy. A scale of 2 millimeters = 10' of arc is required for an accuracy of 1', which means a diagram about 40 inches square. This may be divided into sheets of convenient size, but the chances for error are thereby increased. The necessity for a set of rules to determine the procedure to be employed is also a disadvantage.

Consider a sphere on which are drawn the meridians and parallels properly numbered, and on this sphere draw the astronomical triangle ZPS. (See fig. 1.) Let this triangle be moved over the surface of the sphere in such a manner that the side ZP will always remain on the same meridian and until the vertex Z coincides with the original position of P. Or, in other words, rotate the triangle over the surface of the sphere about an axis through the equator and



- Case 1. L and D same name -  $t < 90^\circ$   
Read  $\beta$  on scale II  
Azimuth from upper pole, E or W as star is E or W of meridian
- Case 2. L and D same name -  $t > 90^\circ$   
Read  $(180^\circ - t)$  instead of  $t$   
Read  $\beta$  on scale I  
Azimuth from upper pole, E or W as star is E or W of meridian
- Case 3. L and D opposite names  
Read  $\beta$  on scale I  
Azimuth from lower pole, E or W as star is E or W of meridian

FIG. 21.—Favé diagram

perpendicular to the meridian ZP. The side ZS, which is the zenith distance, will now coincide with some meridian, and its length may be read off directly. The angle SZP, which is the azimuth, will have its vertex at P and its sides along two meridians. Its value may thus be determined directly from the graduated circles. If an accuracy of a minute of arc is required, such a sphere would be inconveniently large, but any true projection of the sphere could be used instead.

The stereographic meridian projection has been employed by Littlehales,<sup>29</sup> of the United States Hydrographic Office, for this purpose. The projection is for a 12-foot sphere and, since it is very large, has been subdivided into 368 overlapping sheets. These, together with a key diagram, are bound in one volume, which is rather bulky and not very convenient for aircraft use. An accuracy of about 1' or 2' of arc is obtainable in practically all cases, and the procedure is rapid and fairly simple.

<sup>29</sup> "Altitude, Azimuth, and Geographical Position," by G. W. Littlehales. Published by Lippincott.

The Mercator projection has been used in a similar manner by Commander Veater,<sup>30</sup> of the British Navy. The observer's meridian is taken as the equator of the projection, and rotation on the sphere becomes translation parallel to the observer's meridian on the projection. The projection is divided into sections of a convenient size and a key diagram is furnished.

The nomograms and diagrams which have thus far been described are perfectly general in application and may be used for observations of the sun or moon as well as the stars and planets. The changes in the right ascensions and declinations of the fixed stars are very small, and for the purposes of aerial navigation these stellar coordinates may be considered as constants for periods of several years. Thus a variety of diagrams is possible for the reduction of observations of the fixed stars. A few of these are here described for illustration and because under certain conditions they may prove to be of value to the navigator.

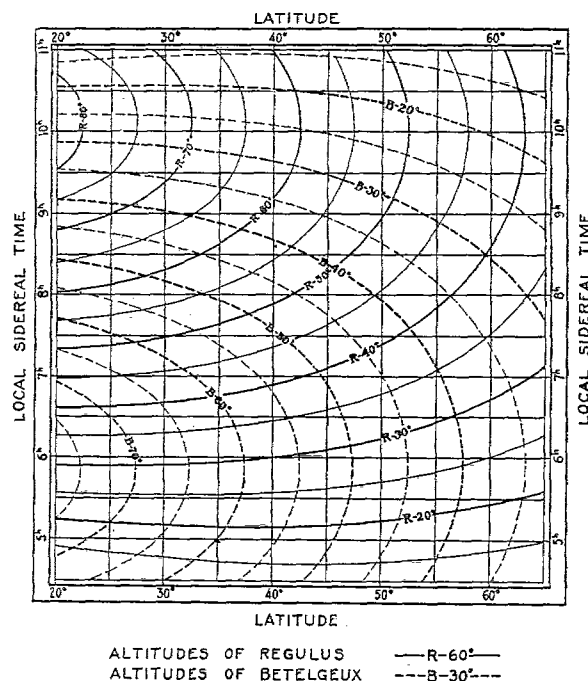


Fig. 22.—Two star diagram

Local sidereal time and latitude are completely determined by the simultaneous altitudes of two stars. If latitudes be taken as abscissæ and local sidereal times as ordinates, curves showing the simultaneous altitudes of two given stars may be constructed, as in Figure 22.<sup>31</sup> From this diagram, and the Greenwich sidereal time as indicated by his chronometer, the navigator can determine his position. No tables are required; even the Nautical Almanac is unnecessary. All computations, including the calculation of hour angles, are eliminated, save the simple addition and subtraction required to find the longitude from the local and Greenwich sidereal times. Suppose, for example, that on a flight in the United States the following observations were made:

Altitude of Regulus, 47°.

Altitude of Betelgeux, 52°.

Greenwich sidereal time, 13 hours 20 minutes.

Find on the diagram the point corresponding to the given altitudes, interpolating between curves where necessary. Read the abscissa of this point giving a latitude of 39° (in round numbers), and the ordinate giving a local sidereal time of 7 hours 30 minutes. Then subtract the local from the Greenwich sidereal time, giving a longitude of 5 hours 50 minutes, or 87°

<sup>30</sup> H. E. Wimperis, loc. cit.

