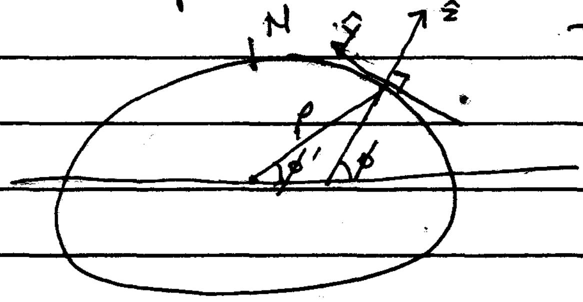
Semidiameter and Horizontal Parallax on an Ellipsoidal Earth.



Take a geocentric coordinate system oriented parallel to the observer's coordinates with the y-axis directed toward the North pole and the z-axis vertical.

Let $\nu = \phi - \phi'$ be the difference between the astronomical and geocentric latitudes then

From Smart, Textbook on Spherical Astronomy, Chapter IX

$$\rho^2 = a^2 \frac{[1 - e^2(2-e^2)\sin^2\phi]}{1 - e^2\sin^2\phi}$$

$$\sin\nu = \frac{e^2 \sin\phi\cos\phi}{[1-e^2(2-e^2)\sin^2\phi]^{\frac{1}{2}}} \; ; \; \cos\nu = \frac{(1 - e^2\sin^2\phi)}{[1-e^2(2-e^2)\sin^2\phi]^{\frac{1}{2}}}$$

a is the Earth's equatorial radius and e its eccentricity

Let R denote the geocentric distance of the Moon then

$$\sin H.P. = \frac{a}{R}$$

The direction of the observer from the Earth's center is

$$\vec{d} = (0, -\cos\nu, \sin\nu)$$

The direction of the Moon from the observer's position is

$$\vec{P}_o = (\sin Z\cos h, \cos Z\cos h, \sin h)$$

The geocentric position of the Moon is then $(\rho\vec{d} + \lambda\vec{P}_0)$ which satisfies the relation

$$R^2 = (\rho\vec{d} + \lambda\vec{P}_0)^2 = \rho^2 + 2\lambda\rho d.P_o + \lambda^2$$

λ is the distance of the Moon from the observer

Hence $\lambda = R\left\{\sqrt{1-\frac{\rho^2}{R^2}\left[1-(\vec{P}_0.\vec{d})^2\right]} - \frac{\rho}{R}(\vec{P}_0.\vec{d})\right\}$

where

$$\vec{P}_0.\vec{d} = \frac{\sin h - e^2\sin\phi(\cos h\cos\phi\cos Z + \sin h\sin\phi)}{\sqrt{1-e^2(2-e^2)\sin^2\phi}}$$

$$\frac{\rho}{R} = \frac{a}{R}\frac{\sqrt{1-e^2(2-e^2)\sin^2\phi}}{\sqrt{1-e^2\sin^2\phi}}$$

The Moon's semidiameter at the observer's position is

$$\sin SD = \frac{r_m}{\lambda} = \frac{r_m}{a}\frac{a}{R}\frac{R}{\lambda} = 0.2725\frac{R}{\lambda}\sin H.P.$$

For a spherical Earth ($e=0$) this reduces to

$$\sin SD = \frac{r_m}{a}\frac{\sin H.P.}{\sqrt{1-\cos^2 h\sin^2 H.P.} - \sin H.P.\sin h}$$

which agrees with other derivations

The parallax adjusted altitude of the Moon h' is given by (i.e. height of the Moon at the Earth's centre)

$$\sin h' = \cos ZD' = \frac{1}{R}(0,0,1).(\rho\vec{d} + \lambda\vec{P})$$

$$= \frac{\rho}{R}\cos\nu + \frac{\lambda}{R}\sin h$$

$$= \sin H.P.(1-e^2\sin^2\phi)^{\frac{1}{2}} + \frac{\lambda}{R}\sin h$$

Taking $\sin^{-1}$ and expanding to first order in $\frac{a}{R}$ and e^2

$$h' = h + \frac{a}{R}\cos h + \frac{a}{R}\frac{e^2}{2}(\cos Z\sin h\sin 2\phi - \cos h\sin^2\phi)$$

Assuming a mean H.P. of $57.7'$ and $e^2 = 0.00669454$ gives $\frac{a}{R}\frac{e^2}{2} = 0.0032°$ which agrees with the Nautical Almanac