§38. Tacking Downwind

When your destination is almost directly downwind of you,¹ you notice that if you point a bit higher, you go faster. You know that you are sailing a little further, but your instinct is that by sailing faster, though a bit further, you will arrive at your destination more quickly than if you sailed the more direct course.

This instinct tells you that there is a relationship between *distance* travelled, *speed*, and *time*. And indeed, with years of experience, a skipper will learn the optimal downwind tacking angle to use for his boat in different wind and sea conditions. But Arvel² Gentry published an important article in 1970³ in which he outlined the math that will allow you to sail downwind efficiently without waiting all those years. He gives, if you will, the math that supports your instincts.⁴

Gentry, who died in 2015, published a number of articles over a period of years in which he brought his experience in aircraft design to bear on areas of interest to sailors.

As of 05 March 2016, you can download a number of these articles⁵ from http://arvelgentry.jimdo.com/ and http://tinyurl.com/AG-Articles .

Assume that the wind is coming from 205°. So if you are sailing at 25°, you are sailing dead downwind.

Assume the great circle route to your destination⁶ is 35°. So then, if you are sailing 10° higher than dead downwind, you are precisely on course for your destination. But will you get there faster if you sail 20° higher, or even 30° higher than dead downwind?

¹ It is seldom *exactly* downwind. At the end of this section, I will explain how this math works to take care of this rare scenario as well.

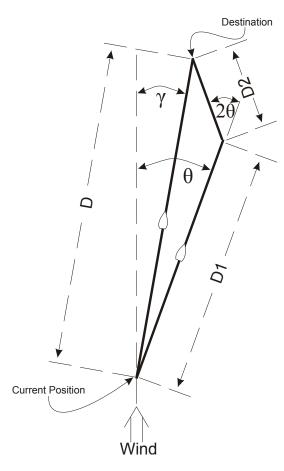
^{...}an old Welsh name.

³ He was, at the time, Chief of Applied Research at Douglas Aircraft. By 1986, he was head of Boeing's aerodynamics computing department, and in his spare time was helping to design a winged keel for one of the America's Cup challengers.

⁴ http://tinyurl.com/TackingDownwind (retrieved 05 March 2016)

⁵...with the blessing of his daughter, who is more interested in seeing her father's legacy carried on than in protecting copyrights. http://tinyurl.com/KimGentry (retrieved 05 March 2016)

⁶ or the rhumb line course, if you are doing coastal sailing.



- γ = angle between line to destination and dead downwind direction
- θ = downwind tacking angle
- D = distance to mark without tacking
- Dθ = D1 + D2, i.e. distance to destination with downwind tacking (further than D but potentially faster to complete)
- 2θ = course to follow after gybe (= twice θ)
- S_{γ} = speed you make on direct course to destination, at angle γ to dead downwind
- S θ = speed you make on course θ , pointing higher from dead downwind than γ

Stating the question more generally, you know γ . You want to know the optimum angle for θ which will get you to your destination the very fastest. You don't care if $D\theta$ is further than D. You care about getting home quick.

The complication in this is that the optimum answer will be different for different hull shapes and sail combinations, and also for different sea states and wind speeds. So then, you must begin by observing the actual speed you make while following different courses. That is to say, it will take several minutes to collect the data before you can calculate your optimum course.

Then, you need a "distance factor" (DF) and a "speed factor" (SF) that you can relate a "time factor" (TF). These are:

$$\mathsf{DF} = \frac{\cos(\gamma)}{\cos(\theta)} \qquad \qquad \mathsf{SF} = \frac{\mathsf{S}\theta}{\mathsf{S}\gamma} \qquad \qquad \mathsf{TF} = \frac{\mathsf{D}\mathsf{F}}{\mathsf{S}\mathsf{F}}$$

The time factor represents the fraction of time it will take to sail on the given course θ compared to the baseline time it will take to sail on course γ . If the TF = 0.750, then it means that sailing course θ will take 75% of the time it would take if you sailed on course γ . Another way of saying is is that course θ is 25% faster under these conditions.

