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PRECISION INDICATION OF THE MERIDIAN

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1. Introduction

In 1850 M. Leon Foucault[1] carried out his famous experiment at the Paris Observatory which demonstrated the rotation of the earth. A long heavy pendulum set to oscillate in a north-south plane, will maintain this plane fixed in direction in space, and, as the earth rotates about its axis, the plane of oscillation of the pendulum appears to an observer on earth to rotate, and will therefore move from the north-south direction at an angular velocity of :—

$\Omega \sin \phi$ (see Fig. 2 (*a*))

where ϕ = latitude of point

Ω = angular velocity of rotation of earth

This experiment is still carried out daily at the Science Museum in South Kensington, London.

Foucault realised that a rapidly rotating heavy wheel mounted in gimbals would maintain its axis of rotation fixed in space but he failed to demonstrate the rotation of the earth in this way because his rotating wheel or " spinner " was not electrically driven. Foucault called his device a "gyroscope."

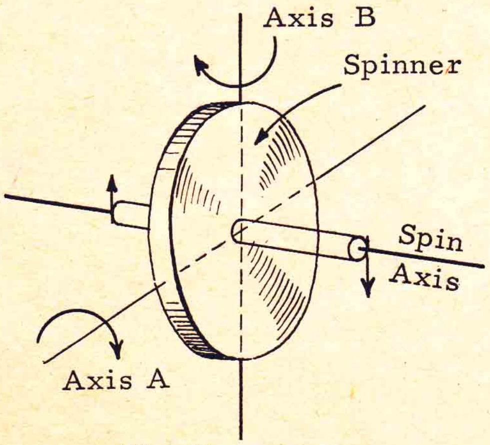
The mathematical theory of the gyroscope had been developed by the beginning of the present century, and during the same period the introduction of steel ships, particularly in the Navy, aroused interest in a north seeking device which could replace the magnetic compass. Sperry, Anschutz and Brown[2] were well-known workers in this field and gyrocompasses were developed capable of indicating the direction of true north to within a degree, in a ship at sea during stormy weather.

The development of the gyroscope as a precise instrument for use in mining started as early as 1914,[3] since the transfer and maintenance of accurate bearings underground has always been a difficult problem, which was not solved until the recent advent of precise gyroscopic north seeking devices. The first instrument capable of establishing bearings to a minute of arc was developed by Dr. Jungwirth in 1948 at the Bergakademie in Clausthal, Germany[4] and it was known as the " Meridian Indicator." It was used in South Africa for the transfer of bearings underground and together with its tripod it weighed 500 lbs. A compressed air generator weighing more than the Meridian Indicator was also required.

Under the direction of Professor O. Rellensmann, development continued at the Bergakademie, and in 1959, J. I. McLelland[5,6] announced a new instrument called the Gyrotheodolite K.T.1.

It weighed 100 lb. and power for its operation was supplied by a 24V lead plate battery. This instrument was used in South Africa by Professor G. B. Lauf of the University of the Witwatersrand, and after various improvements had been made Lauf reported[7] that bearings accurate to a few seconds or arc could be transferred underground, the field-work at a station underground being completed in 2–3 hours. The gyrotheodolite is manufactured by Otto Fennel GmbH and Co., of Kassel, Western Germany.

As a result of recent advances in the manufacture of robust highly accurate gyroscopes for airborne inertial navigation systems, a new instrument, the Precision Indicator of Meridian (PIM) has been developed in England by English Electric Aviation, which now forms part of the British Aircraft Corporation. It uses a floated gyroscope, a type pioneered by Dr. C. S. Draper at the Massachusetts Institute of Technology.[12] A prototype of PIM was exhibited at the Farnborough Air Show in 1960. PIM was developed originally to a military specification which demanded a determination of north to a minute of arc within 30 minutes of the time of arrival at site. Extensive tests all over the world have shown that this requirement can be met with ease. In 1963, Sutton and Thomas[8] of the Mining Department (Royal School of Mines), Imperial College of Science and



The Gyroscope

FIGURE 1.

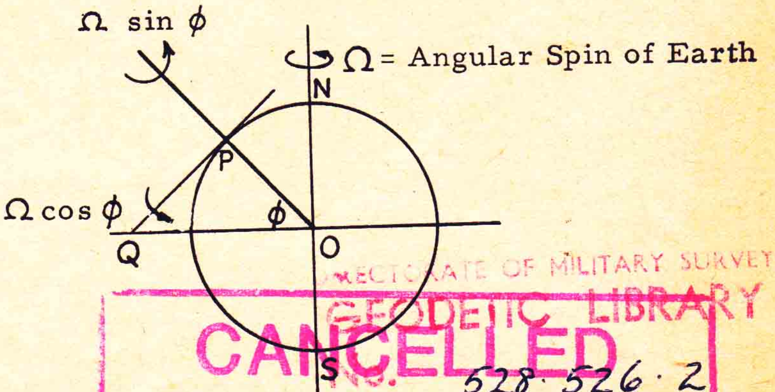


FIGURE 2 (*a*).

Technology, checked a prototype of PIM against bearings established independently by a geodetic theodolite reading directly to 0·2″ of arc. The maximum discrepancy was 2″ of arc, and each bearing determined by PIM took 2–3 hours from time of arrival at the point of observation. A production model of PIM is now being manufactured by the Precision Products Group, Guided Weapons Division of the British Aircraft Corporation.

Another type of north seeking device is currently under development[9] by the Astro-Space Laboratories of Huntsville, Ala., in the USA. The standard deviation of a single reading is stated to be 23·7″ of arc after an extensive series of tests carried out by the manufacturers on a prototype. This is rather more than the corresponding standard deviations now being achieved using PIM and the gyrotheodolite, but a more sensitive model is being designed.

A brief description will now be given of the three latest systems mentioned above, and this will be followed by a more detailed discussion of PIM, particularly of the latest production model which has been used extensively all over the world during the past year.

