The general running fix requires finding positions $P_{1}$ and $P_{2}$ that satisfy the following conditions

1) $P_{1}$ lies on $\mathrm{LoP}_{1}$
2) $P_{2}$ lies a distance $D$ from $P_{1}$ along a rhumb line course with bearing $C$.
3) $P_{2}$ lies on $\mathrm{LoP}_{2}$

The required fix is given by $P_{2}$. In general it is to be expected that two geographically widely separated sets of points can be found to satisfy these conditions. A similar situation arises in double altitude sights. A reasonable initial estimate of position ensures that the correct solution is selected.

Assume a value $L_{1}$ for the latitude of position $P_{1}$ and find the corresponding longitude, $\lambda_{1}$, such that $P_{1}$ lies on $\mathrm{LoP}_{1}$ using the equation

$$
\lambda_{1}=-\mathrm{GHA}_{1} \pm \cos ^{-1}\left(\frac{\cos Z D_{1}-\sin \delta_{1} \sin L_{1}}{\cos \delta_{1} \cos L_{1}}\right)
$$

The upper (lower) sign applies when the object lies to the observer's west (east). This formula follows from the familiar cosine rule of spherical trigonometry.

Compute the position $P_{2}$, with latitude $L_{2}$ and longitude, $\lambda_{2}$, located a distance $D$ along a rhumb line course of bearing $C$ from $P_{1}$. This is the direct Mercator sailing problem which can be solved, in principle, using the equations

$$
\begin{gather*}
L_{2}=L_{1}+\left(\frac{D}{a}\right) \cos C  \tag{1}\\
\lambda_{2}=\lambda_{1}+\left(\mathrm{MP}\left(L_{2}\right)-\mathrm{MP}\left(L_{1}\right)\right) \tan C \tag{2}
\end{gather*}
$$

Here $a$ is the Earth's radius and $\operatorname{MP}(\phi)$ is the meridional part

$$
\mathrm{MP}(\phi)=\ln \left(\tan \left(\frac{\pi}{4}+\frac{\phi}{2}\right)\right)
$$

When $L_{2}=L_{1}$ eq.(2) becomes

$$
\lambda_{2}=\lambda_{1}+\left(\frac{D}{a}\right) \sec L_{1}
$$

Having obtained $L_{2}$ and $\lambda_{2}$ for position $P_{2}$ from eq.(1) and (2) compute the quantity

$$
\mathrm{f}=\sin \delta_{2} \sin L_{2}+\cos \delta_{2} \cos L_{2} \cos \left(\lambda_{2}+\mathrm{GHA}_{2}\right)-\cos Z D_{2}
$$

$P_{2}$ lies on $\mathrm{LoP}_{2}$ when $\mathrm{f}\left(L_{1}\right)=0$ and the 3 conditions listed above will all be satisfied. The starting value $L_{1}$ can be adjusted iteratively using standard methods for finding the root of a function of one variable. Convergence is expected to be rapid provided the intersection angle between the LoP's is not too small which is a standard requirement for reliable fixes.

In this procedure the path between $P_{1}$ and $P_{2}$ is a single rhumb line but it could equally be constructed from multiple rhumb line legs by the repeated application of eq.(1) and (2).