<sup>31</sup> Proposed by the author.

30' west. The position, a point in southeastern Illinois, can now be plotted on the map. If the sky happens to be overcast, and only one star is visible, two latitudes (or local sidereal times) may be assumed and the corresponding coordinates found from the diagram. The two points thus found will determine a position line on which the aircraft must be located.

For practical purposes such diagrams should have an accuracy of 2' or 3' of arc. This may be attained by a scale of about 1 inch to 1° of latitude and correspondingly for the local sidereal time. Then a latitude range of 20° (covering practically all of the United States) would require a diagram 20 inches in width. Using five or six pairs of stars suitably chosen, diagrams would be available for all nights of the year over this latitude range. These might be conveniently placed on one sheet of paper placed on rollers in a case similar to the usual form of map case.

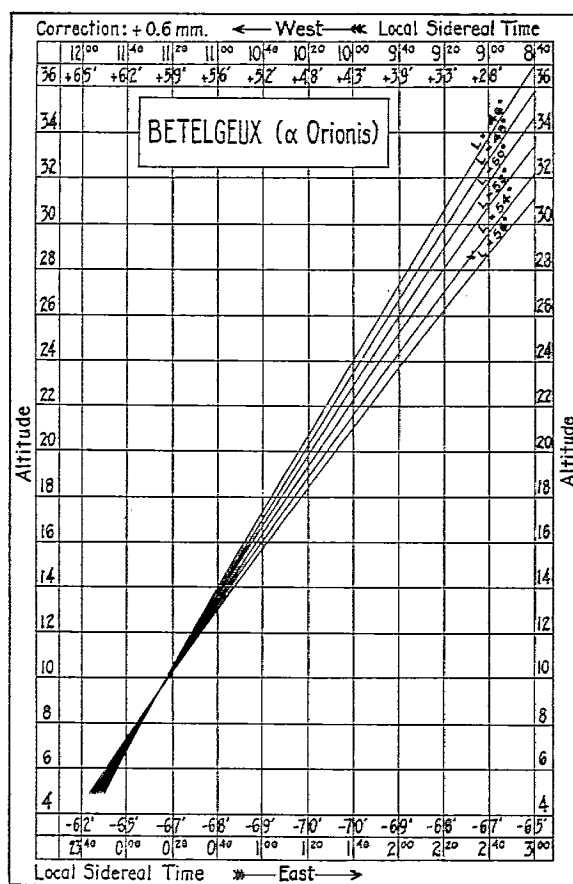


FIG. 23.—Leick diagram

The advantages of this form of diagram are the complete elimination of computations and supplementary tables, and the simple and very rapid procedure. The chief disadvantage lies in the fact that the diagram can be used only at night. Where space is limited, the diagram can be subdivided to cover a range of 10° of latitude in each roll, having a width of about 10 inches, which is not too large for even the smallest airplanes on which its use would be necessary or desirable.

The diagram shown in Figure 23 was published by Leick<sup>32</sup> in 1911. Given the altitudes of Polaris and one other star, local sidereal time, and hence longitude and latitude may be determined. An example (given by Leick) will illustrate the use of the diagram.

<sup>32</sup> Annalen der Hydrographie und Maritimen Meteorologie, Vol. 39, No. 6, June, 1911. "Ein Verfahren zur Auswertung astronomische Ortsbestimmungen im Ballon bei Nacht," by Dr. A. Leick.

Given:

Altitude of Polaris =  $h_1 = 55^\circ 35'$ .

Altitude of Betelgeux =  $h_2 = 20^\circ 43'$  (east of meridian).

Greenwich sidereal time = 0 hours 19.8 minutes.

Along the left or right hand scales find  $h_2 = 20^\circ 43'$ . Follow the horizontal to the latitude curve  $L = h_1 = 55^\circ 35'$ , and from this point pass vertically downwards and find  $Dh = -70'$ . Then the latitude of the observer =  $L = h_1 + Dh = 54^\circ 25'$ . Now enter the left or right hand scales as before, with  $h_2$ , and pass horizontally to the latitude curve  $L = 54^\circ 25'$ , and from this point downward to the time scale, from which find the local sidereal time  $t_x = 1$  hour 32.8 minutes. Then the longitude of the observer =  $\lambda = t_x - t_0 = 1$  hour 13 minutes or  $18^\circ 15'$ . It is to be noticed that readings of the upper scales require corrections. The point on the scale as found by the solution should be transferred 0.6 millimeter in the direction of the arrow, and the scale reading of this new point is the required result.