2. The Gyroscope and Earth Spin

In Figure 1, the spinner or rotor of a gyroscope is a heavy wheel driven electrically at an angular velocity of 20,000–30,000 revs. per minute. Axis A is at right angles to the spin axis of the spinner and is sometimes referred to as the Input Axis. Axis B is at right angles both to Axis A and the spin axis, and is referred to as the Output Axis. If the angular momentum of the spinner is very high (*i.e.*, a heavy wheel rotating at a high rate of revolution) then the spin axis tends to maintain its direction in space. If an attempt is made to tilt the spin axis in space by rotating it about Input Axis A, resistance to this rotation will be experienced, and the spinning system will precess or rotate slowly about Output Axis B, the rate of precession being proportional both to the Input rate of rotation and to the angular momentum of the spinner.

Figure 2 (*a*) shows the earth rotating about its polar axis NOS with an angular velocity Ω. At a point P in latitute ϕ, the earth's angular velocity can be resolved into two components :—

Ω Sin ϕ about the vertical line OP at P
Ω Cos ϕ about the horizontal north-south line QP at P

Figure 2 (*b*) is a plan of the earth at P, showing a north-south line (*i.e.*, a meridian) and its component of earth spin ($\Omega \cos \phi$). The spin axis of a gyroscope is held in a horizontal position making an angle θ with north, and the horizontal component of earth spin can now be resolved into two further components—

($\Omega \cos \phi$) $\cos \theta$ about the spin axis
($\Omega \cos \phi$) $\sin \theta$ about the Input axis of the gyroscope.

This second component causes the spin axis to rotate in space, and if the gyroscope system is free to rotate about a vertical line at P (corresponding to the Output axis) it will precess, and the couple causing the precession is proportional to :—

$$H \Omega \cos \phi \sin \theta \qquad \text{.....................(1)}$$

where H is the angular momentum of the spinner. This precessional couple makes the spin axis turn about the vertical at P in such a way that the spin axis orients itself with the north-south direction at P. In this way, a gyroscope held with its spin axis horizontal, becomes a north seeking device.

3. The Gyrotheodolite

The Gyrotheodolites (K.T.1, K.T.2 and T.K.3) are all pendulous systems in which the horizontal or spin axis is maintained horizontal by means of gravity. In Figure 3, for example, the weight U keeps the spin axis W horizontal, thus causing it to rotate in space with an angular velocity $\Omega \cos \phi \sin \theta$, and making the whole system precess about vertical axis X in such a manner that spin axis W seeks north. Marine gyro-compasses are constructed in this way.

Figure 4 illustrates the Gyrotheodolite in which the spin axis AB and the spinner G are contained in a sealed container suspended by a special Nivaflex tape XY, to the vertical axis of a theodolite: if the theodolite is rotated, the gyroscope and its case turn with it, and the amount of rotation can be

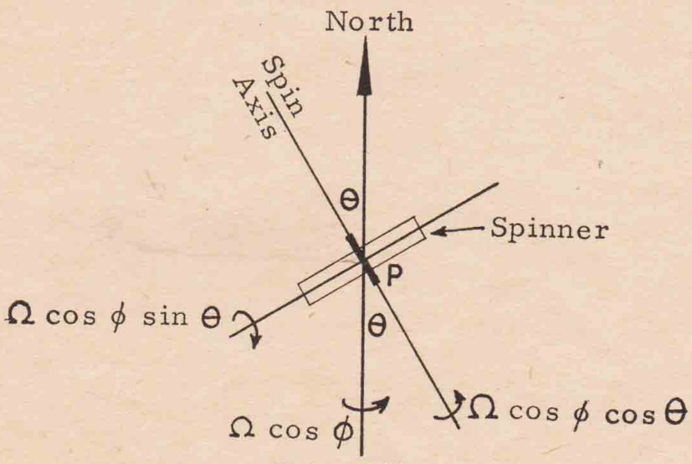


Figure 2 (*b*).

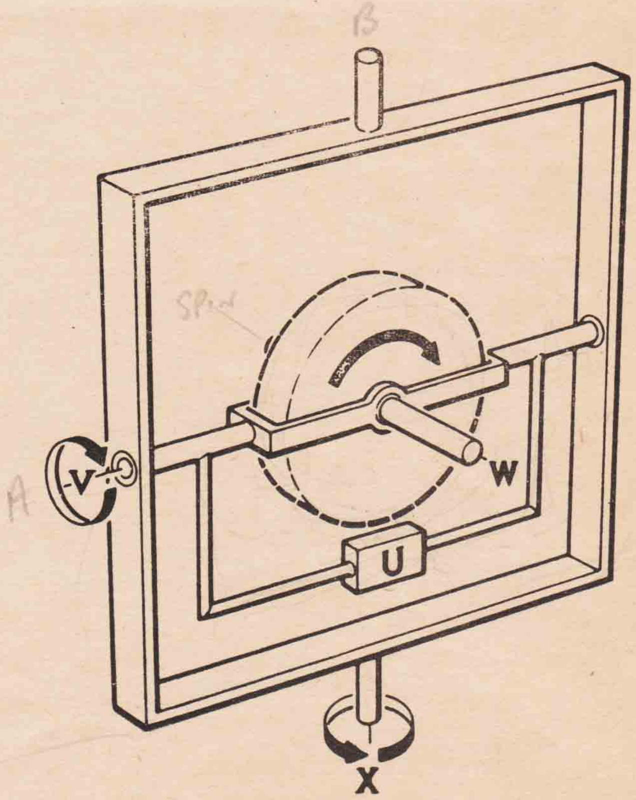


Figure 3.

measured on the horizontal circle. In order to take measurements to an axis fixed to the gyroscope container, a mirror is attached to the container and this is viewed through an auto-collimating telescope. If the gyroscope rotates about a vertical axis, its movement can be followed by keeping a mark, reflected from the mirror, central in the reticule of the auto-collimator—this process is called tracking. The auto-collimating telescope is attached to the theodolite.

If the gyroscope, with the rotor not spinning, is turned about axis XY, it will oscillate as a conical pendulum as the suspension tape twists and untwists. The motion can be tracked by the auto-collimator and the extreme positions of swing recorded on the horizontal circle.

