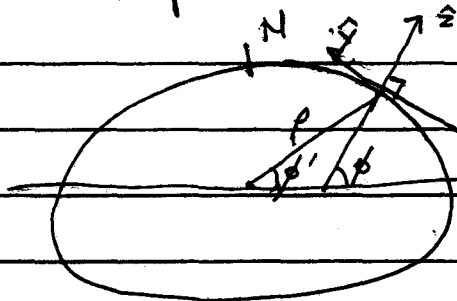


Semidiameter and Horizontal Parallax on an Ellipsoidal Earth.



Take a geocentric coordinate system oriented parallel to the observer's coordinates with the y-axis directed toward the North pole and the z-axis vertical.

Let $\nu = \phi - \phi'$ be the difference between the astronomical and geocentric latitudes then

From Smart, Textbook on Spherical Astronomy, Chapter IX

$$\rho^2 = a^2 \frac{1 - e^2(2 - e^2) \sin^2 \phi}{1 - e^2 \sin^2 \phi}$$

$$\sin \nu = \frac{e^2 \sin \phi \cos \phi}{[1 - e^2(2 - e^2) \sin^2 \phi]^{1/2}}; \quad \cos \nu = \frac{(1 - e^2 \sin^2 \phi)^{1/2}}{[1 - e^2(2 - e^2) \sin^2 \phi]^{1/2}}$$

a is the Earth's equatorial radius and e its eccentricity
Let R denote the geocentric distance of the Moon then

$$\sin \text{H.P.} = \frac{a}{R}$$

The direction of the observer from the Earth's center is

$$\vec{d} = (0, -\sin \nu, \cos \nu)$$

The direction of the Moon from the observer's position is

$$\vec{P}_0 = (\sin Z \cos h, \cos Z \cos h, \sin h)$$

The geocentric position of the Moon is then

$(\rho \vec{d} + \lambda \vec{P}_0)$ which satisfies the relation

$$R^2 = (\rho \vec{d} + \lambda \vec{P}_0)^2 = \rho^2 + 2\lambda \rho \vec{d} \cdot \vec{P}_0 + \lambda^2$$

λ is the distance of the Moon from the observer

$$\text{Hence } \lambda = R \left\{ \sqrt{1 - \frac{\rho^2}{R^2} [1 - (\vec{P}_0 \cdot \vec{d})^2]} - \frac{\rho}{R} (\vec{P}_0 \cdot \vec{d}) \right\}$$

where

$$\vec{P}_0 \cdot \vec{d} = \frac{\sinh h - e^2 \sin \phi (\cosh h \cos \phi \cos Z + \sinh h \sin \phi)}{\sqrt{1 - e^2 (2 - e^2) \sin^2 \phi}}$$

$$\frac{\rho}{R} = \frac{a}{R} \frac{\sqrt{1 - e^2 (2 - e^2) \sin^2 \phi}}{\sqrt{1 - e^2 \sin^2 \phi}}$$

The Moon's semidiameter at the observer's position is

$$\sin SD = \frac{r_m}{\lambda} = \frac{r_m}{a} \frac{a}{R} \frac{R}{\lambda} = 0.2725 \frac{R}{\lambda} \sin H.P.$$

For a spherical Earth ($e=0$) this reduces to

$$\sin SD = \frac{r_m}{a} \frac{\sin H.P.}{\sqrt{1 - \cos^2 h \sin^2 H.P.}} = \frac{\sin H.P. \sinh h}{a \sqrt{1 - \cos^2 h \sin^2 H.P.}}$$

which agrees with other derivations

The parallax adjusted altitude of the Moon h' is given by (i.e. height of the Moon at the Earth's center)

$$\sin h' = \cos Z D' = \frac{1}{R} (0, 0, 1) \cdot (\rho \vec{d} + \lambda \vec{P})$$

$$= \frac{\rho}{R} \cos \nu + \frac{\lambda}{R} \sinh h$$

$$= \sin H.P. (1 - e^2 \sin^2 \phi)^{\frac{1}{2}} + \frac{\lambda}{R} \sinh h$$

Taking \sin^{-1} and expanding to first order in a and e^2

$$h' = h + \frac{a}{R} \cosh h + \frac{a}{R^2} e^2 (\cos Z \sinh h \sin \phi - \cosh h \sin^2 \phi)$$

Assuming a mean H.P. of 57.7' and $e^2 = 0.00669454$ gives

$$\frac{a}{R^2} e^2 = 0.0032^\circ \text{ which agrees with the Nautical Almanac}$$