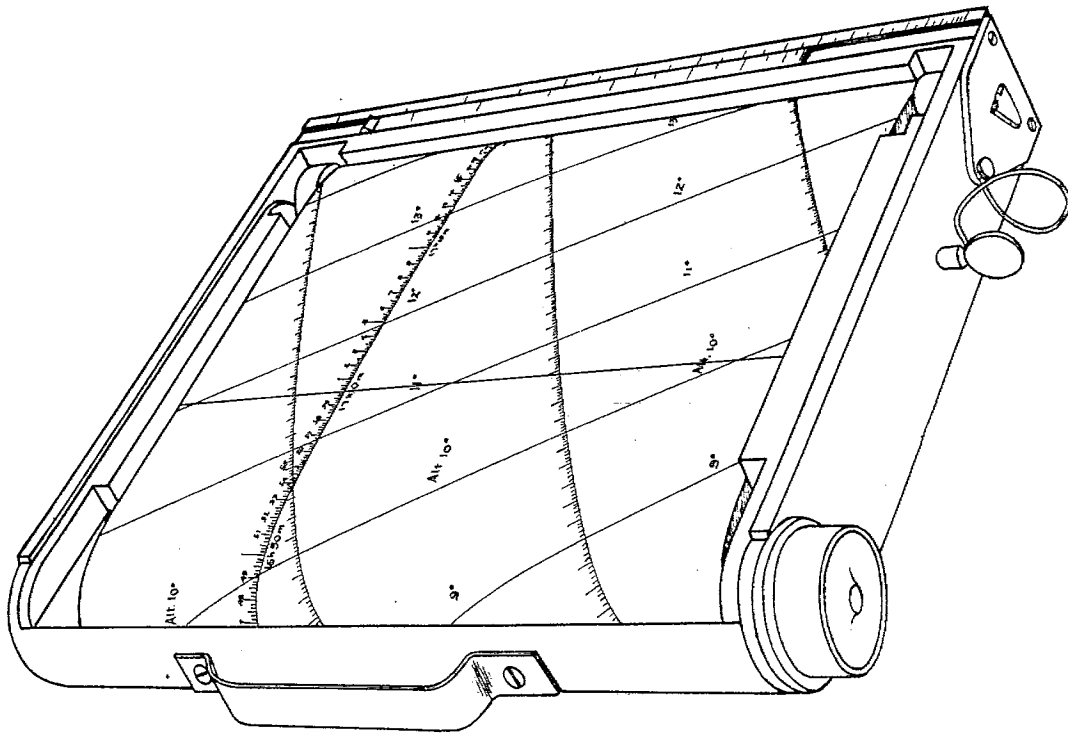


FIG. 24.—Baker navigating machine

#### 4. GRAPHICAL SOLUTIONS.

At any instant the curves of equal altitude for a given star may be considered as a system of circles about the substellar point as center. These circles, together with their great circle orthogonals, may be imagined as a network moving over the surface of the earth from east to west with the change of time and slowly north or south according to the changes in declination of the star. Now if the navigator has a map and a transparent sheet on which are plotted the curves of equal altitude (to the same projection and scale as the map), he can locate the sheet of curves on the map according to the geographical position of the star, and the curve corresponding to his observed altitude will then be his position line. This principle has been used by Commander Baker<sup>33</sup> of the British Navy in the construction of the Baker navigating machine. (Fig. 24.) The navigating machine is composed of a case holding a Mercator map and a transparent diagram mounted on rollers in such a manner that it may be moved over the surface of

<sup>33</sup> Transactions of the Royal Aeronautical Society, No. 2, London. "Position fixing in aircraft during long-distance flights over the sea," by T. Y. Baker and L. N. G. Filon.

the map. On the diagram are drawn the curves of equal altitudes and their great circle orthogonals. The westward motion of these curves is reproduced by rolling the diagram over the map, a line on the diagram being made to follow a certain parallel on the map. A time scale on the diagram can be set against some meridian on the map, thus locating the diagram in the proper position. The north and south movement of the curves due to changes in the declination of the star can not be reproduced in this manner, since the spacing and shape of the curves would alter in the Mercator projection.

A single diagram suffices for a fixed star. A number of star diagrams are placed on the same sheet, such parts as do not allow of good cuts being omitted. Thirteen diagrams are furnished for the sun, constructed at  $4^\circ$  intervals in declination from  $24^\circ$  south to  $24^\circ$  north. The position line is obtained by inserting the proper diagram in the machine and setting the time scale against the proper meridian. Then the curve of altitude, or the necessary portion of it, corresponding to the observed altitude is transferred to the map by the use of a bit of carbon paper. If the declination of the star differs materially from that of the diagram, which in general is the case for observations of the sun, a correction must be made. If the difference is less than  $2^\circ$ , a first order correction is sufficient, as the error will never be more than a minute or two. This correction is computed with the aid of a special slide rule attached to the machine.

When the St. Hilaire method is used with Mercator charts, the intercept, or the distance between the assumed or dead reckoning position and the position line, must be small in order to avoid errors due to the form of projection. About 1901 Favé<sup>34</sup> developed a method which is essentially an extension of the St. Hilaire method, the restriction being avoided by the use of the stereographic projection. A map covering about  $30^\circ$  of the earth's surface is employed, the central point of the map being also the center of projection. (See fig. 25.) This central point is used in place of a dead reckoning or an assumed position. Tables of altitudes and azimuths, computed for the central point, are furnished to eliminate computations by the navigator.

The map is constructed on a transparent sheet of paper or celluloid. Meridians and parallels are shown at 10-minute intervals. (Figure 25 has been simplified for clearness.) A large circle about the central point is drawn and graduated in degrees to represent azimuths. On a separate sheet are plotted the curves of equal altitude, at intervals of degrees or convenient submultiples, the curves being shown as they would be projected at the center of the map. A straight line, the azimuth index, is drawn through the centers of the arcs.

If the observed altitude is the same as the tabulated altitude for the central point, the map is placed over the curves so that the curve corresponding to the altitude passes through the central point. The map is then turned until the azimuth index passes through the proper azimuth graduation as determined by the tables. The position line is now properly located and may be traced on the map.