The mean position of swing when the tape is free of twist is calculated (see below) from these extreme readings and this mean reading is set into the instrument. The gyroscope is now started up and it precesses towards north as explained above. Its motion is tracked continuously by the auto-collimator, so that the theodolite and the suspension tape rotate with the container and the tape transmits no torque to disturb the precession. The container now oscillates about north, and once again the extremes of swing are recorded on the horizontal circle, the mean position indicating the direction of true north.

Lauf[7] has shown that both the above oscillations are damped simple harmonic motions whose differential equation is given by

$$K_1 \frac{d_2\theta}{dt^2} + K_2 \frac{d\theta}{dt} + K_3\theta = 0 \quad \text{...............(2)}$$

Where K_1, K_2 and K_3 are constants and θ is the angular deflection from the mean position at any instant. In the case of the second oscillation

$$K_3 = H \,.\, \Omega \cos \phi$$

and since when θ is small

$$\sin \theta = \theta$$

the term $K_3\,\theta$ is the restoring couple due to precession and is given by equation (1).

The damping constant K_2 is always small, but it causes the oscillations on each side of the mean to decrease exponentially so that the mean of two swings is not the mean position.

If r_1, r_2, r_3 are three consecutive horizontal circle readings, two on one side and one on the other side of the mean, then the mean reading can be calculated as follows :—

$$r_m = \frac{(r_1 + 2r_2 + r_3)}{4}$$ This is known as a " Schuler" mean.

Seven consecutive readings give five consecutive Schuler means, and these are then meaned to obtain the final mean reading. The Schuler mean assumes that damping is sufficiently small for three consecutive swings to be reduced linearly (approximately), instead of exponentially as theory requires. The following mean, based upon five consecutive readings, is more accurate since only fourth powers and above are neglected in the approximation to the exponential.

$$r_m + \frac{(r_1 + 4r_2 + 6r_3 + 4r_4 + r_5)}{16}$$

Seven consecutive swings will give three means similar to the above.

If angles are measured sexagesimally, all numerical reductions, such as the above, should be made on a desk calculator where they can be worked out directly without writing down intermediate results using the method of " partial complements " (reference (10) Chapter 5).

It should be noted that any variation in the angular momentum (H) of the spinner affects the constant K_3 and hence the differential equation of motion (2) so that the observed swings are disturbed. The tracking of the precessing gyroscope must also be continuous, otherwise unknown torques are transmitted from the tape. The need for continuous tracking is the main disadvantage of the Gyrotheodolite, and Lauf and others have stated that it requires exceptional skill to produce a result correct to a few seconds of arc. It should be noted that tracking must be continued whilst the horizontal circle* is read. Photo-electric tracking would seem to be the answer.

The two periods of oscillation of the Gyrotheodolite K.T.1. are :—

1 minute 20 seconds—gyroscope not spinning
26 minutes at 51° N or S—gyroscope spinning

Lauf observes at least seven reversals. Non-spinning means are determined before and after each spin-up of the gyroscope.

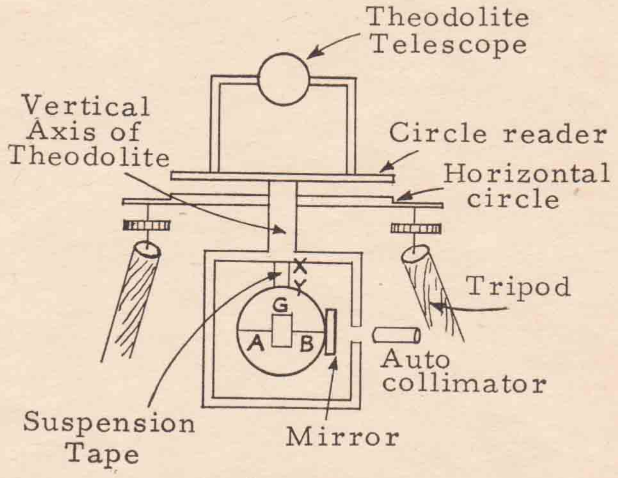
An example of the reduction of seven observed swings will be found in Appendix A.

4. The Precision Indicator of the Meridian (PIM)

This instrument differs in many respects from the pendulous systems described above. In Figure 5, the outer case of the gyroscope is attached rigidly to the vertical axis of the theodolite. It rotates with the telescope, and the amount of rotation can be measured on the horizontal circle. This outer case is not visible, since it is protected by an outer cover. The outer case contains an axis XY which is a continuation of the vertical axis of the theodolite, and the sealed inner case, which contains the gyroscope spinner G, rotates freely about axis XY.

The space between the inner and outer cases (0·01 inch) is filled with a special fluid " Fluorolube " which is maintained thermostatically at a temperature of 71°C, and this floats the inner case at neutral buoyancy so relieving the pivots at X and Y : this is the principle of the floated gyroscope. The viscous fluid also provides damping and protection against shock. The pivots at X and Y are manufactured with extreme care, pins and jewelled bearings being selected within a tolerance of 0·000015 inch. In this way stiction* around the axis XY is virtually eliminated. The pins are

* Stiction = static friction.



Gyrotheodolite KT1

FIGURE 4.

made of tungsten carbide, and, in the unlikely event of a pivot being overloaded, the pins will break rather than bend.

The spinner G is the rotor of an inside out hysteresis motor which is driven at a synchronous speed of 24,000 revs/minute by a 400 c/s 3 phase transistorised generator. The supply frequency is crystal controlled and the amplitude of oscillation is also cootrolled in order that the angular momentum of the spinner remains constant to a high order of accuracy. Temperature control of the flotation fluid referred to above, is by means of a proportional controller which is in the form of a high frequency switching system which does not disturb the remainder of the instrument and temperature is maintained to within $\pm$ 0·1°C. The spinner itself, with an angular momentum of 2×10^6 gr.cm²/sec. and a diameter of 2 inches, is balanced dynamically to a fraction of a dyne cm. The complete inner case is also carefully balanced, and it is filled with helium at a pressure slightly above atmospheric pressure. Prototypes of PIM used a Honeywell gyroscope manufactured in the USA which was originally designed for another purpose, but production models of PIM contain a gyroscope designed and manufactured by B.A.C. specifically for north finding.