Example with a TEOG 40 foot sloop, where γ is 10° higher than dead downwind.

That is, if the wind is coming from due south, 180° , dead downwind would be 0° . Pointing 10° closer to the wind than 0° would be sailing at 10° (or giving equal answers, sailing at 350° - but we are not interested in true bearings; only bearings relative to dead downwind).

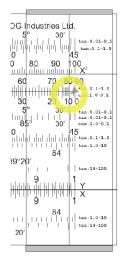
Course to	Course sailed	Distance	Speed on	Speed	Time	Ideal
Dest. relative	relative to	Factor	course θ	Factor	Factor	Course
to dead	dead	DF		SF	TF	
downwind	downwind					
γ	θ	<u>cos(γ)</u>	Sθ	<u>S0</u>	<u>DF</u> SF	
		$\cos(\theta)$		Sγ	SF	
10	10	1.000	2.2	1.000	1.000	
			(i.e. Sγ)			
	20	1.048	2.6	1.182	0.887	
	30	1.137	3.2	1.455	0.781	
	40	1.286	3.8	1.727	0.745	*
	50	1.532	4.2	1.909	0.802	

Conditions: 4 knot wind; small waves with glassy crests; no breaking waves.

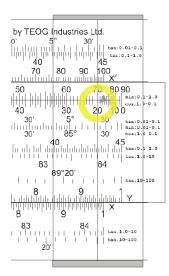
Let's unpack this. If you are sailing directly at your destination, pointing 10° higher than dead downwind, your speed is 2.2 knots. Of course, at that angle $\cos(\gamma)/\cos(\theta) = \cos(10^\circ)/\cos(10^\circ) = 1$. You will find that your speed factor and time factor are both 1. This is your baseline result. You want to see if you can do better than this.

The DF for 20° is $\cos(10^{\circ}) \div \cos(20^{\circ}) = 1.048$

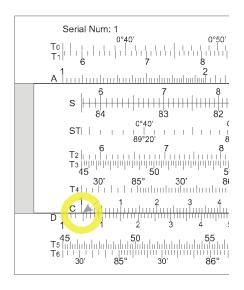
Position your cursor over cos(10°).



Now move the hash mark for $cos(20^\circ)$ underneath the cursor, to do a division.



Read the answer on the D scale, under the left index on the C scale.



Do this for the other values of θ you want to check: 30°, 40°, and 50°.

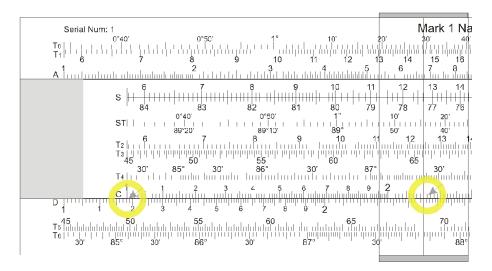
Now, settle your craft in for a few minutes on each course, starting at 10° (i.e. γ , pointing directly at your destination), 20° , 30° , 40° , and 50° . You will be using the masthead wind vane and your instruments to ensure that you are on the proper course. In each case, allow a few minutes to pass for your speed to settle down. In 4 knots of wind, your speeds should be quite consistent.

As you move into higher winds, and bigger waves, your speed will vary more from moment to moment.⁷ Do your best to get an average speed to the nearest tenth of a knot.

Now that you have the first four columns of your chart for 4 knots of wind filled in, you can calculate SF. This is a straightforward division. When $\theta = 20^{\circ}$ and your speed is 2.6 kn,

⁷ You will slow down as a big wave smashes into your bow.

then SF = 2.6 kn/2.2 kn. Position 2.2 on C over 2.6 on D, and read the answer on D under the left index of C = 1.182.



Carry this on to calculate the speed factor values for the other course values you want to evaluate.

Now, fill in the time factor column, where TF = DF/SF. When θ = 20°, this is 1.048/1.182 = 0.887.