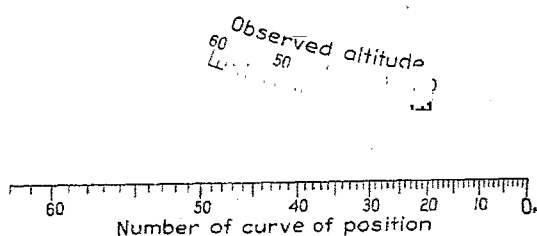
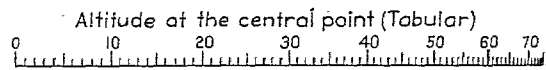
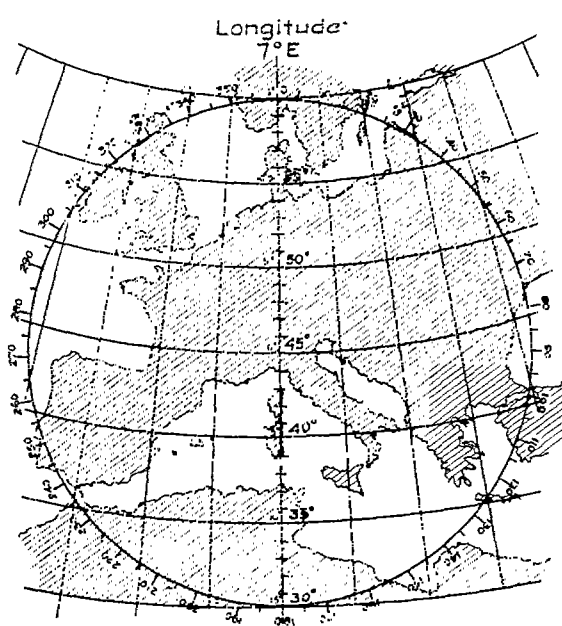
In the general case the observed altitude will differ from the tabular altitude, and a slightly different procedure is necessary. The map is oriented over the sheet of curves according to the tabular azimuth and so placed that the curve corresponding to the tabular altitude passes through the map center. The curve corresponding to the observed altitude would be the required position line if there were no distortion in the projection. A correction is necessary, which may be determined by means of the nomogram in Figure 26. The tabular and observed altitudes are found in the corresponding scales, and the intersection of a straight line through these points with the third scale indicates the number of the curve of position to be used. The intersection of the curve corresponding to the observed altitude and the azimuth index is traced on the map, and then the map is moved along the azimuth index until the curve found by the nomogram passes through this point. This curve is the position line sought.

Altitudes and azimuths for the fixed stars may be tabulated for the central point. Since interpolations are necessary, Favé advocates the use of so called "graphical tables." A rec-

<sup>34</sup> III<sup>e</sup> Congrès International d'Aéronautique, Vol. 3, 1906. "Recherches \* \* \* Détermination du Point en Ballon," by Favé.



tangular net is drawn, the vertical lines being spaced according to minutes of time on a convenient scale. Two horizontal lines are drawn for each hour of sidereal time and graduated according to altitudes and azimuths, respectively. Thus the interpolation can be made graphically, and no numbers need be written down. A single page of convenient size is sufficient.



Section A-A

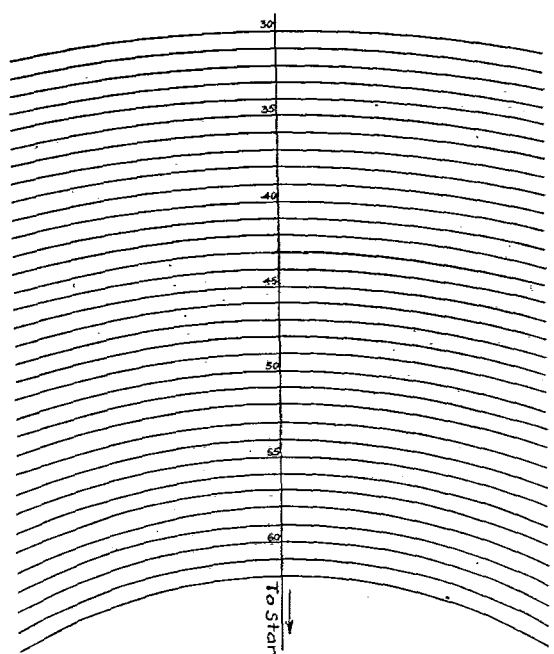


FIG. 25.—Favé chart

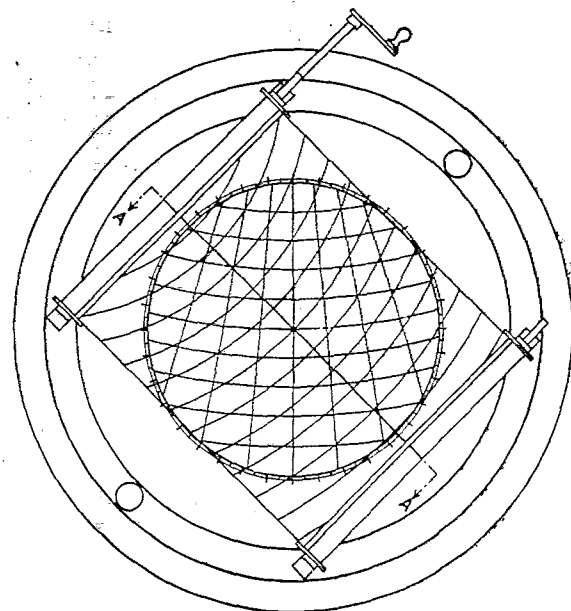


FIG. 27.—The Brill instrument

for any one star. Since the declination of the sun varies continuously, any tables of altitude and azimuth would be bulky and would require double interpolation. Therefore Favé suggests that the diagram (fig. 21) be used to determine the altitudes and azimuths of the sun and also of the moon and planets.