The inner and outer cases in Figure 5 are linked electromagnetically. If the inner case rotates about gimbal axis XY, an A.C. pick-off located at one end of the gimbal axis transmits a signal proportional to the angular displacement of the inner container from a fixed reference position. This signal is amplified, and fed back to a torque motor also located on the gimbal axis and the inner container is, in this way, restrained so that it rotates about axis XY by no more than one or two seconds of arc. To all intents and purposes, therefore, the inner and outer containers rotate together and oscillation of the inner container is prevented by the torque motor. Thus, although axis XY is completely free of restraints, rotation about it is limited electronically to a few seconds of arc. For this reason power can be supplied to drive the spinner by means of short wires linking inner and outer containers : all such connections are, however, *via* slender ribbons of silver alloy 0·007 inch wide and 0·0005 inch thick.

The torque required to prevent rotation of the inner case, balances the couple which is due to precession, and if the theodolite is levelled carefully so that axis XY is vertical, the balancing torque is proportional to :—

$$H. \ \Omega \cos \phi \sin \theta \quad \text{......................(1)}$$

as explained previously. The current supplying the torque motor is passed through a simple milliameter, and this is used as a measure of the torque.

The outer case of the gyroscope is attached rigidly to the specially extended vertical axis of a Hilger and Watts Microptic No. 2 single second double reading theodolite. The special tripod is rather low, and for this reason a diagonal eyepiece is fitted permanently to the telescope. Plate I is a view of the complete sighting unit mounted on a tripod, total weight 41 lb.

The Control Unit illustrated in Plate II contains all the power supply systems apart from batteries ; the meter which measures the balancing torque is on the right. The control unit accepts any low impedance d.c. supply voltage between 21·5 and 30 volts and 24-volt aircraft batteries can be used : external variations of temperature between −31°C to +55°C can be accepted, and any fault can easily be detected by an unskilled operator and corrected by the insertion of a new module. The Control Unit is in fact an analogue computer which solves continuously the differential equation

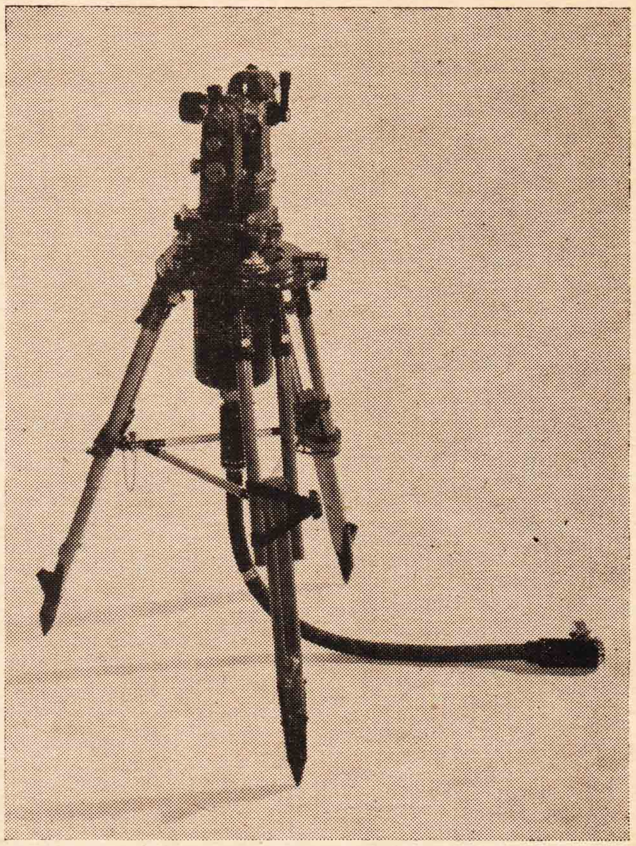


PLATE I.—*PIM Sighting Unit.*

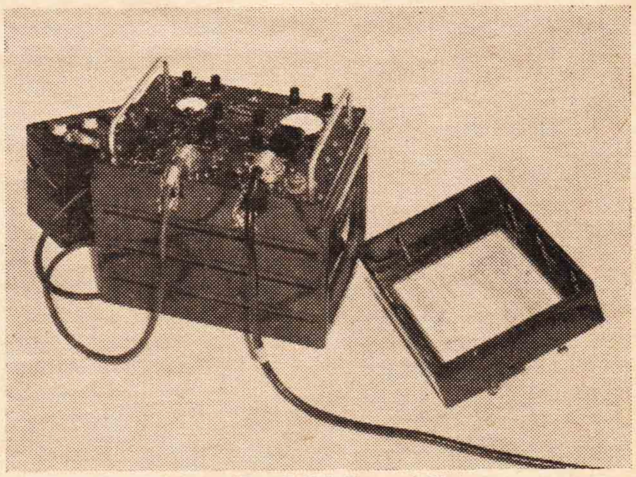


PLATE II.—*PIM Control Unit.*

given in Appendix D, and the time constants are such that to all intents and purposes oscillation of the gimbal is prevented. The complete equipment has been designed for rough usage, and handling by unskilled operators. Plate III shows PIM in use underground in New Zealand in very wet conditions.

The sighting unit (Plate I) is set up over a point and carefully centred and levelled. The control unit is set up nearby and linked to the sighting unit by a single multicore cable. The gyroscope is switched on and allowed to warm up, which takes 5 or 10 minutes depending upon ambient conditions : the operator cannot use the instrument unless it is operating correctly at the right temperature—interlocking controls attend to this. During the waiting period the Reference