Position the cursor on 1.048 on the D scale, and position 1.182 on the C scale under the cursor, and read the answer under the right index of the C scale.

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The smaller the time factor value, the quicker you will arrive at your destination. In this case, sailing 40° higher than dead downwind will get you to your destination in 74.5% the time it would take you if you sailed directly for your destination.

You should understand how to work the slide rule on this problem, but you may be struggling to wrap your head around this whole concept of calculating tacking downwind angles. It may help to use this same TEOG 40 example at some different wind speeds.

Conditions: 8 knot wind; scattered whitecaps.

Course to	Course sailed	Distance	Speed on	Speed	Time	Ideal
Dest. relative	relative to	Factor	course θ	Factor	Factor	Course
to dead	dead	DF		SF	TF	
downwind	downwind					
γ	θ	<u>cos(γ)</u>	Sθ	<u>S0</u>	<u>DF</u> SF	
		$\cos(\theta)$		Sγ	SF	
10	10	1.000	4.3	1.000	1.000	
			(i.e. Sγ)			
	20	1.048	5.3	1.233	0.850	
	30	1.137	6.0	1.395	0.815	*
	40	1.286	6.7	1.558	0.825	
	50	1.532	7.0	1.628	1.062	

Conditions: 12 knot wind; numerous whitecaps.

Course to	Course sailed	Distance	Speed on	Speed	Time	Ideal
Dest. relative	relative to	Factor	course θ	Factor	Factor	Course
to dead	dead	DF		SF	TF	
downwind	downwind					
γ	θ	<u>cos(γ)</u>	Sθ	<u>S0</u>	<u>DF</u> SF	
		$\cos(\theta)$		Sγ	SF	
10	10	1.000	6.3	1.000	1.000	
			(i.e. Sγ)			
	20	1.048	7.3	1.233	0.850	*
	30	1.137	7.8	1.395	0.815	
	40	1.286	8.2	1.558	0.825	
	50	1.532	8.2	1.628	1.062	

Conditions: 16 knot wind; 7 foot waves, ubiquitous whitecaps, some spray.

Course to	Course sailed	Distance	Speed on	Speed	Time	Ideal
Dest. relative	relative to	Factor DF	course θ	Factor SF	Factor	Course
to dead downwind	dead downwind	DF		эг	TF	
γ	θ	<u>cos(γ)</u>	Sθ	<u>S0</u>	DF	
1	U U	$\cos(\theta)$	00	<u>σ</u> Sγ	<u>DF</u> SF	
10	10	1.000	7.9	1.000	1.000	*
			(i.e. Sγ)			
	20	1.048	8.0	1.013	1.035	
	30	1.137	8.2	1.038	1.096	
	40	1.286	8.2	1.038	1.239	
	50	1.532	8.2	1.038	1.476	

What you are observing here is that when winds are light, and you are nowhere near your maximum hull speed, pointing higher than your course by as much as 30° could be a good idea.

But as your boatspeed goes up, progressively closer and closer to your maximum hullspeed, then your ideal course moves closer to pointing directly at your destination. With this particular TEOG 40, once the wind gets to 16 knots, going faster through the water really isn't an option. 8.2 knots is as fast as you are going to go.⁸ Your best plan is to point directly at your destination.

Conditions: 24 knot wind; 12 foot waves, the boat is surfing with speeds up to 12 knots.

All these equations go right out the window when you start to surf. Hang onto your hat and have fun!

Use the same equations on those occasions when your destination is dead downwind. Just use $\gamma = 0^{\circ}$, and work the rest of your table as normal.

Inshore racing sailors would do well to start a notebook and gather data for a wide range of wind speeds and sea conditions, the better to know how their boat and sails perform.

⁸ If you think that this is not quite up to what the equation for maximum hull speed would suggest, you need to recall that the hull speed equation deals in LWL (length at water line) and not LOA (length over all). The TEOG 40 has a LOA of 40 feet. Its LWL is shorter.