The map, curves, and tables may be used for any longitude provided the latitude of the central point remains unchanged. Favé recommends the use of five sets for latitudes of  $0^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $75^\circ$ , and  $90^\circ$ , respectively. Later, Favé improved his method by putting it into instrumental form. Instead of the sheet of curves, a special protractor and a curved ruler were devised by means of which the position lines could be located mechanically.

The principles of the Favé method have been utilized by Brill<sup>35</sup> in Germany in the construction of the navigating instrument shown in Figure 27. A circular map in zenithal projection covering about  $10^\circ$  of latitude is engraved on a sheet of transparent celluloid mounted in the frame of the instrument. Two sets of position line curves drawn on tracing linen are furnished. These are mounted, one above and the other below the map, on rollers, by means of which they may be moved across the map. The rollers are fixed to rings which can be rotated about the central point of the map, and the azimuth settings are made in this manner. Tables of altitudes and azimuths of suitable fixed stars computed for the central point of the map are given. The instrument is primarily intended for reducing the observations on fixed stars.

The Voigt<sup>36</sup> or "Orion" instrument, shown in Figure 28, was produced at the Motorluftschiff-Studien-Gesellschaft at Hamburg, in Germany. This instrument is also based on the same principles as the Favé method. The instrument consists of a rectangular metal frame, about  $18\frac{1}{2}$  inches by  $14\frac{1}{2}$  inches, on which is mounted the map and the mechanism for obtaining position lines. A circular map in azimuthal projection is engraved on a sheet of aluminum which is pivoted at its central point. Meridians and parallels are shown at one-half degree intervals. The central meridian is subdivided at two-minute intervals and graduated in degrees of latitude and also in degrees in both directions from the central point. A scale of azimuths is engraved around the circumference of the map, and an azimuth index fixed to the frame is provided. Three maps are furnished, each covering a latitude range of  $10^\circ$ , the central points being at latitudes  $42^\circ$ ,  $50^\circ$ , and  $55^\circ$ , respectively. The position line mechanism is mounted on a bridge engaging by means of gears in two racks on the left and right hand edges of the frame. This bridge may be moved over the surface of the map and clamped in any desired position. On the bridge is a flexible metal ruler which may be set to the proper curvature by suitable gearing. The curvature for any position line is indicated by a circular scale graduated in degrees of altitude.

The procedure is rapid and simple. The altitude of a star is measured at a given time, and from the tables which are furnished the altitude and azimuth of the same star from the central point at the time of observation are found. The curvature of the ruler is adjusted by setting the altitude scale to the observed altitude. The map is set to read zero azimuth. Then the ruler is moved until its distance from the central point is equal to the difference between the observed and tabular altitudes, being north or south of the central point according as the observed altitude is less or greater than the tabular altitude. The map is rotated until the azimuth scale is set to the tabular azimuth. The ruler is now in position for drawing the position line. A similar procedure for a second star gives a second position line, and the intersection of the two is the observer's position.

The toposcope,<sup>37</sup> a recently proposed French instrument, is almost identical with that of Brill. A single sheet of curves is supplied, the curves being cut through the paper so that the position line may be marked directly on the map with a pencil.

Littlehales<sup>38</sup> of the United States Hydrographic Office has suggested the use of the American polyconic projection. At the center of the map, which may cover an extensive area, is drawn a compass diagram with radial lines to a graduated circle near the limits of the map. A series

<sup>35</sup> *Illustrierte Aeronautische Mitteilungen*, Vol. XIII, No. 22, Nov. 3, 1909. "Ein Verfahren zur Auswertung astronomische Positionsbestimmungen," by Dr. Alfred Brill

<sup>36</sup> *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Vol. II, No. 9, May 13, 1911. "Die mechanisch-graphische Lösung des Höhenproblems mit dem Voigtschen Instrument," by Hans Boykow.

<sup>37</sup> *Rapports du I<sup>er</sup> Congrès International de la Navigation Aérienne*, Vol. II, Paris, December, 1921. "Le 'Toposcope,' (chercheur du Point)," by Vucetic.

<sup>38</sup> *Proceedings of the U. S. Naval Institute*, March, 1918. "The chart as a means of finding geographical position by observations of celestial bodies in aerial and marine navigation," by G. W. Littlehales.

of concentric circles spaced according to angular measure is also drawn. The difference between observed and tabular zenith distances is laid off in the proper direction and a straight line drawn perpendicular to the azimuth to represent the position line. Since the position line is in reality a curved line, there is an error due to the use of a straight line, which may reach a considerable value when the line is long, unless allowance is made for curvature. For the trans-Atlantic flight of the NC flying boats, Littlehales prepared a similar chart covering practically all of the North Atlantic Ocean.<sup>39</sup> This chart was drawn according to the Lambert zenithal projection. Tables of altitudes and azimuths and a protractor or template for drawing position lines conveniently and accurately were provided.