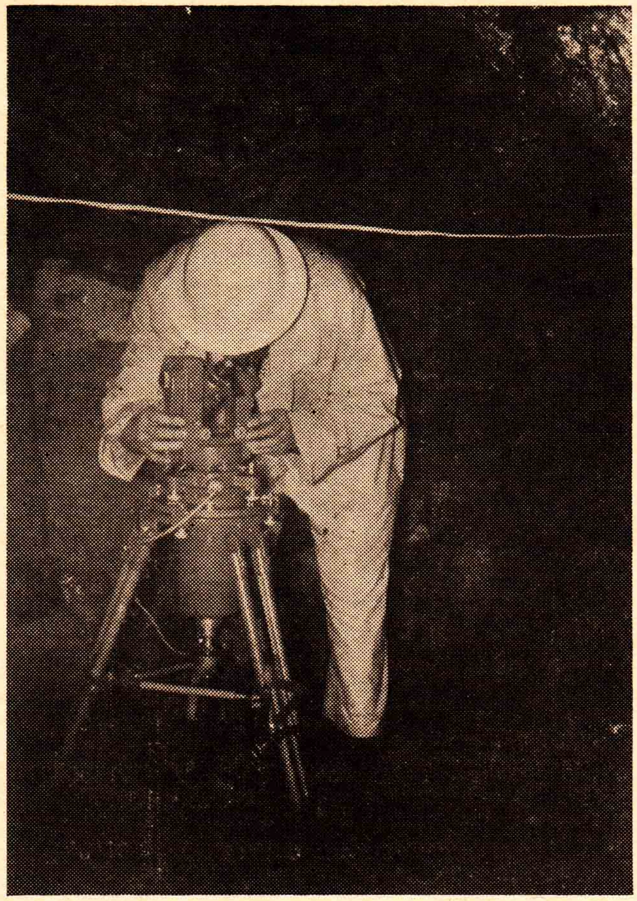


PLATE III.—*PIM in use underground in wet conditions.*

Object can be sighted. The telescope is now rotated until the meter in the control unit indicates maximum deflection. Since the meter reading is proportional to :—

$$H. \ \Omega \cos \phi \sin \theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

this occurs when $\sin \theta = \pm 1$, since θ is the only variable and the spin axis of the gyroscope is now oriented east-west (*i.e.*, $\theta = 90°$ or $270°$). The theodolite telescope whose axis is set at right angles to the spin axis is oriented north-south. This orientation need only be approximate, since near 90°, $\sin \theta$ is insensitive to changes in θ. The gain control of the amplifier is now adjusted so that the meter reads its maximum reading of $\pm$ 25 divisions. In this way calibration for latitude in formula (1) is accomplished in a few minutes. The above adjustments are made with the meter switched to the least sensitive of its three reading ranges, and the theodolite is now turned through appximately 90° so that it points east or west and the meter reads zero. By switching the meter to its more sensitive ranges the telescope is brought almost exactly east or west and the meter reads the value of θ in minutes of arc. Before reading the meter in its most sensitive range, the gyroscope is allowed two or three minutes to settle : during this period the observer can bring his booking up to date and he remains near the control unit which is remote from the sighting unit. The observer's most difficult task during this complete operation is sighting the reference object, which is normal manipulation of a theodolite. The final circle reading of the theodolite, usually adjusted to an exact multiple of 10′, corrected by the meter reading (θ') gives the true direction of east or west. Instrumental errors of the sighting unit must now be eliminated and these will be discussed later.

Thus summarising : PIM uses a robust floated gyroscope which indicates on a meter the deviation of its spin axis from north. The whole gyroscope turns with the theodolite telescope, and the inner container can only rotate a few seconds of arc with respect to the outer container. Each reading of the meter requires a settling time of 2 or 3 minutes, and readings can be continued until the required accuracy is achieved. At least four readings are required in order to obtain a result free of systematic errors (see below) and four such readings are referred to as a " quartet."

It will be found in practice that the method described above of bringing the spin axis, approximately to north and then reading the error on a meter is exceptionally simple and rapid and is far quicker than the alternative method of reducing the meter readings to zero.

5. The Astro-Space North Seeking Gyroscope

A description of this instrument, which is still under development has been given by Philip J. Klass.[9] Basically it consists of two spheres. An outer sphere whose axis of rotation is vertical and which is driven at 6,000 revs./min., and an inner sphere which has a hole, plugged at both ends, drilled through its centre to form an axis of rotation and it follows the rotation of the outer sphere. When both spheres have been brought up to speed, air at 14 p.s.i. is pumped into the 0·001 in. clearance space between them (Figure 6) so that the inner sphere floats on a thin film of air.

The inner sphere behaves as a gyroscope and attempts to maintain its axis fixed in space as the outer sphere rotates with the earth. The axis of the inner sphere drifts away from the axis of the outer sphere and the driving torques transmitted through the viscous film of air become unbalanced, and cause the inner sphere to precess until it follows precisely the earth rotation of the outer sphere, and it settles with its axis LM forming an east-west plane with the vertical axis of the outer sphere XY. The small angle of displacement between the two axes remains constant at a given latitude and the line at right angles to the plane containing the axes is the north-south line required.

A mirror mounted at the end of the axis of the inner sphere is observed in an auto-collimator attached to a theodolite which is also mounted on the vertical axis of the outer sphere. In this way the direction of the inner axis can be referred to the theodolite. The angle between the two axes is small, amounting to 24′ of arc in latitude 40°, but subsequent models have a displacement angle of about 1 degree. The standard deviation of a single reading is stated to be 23·7″ of arc.

There are, as yet, no reports available of the Astro-Space Gyroscope in use in the field.

6. Systematic and Random Errors

The following discussion refers to PIM, the instrument with which the writer has been connected. Similar errors are found in all gyroscopic instruments but they will be eliminated in a different manner.

(*a*) *Index Error*

The input axis of the gyroscope (at right angles to the spin axis, see Figure 1) may not be aligned with the line of sight of the telescope and this index error is measured at the factory by sighting along a line whose astronomical bearing is known. A production model of PIM which has travelled all over the world for demonstration purposes, which has been dropped

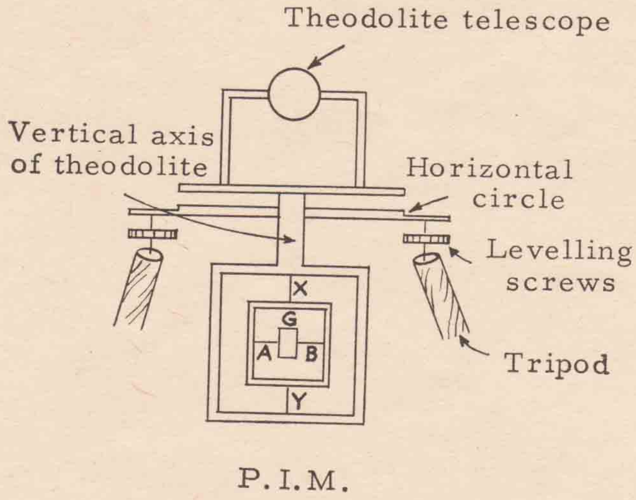


FIGURE 5.

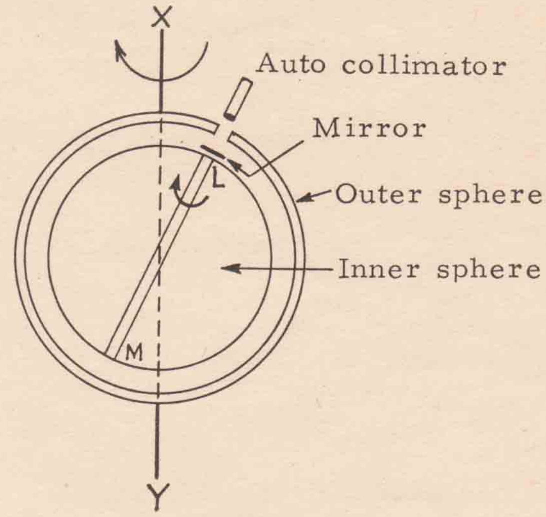


FIGURE 6.

at least once, and has been subjected to a good deal of rough usage in transit, had its index error checked at regular intervals and there was no measurable change (to seconds of arc) during the course of a year.

If the index error is not known, PIM can still be used to determine relative bearings from a given baseline, and for many surveying purposes, in mining for example, this is the usual method. This is the method used by Professor Lauf in South Africa.

(*b*) *Systematic Bias*

Zero readings of the meter are obtained when the spin axis still makes a small angle with the north-south line. This bias is produced by small residual torques and restraints in the system and although it may vary slightly from spin-up to spin-up, during a single run it remains constant. Systematic Bias is eliminated by taking a reading first, with the telescope pointing West (*i.e.*, West Heading) in which case the spin axis is oriented North-South, and second, with the telescope pointing East (*i.e.*, East Heading) in which case the spin axis will be turned horizontally through 180° and systematic bias will change sign and be eliminated in the mean reading.

Whilst the first West and East Headings are being made care is taken that the theodolite is always turned counter clockwise (c/c). A second pair of headings is now observed in which the telescope is always turned clockwise (c), and in this way errors caused by any small sets in the conducting ligaments for example, are eliminated. Four such observations, occupying 15–20 minutes, form a quartet of readings, and their mean value, corrected by index error, is accepted as the direction of true north.

Appendix B shows the complete reduction of two quartets, where it should be noted that the West and East Headings set into the theodolite are exact multiples of 10′ and remain constant, for as many quartets as are observed without changing the RO reading.

(*c*) *Random Drift*

All gyroscopes suffer from random drift. This causes the spin axis to wander slowly and at random about a mean position, in the case of north-seeking gyroscopes. For this reason all errors may not be eliminated by observing a single quartet, but repeated observation of quartets will soon establish a statistical pattern and eight or nine quartets have given a standard deviation of less than 10″ using production models of PIM, so that the mean has a standard deviation of only 3″–4″.

(*d*) *Settling Time*

After the theodolite has been moved it is important to allow the spinner time to settle in its axis of rotation. This settling time may vary between counter clockwise and clockwise rotation of the theodolite, since the precession caused by the vertical component of earth spin ($\Omega \sin \phi$—see Figure 2 (*a*)) is resisted by the bearings of the spin axis and the axis settles in a given position which is disturbed differently by clockwise and counter clockwise rotation. The settling time is 2–3 minutes and this accounts for most of the time taken to make accurate observations.

7. Field Trials of PIM

In Appendix C, a summary is given of bearings established by PIM in many parts of the world. Particular note should be taken of the column headed " Mean Difference from True Bearing " since this indicates independent checks on the PIM results and these are all 10″ or less except in two instances when only a single pair of headings was observed. The following remarks refer in greater detail to these and other recent results.

(i) *Tywarnhale Mine and Dragonby Mine*

The writer together with his colleague Mr. D. Sutton has already reported these results in detail.[8] Bearings established with great care using a geodetic theodolite were checked using a PIM prototype. Two underground stations were selected where inclined shafts made it possible to carry accurate bearings underground.[11] The checks produced the following discrepancies :—

0″ 0″ 1″ 2″

(ii) *London Underground*

Early in 1964, eight bearings were observed along the line of the new Victoria Tunnel for the purpose of checking existing surface and underground control. Surface surveying had been undertaken under difficult circumstances through the streets of London, and underground *via* lift shafts, passages and stairways used by the public. Agreement was excellent with six of these bearings. Disagreements were larger, in two cases, than the standard error of the PIM results would indicate, although these differences were not serious in view of the complexity of the traverses. At one of the points, where there was agreement with surface traversing, an additional check by astronomy was made quite recently and agreement with the PIM result to one second of arc was achieved.

(iii) *National Coal Board—Clipstone Colliery*

H. R. Herbert, Chief Surveyor, East Midlands Division of the NCB, organized a demonstration of PIM at Clipstone Colliery, and, according to his report, only two or three quartets were observed at each station. Since PIM is not yet flame-proofed, special permission was required to take the apparatus underground.

Three base-lines were chosen, one on the surface, one at a depth of 640 yards (=1,920 feet) and the third at a depth of 935 yards (=2,805 feet). The whole operation, surface and underground, including travelling time was only 7 hours and Mr. Herbert's report can be described as very favourable, particularly with regard to speed of performance, mobility and ease of operation.