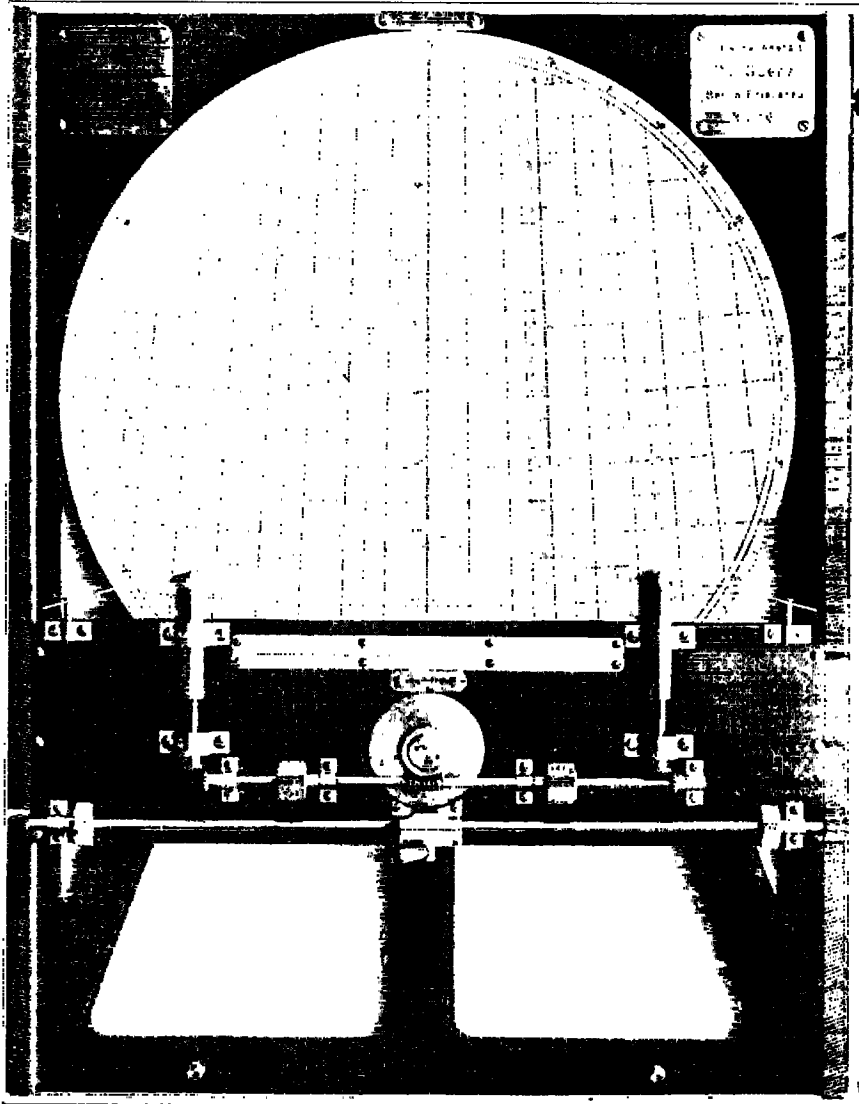


FIG. 28.—Voigt ("Orion") instrument

#### 5. MECHANICAL SOLUTIONS

Several instruments for the mechanical solution of spherical triangles,<sup>40</sup> as well as a number of slide rules for calculating azimuths,<sup>41</sup> have been devised. Most of these instruments are, for

<sup>39</sup> Aviation, July 1, 1920. "Some lessons of the trans-Atlantic flight," by Commodore H. C. Richardson, U. S. N.

<sup>40</sup> Wimperis and Horsley "Nomogram" Slide Rule. H. E. Wimperis, loc. cit.

<sup>41</sup> Annalen der Hydrographie und Maritimen Meteorologie, Vol. 38, No. 10, October, 1910. "Der Azimutstab von R. Nelting," by Dr. E. Kohlschütter.

one reason or another, not suitable for the purposes of aerial navigation; and the discussion will be limited accordingly to those instruments which appear to be practicable.

The line-of-position computer <sup>42</sup> (fig. 29) was invented by Prof. Charles Lane Poor, of New York. The computer is a slide rule based on the cosine-haversine formula (Formula III, on page 28). Concentric circular scales are engraved on a metal disk about 15 inches in diameter. A circular sheet of transparent celluloid and an arm, each bearing a radial index line, are pivoted