Results are as follows :—

Base Line	NCB Bearing	PIM Bearing	Difference
Surface	159° 06′ 50″	159° 07′ 04″	−14″
Top Hard	218° 54′ 54″	218° 55′ 42″	−48″
Low Main	296° 15′ 42″	296° 16′ 36″	−54″

The only absolute check is on the surface where agreement is good, and it is interesting to note that a check PIM observation made on the surface on the following day agreed with the previous PIM result to 2″ of arc. Bearings carried down plumb wires are known to be unreliable and Lauf[7] quotes a number of examples, some from the NCB, where errors as high as 12′ of arc have been reported. Sheppard[11] gives experimental evidence that the centre of a plumb line need not be the point which should be sighted, and over short plumb planes this may lead to considerable error. A twist between surface and Top Hard (1,920 feet) amounting to 50″ accounts for both discrepancies, since PIM gives excellent agreement in the relative angle between Top Hard and Low Main (885 feet).

Mr. Herbert is of the opinion that there would be a considerable saving in time when carrying out correlation work if a meridian indicator were used.

(iv) *Canada*

In July 1964, a production model of PIM was purchased by a Canadian mining organization. They report that it has been used since receipt for establishing underground bearings on various mines.

In order to check its accuracy, an astronomic bearing was established at a PIM base station and agreement to 2″ of arc was achieved.

(v) *Lake Manapouri, New Zealand*

A number of independent checks have been obtained in Australia and New Zealand, but the most interesting result is in connection with a huge hydro-electric scheme at Lake Manapouri. A bearing was required near a machine hall at a depth of 700 feet in order to commence driving a spiral access tunnel $1\frac{1}{4}$ miles in length. Conventional methods could not be used because the base-line for plumbing was only 3 feet and seepage water from the lake was continually falling down the exploration shaft. PIM was used to supply a bearing thought to be accurate to within 20″, the whole exercise occupying $2\frac{1}{2}$ hours including the time taken to lower the equipment down the shaft and return it to the surface.

8. Conclusion

There appears to be no doubt that reliable instruments are now available which can be used to determine the direction of the meridian within a few seconds of arc. They are already being used by the Armed Forces, and in a number of countries mining companies are using them extensively to solve the problem of establishing accurate bearings underground.

In surface surveying meridian indicators can also be used to check bearings, and rapid resections are possible using only two sights. Astronomic bearings, of course, give the same results, but the difficulty here lies in selecting a suitable time for observation. It should be noted that PIM, for example, incorporates a single second theodolite, which can be used as such, so that the reliability of the gyroscope equipment can be checked independently by astronomy, at any suitable time. The equipment then becomes available for providing check bearings when, as is often the case, astronomy is impracticable, and this can be done at any point above or below the surface of the earth.

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APPENDIX "A"

CALCULATION OF THE MEAN POSITION OF A DAMPED SIMPLE HARMONIC OSCILLATION

Observed Swings				*3 Point (Schuler)*			*4 Point*			*5 Point*		
deg.	min.	sec.		deg.	min.	sec.	deg.	min.	sec.	deg.	min.	sec.
58	02	34	Right									
53	07	07	Left									
58	00	14	Right	55	34	15						
53	08	37	Left	55	34	03	55	34	09			
57	58	14	Right	55	33	56	55	33	59	35	34	04
53	10	57	Left	55	34	01	55	33	58	55	33	59
57	56	46	Right	55	34	13	55	34	07	55	34	02
				55	34	06	55	34	03	55	34	02

3 Point Mean :—r_m (Schuler) $= \dfrac{(r_1 + 2r_2 + r_3)}{4}$

$$= \frac{58\ 02\ 34 + 2(53\ 07\ 07) + 58\ 00\ 14}{4} = 55\ 34\ 15 \text{ etc.}$$

5 Point Mean :—r_m $= \dfrac{(r_1 + 4r_2 + 6r_3 + 4r_4 + r_5)}{16}$

$$= \frac{58\ 02\ 34 + 4(53\ 07\ 07) + 6(58\ 00\ 14) + 4(53\ 08\ 37) + 57\ 58\ 14}{16} = 55\ 34\ 04 \text{ etc.}$$

The two formulae given above are special cases of a more general result. For example :—

If $r_1\ r_2 \ldots r_7$ are seven consecutive turning points,

$$r_m = \frac{r_1 + 6r_2 + 15r_3 + 20r_4 + 15r_5 + 6r_6 + r_7}{64}$$

The coefficients of "r" are the coefficients in the Binomial expansion

$$(a + b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + \ldots \text{ etc.}$$

and the denominator is the sum of the coefficients.

The exponential is replaced by a polynomial :—

$$e^{-a} = \left(1 - a + \frac{a^2}{2} - \frac{a^3}{6} + \frac{a^4}{24} - \frac{a^5}{120}\right)$$

For four successive turning points

$$r_m = \frac{r_1 + 3r_2 + 3r_3 + r_4}{8} \qquad \left\{ \begin{array}{l} (a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \\ e^{-a} = 1 - a + \dfrac{a^2}{2} \end{array} \right\}$$

All these means are calculated very easily by using partial complements (Reference 10 Chapter 5).

On the whole, a succession of four point means may be the best since each one gives two turning points on each side of the mean which is more symmetrical, and the approximation is quite accurate when "a" is small, as is usually the case.

APPENDIX "B"

A RECORDING METHOD FOR USE WITH PIM

Equipment S/No. :— Site :— Date :—

Operator :— R.O. :— Index Correction :—01° 17′ 26″

Quartets	1			2			3
	deg.	min.	sec.	deg.	min.	sec.	
West Heading ...	270	10	00				
East Heading ...	90	00	00				
Sum	360	10	00				
Mean	180	05	00				
North	00	05	00				
Index Correction ...	− 01	17	26				
Apparent North ...	358	47	34	358	47	34	
Mean Correction* ...	− 00	04	26	− 00	04	19	
True North ...	358	43	08	358	43	15	
R.O. Circle reading	279	35	46	279	35	46	
True Bearing of R.O.	280	52	38	280	52	31	

Meter Readings	Meter	Angular Equivalent	Meter	Angular Equivalent	Meter	Angular Equivalent
c/c West	R2½	−02′ 30″	R3	−03′ 00″		
East	L7¼	−07′ 15″	L6½	−06′ 30″		
c West	R3¼	−03′ 15″	R3¾	−03′ 45″		
East	L43	−04′ 45″	L4	−04′ 00″		
Sum		−17′ 45″		−17′ 14″		
Mean Correction* ...		−04′ 26″		−04′ 19″		

Note : *Transfer Mean Correction from last line to main table above.