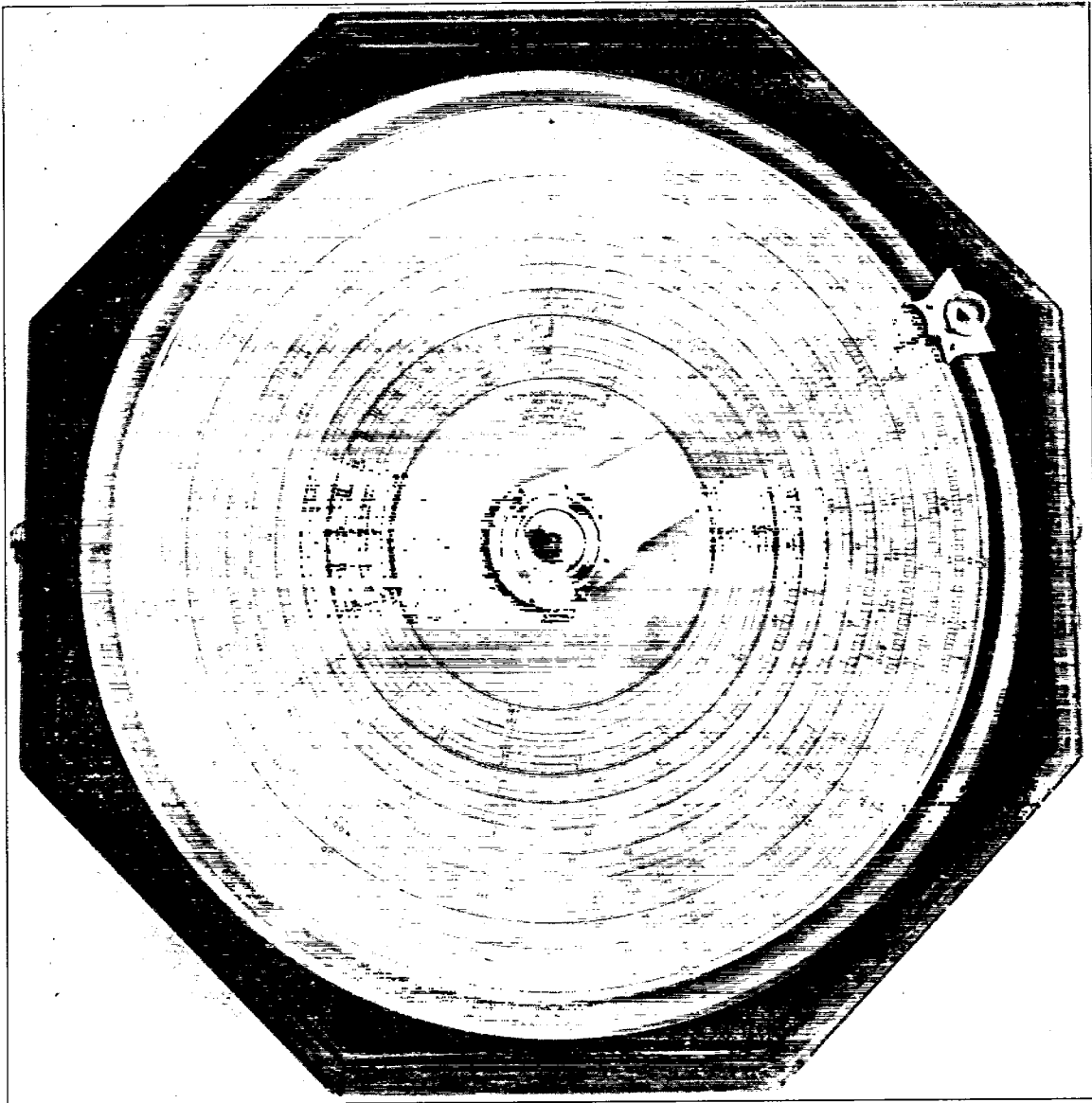


FIG. 29.—Line-of-position computer

at the center of the metal disk. The arm is furnished with a clamping screw so that it may be clamped to the celluloid sheet. There are eight graduated circles, of which the inner is used for determining azimuths and the remaining seven for determining altitudes. The altitude scale is graduated at 10' intervals, but the divisions are large enough so that single minutes may be estimated. The instrument is sufficiently accurate and rapid for aerial navigation, but the number of scales is confusing and several rules are necessary for determining the procedure to be

<sup>42</sup> "Simplified Navigation," by Chas. Lane Poor. New York, 1918. The Century Co.

employed under certain conditions. It is also rather large and would be inconvenient in a restricted space.

If the astronomical triangle be divided into two right triangles, as in Figure 19, the following simple formulas may be used in solving for the altitude and azimuth:

$$\tan y = \frac{\tan d}{\cos H}$$

If  $l$  and  $d$  are the same name,  $Y = (90^\circ - l) + y$ .

If  $l$  and  $d$  are opposite names,  $Y = (90^\circ - l) - y$ .

$$\tan A = \frac{\cos y \tan H}{\cos Y}$$

$$\tan a = \cos A \tan Y,$$

where  $a$  = altitude.

$A$  = azimuth.

$l$  = latitude.

$d$  = declination.

$H$  = hour angle.

and  $y$  and  $Y$  are auxiliary angles.

The Bygrave slide rule<sup>43</sup> (fig. 30) is an English instrument which solves the astronomical triangle by the above formulas. (The notation is that used on the rule and is shown in fig. 30.) The rule consists of three concentric tubes. The inner bears a scale of logarithmic tangents, the intermediate tube a scale of logarithmic cosines, and the outer tube two pointers, one for each scale. Complete instructions are given on the instrument itself. The slide rule is about 9 inches in length and  $2\frac{1}{2}$  inches in diameter. An accuracy of 1' or 2' of arc is attainable in almost every case. The procedure is straightforward, simple, and rapid; and since there are but two scales, the chances for error are reduced to a minimum. The procedure must be changed when the azimuth lies between  $85^\circ$  and  $95^\circ$ , when the hour angle is less than 20' of arc, or when the declination is less than 30'. The rules for these exceptional cases are not involved and are printed on the instrument, so that no difficulties need be anticipated.