APPENDIX "D"

RATE GYRO SERVO LOOP

A single axis integrating gyro can be used as a precise rate gyro by feeding the amplified output signal from the pick-off to a torque motor on the gyro gimbal. The torque motor current required to restrain the gimbal will be an indication of the input rate (+ bias error).

Due to a rate about the gyro input axis the gimbal reacts according to the following equation :—

$$I_g\ddot{\theta}_g + C\dot{\theta}_g + AK_1K_2\theta_g = H\dot{\theta}_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where

I_g	=	Gimbal moment of inertia about gyro output axis
θ_g	=	Displacement of gimbal relative to gyro case
C	=	Viscous restraint coefficient
A	=	Amplifier gain
K_1	=	Pick-off scale factor
K_2	=	Torque motor scale factor
H	=	Angular momentum of gyro spinner
$\dot{\theta}_i$	=	Turning rate about the input axis

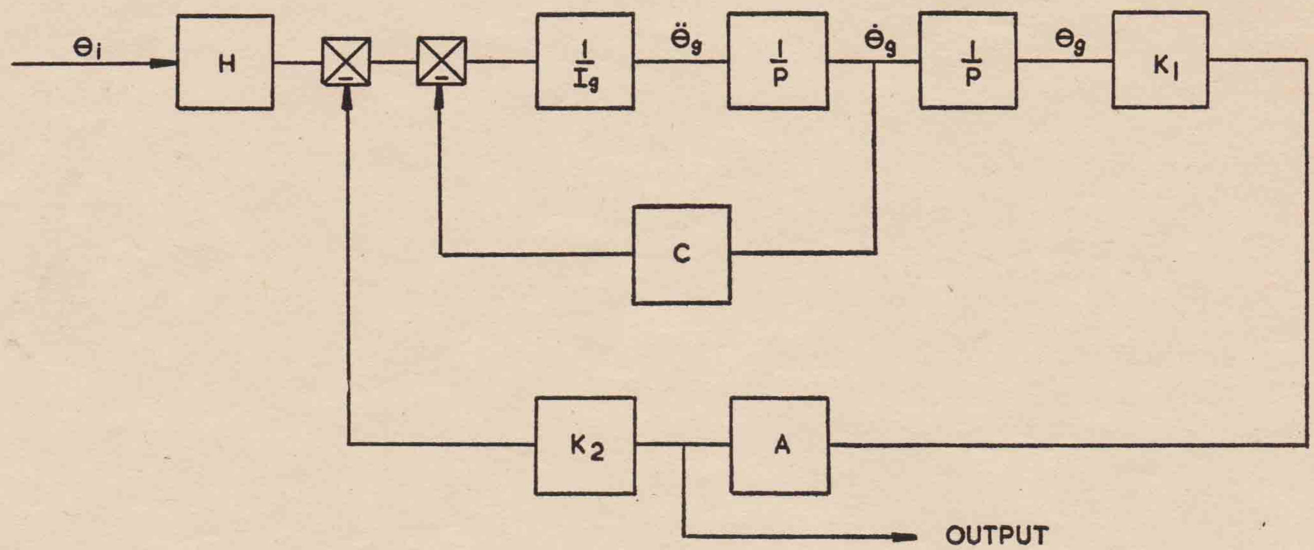
Expressing this in terms of the operator "p"

$$(I_gp^2 + Cp + AK_1K_2)\,\theta_g = H\dot{\theta}_i$$

$$\theta_g = \frac{H\dot{\theta}_i}{I_gp^2 + Cp + AK_1K_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

This can also be expressed in block diagram form (see figure on page 9). The system has two characteristic time constants defined by the above equation. The time constant associated with the gyro inertia and damping will be of the order of 5 milli-seconds and for the North Finding instrument can be neglected by comparison with the servo time constant.

Appendix D (*Cont.*)



APPENDIX "C"

SUMMARY OF SOME PIM RESULTS

Date	Place	No. of Quartets	Mean Diff. from True Bearing	Spread	Std. Dev. of a Single Observation	Weather Conditions: Wind Knots	Weather Conditions: Temp. ° Cent.	Weather Conditions: Remarks	
23.11.62	Canada	12	6″	1′ 20″		3	0	Light snow fall	Surface Demonstration
11.12.62	Switzerland	7	4″		16·6″	5	0		Swiss Army
28. 5.63	Cornwall	14	0″		9″				R. School of Mines Underground Bearing
30. 5.63	Scunthorpe	14	2″		5″		11	High Humidity	
5.12.63	Paris	1 pair	40″						NATO Demonstration.
5. 1.64	London	8		32″	12·9″				Underground Bearing for London Transport
5. 1.64	London	8		18″	6·8″				Ditto
19. 1.64	London	6	1″	7″	2·9″				Surface Bearing for London Transport
20. 1.64	London	6		25″	10·1″			Light rain	Ditto
20. 1.64	London	6		24″	9″				Ditto
20. 1.64	London	6		16″	6·8″				Ditto
21. 1.64	London	6		30″	10″				Ditto
21. 1.64	London	6		28″	12·2″				Ditto
28. 1.64	Finland	1	3″			25	−20	Ice, light snow	Finnish Army
4. 2.64	Norway	1 pair	17″				− 5	Ice	Norwegian Army
20. 2.64	Canada	2	1″	55″					Canadian Government
24. 2.64	Canada	3	10″						Ditto
7. 3.64	Sweden	2	10″						Mining Interests—Underground
30. 4.64	Cornwall	7		91″	27·5″				R. School of Mines
5. 5.64	Wantage	4	2″	6″	2·7″	50			3 Quartets within 1″ of trig. Bearing—Military Survey
2. 7.64	New Zealand	7		41″	15″				Civil Eng. Interest—New Zealand Government
4. 7.64	New Zealand	4		33″	13·7″			High Humidity	Ditto—Underground