#### THE REDUCTION OF OBSERVATIONS ON BOARD AIRCRAFT

In marine navigation, a high degree of accuracy is essential in the reduction of astronomical observations. Other factors, however, are of equal or greater importance in aerial navigation. Since aircraft travel at great speeds, position must be fixed at

frequent intervals, at least every hour, and perhaps even more often. Also the aerial navigator must do all of the work of navigating himself, and he can spare but a few minutes of very valuable time for any one operation. Any method, therefore, which is to be of any use in aerial navigation must be rapid. The element of time is of prime importance.

Simplicity follows as a second essential. To be rapid, a method of computation must be as straightforward and direct as possible, with the number of operations reduced to a minimum. Furthermore, the same method and the same procedure should be applicable on all occasions; there is no time to choose between methods. Also the necessary equipment of maps, instruments, books of tables, and so on, must be reduced to the lowest terms.

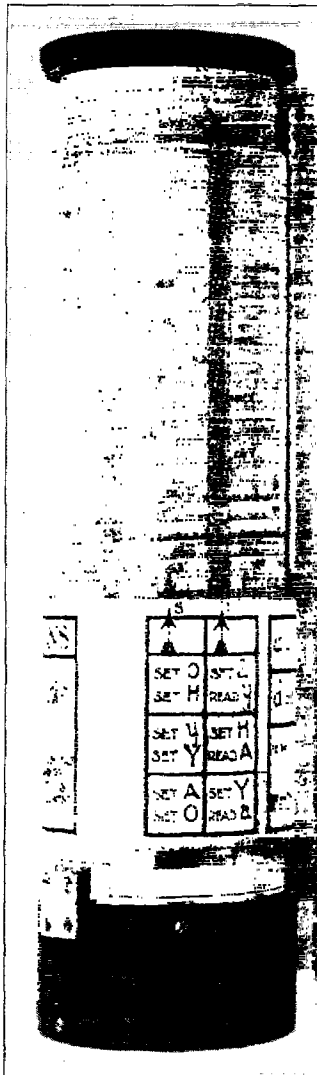


FIG. 30.—Bygrave slide-rule

<sup>43</sup>The Engineer, Mar 3, 1922. "A Position Line Slide Rule."

Accuracy is required, although not to the same degree as at sea. If an observation has a possible error of 5' of arc and the reduction of the observation introduces another possible error of the same amount, the final result may be off 10'. An accuracy of 2' or 3' of arc is desirable in aerial navigation, and a higher degree of precision is warranted if the speed and simplicity of the work are not impaired.

A further requirement is that of convenience. When space and time are limited, the navigator can not handle cumbersome books of tables, inconveniently arranged and requiring interpolations for every quantity to be found. His tools, whether books, charts, or instruments, must not only be convenient in themselves but must be arranged in systematic order.

The above considerations apply most particularly to airplanes. Here the time factor is of greatest importance and the navigator works under the greatest disadvantages. Experience alone will demonstrate the most suitable method. It would seem, however, that the Bygrave slide rule is admirably adapted for the purpose, since it fulfills all the conditions of a practicable method. Printed forms for computation arranged to facilitate the work are a necessity. These forms after use may be kept as more or less permanent records of the work.

In airships, more space is available and consequently the navigator may provide himself with more equipment. It would seem that the Bygrave slide rule and printed computation forms would be advantageous for all ordinary purposes. However, supplementary methods may be provided for use on special occasions, if the value of these methods fully compensates for the added equipment required. Additional tables and the two-star diagrams, for example, may prove to be helpful.

#### IX. MAPS AND ACCESSORIES

The aerial navigator requires two distinct types of maps. His problem is somewhat similar to that confronting the marine navigator who has one set of charts covering rather large areas of the sea on which he lays out his route and plots his courses and position lines, and another set drawn to larger scales presenting the features of coast lines, bays, and straits, and harbors in more or less detail as may be necessary.

Thus for the purposes of piloting, or, in other words, the recognition of his position by the topographical features of the country over which he is flying, the aerial navigator employs route maps. These are usually in the form of long strips showing an area from 50 to 100 miles in width from the point of departure to the destination. The route map pictures the distinctive features of the land surface as completely as possible. On the one hand, the natural features, as rivers, lakes, coasts, and mountains, are clearly indicated, together with information regarding forests, the elevation of the ground surface, and terrain dangerous for landing. On the other, railroad lines, highways, cities, and towns, lighthouses and beacons, and landing fields of all kinds are indicated. In short, the route map must show features of the country which may aid the navigator in keeping to his course and help him to find safe landing fields for emergencies as well as those fields at which he wishes to land. The scales used vary; the most common are 1 to 200,000 and 1 to 500,000. Much work has been done with a view to the perfecting of the route map, but, although in everyday use, no generally recognized standard has as yet been produced.

For long-distance flights over the sea and over the land, in particular, regions devoid of distinctive landmarks, the route map is not sufficient in itself. General maps covering larger areas are required. These may serve several purposes; for example, by their aid the proper route may be selected; on them can be plotted the course pursued as determined by dead reckoning and the position lines found by radio bearings or astronomical observations. The route maps are not suitable for these purposes, as the areas shown are too limited, the scale is unnecessarily large, and the great amount of detail is confusing.

To be of service in the plotting of astronomical observations, the general map should have certain characteristics, and it so happens that these are convenient, if not essential, for the other uses to which the map may be put. The meridians should be straight lines, or very nearly so, in order to facilitate the plotting of azimuths. Furthermore, all great circles should be