employed under certain conditions. It is also rather large and would be inconvenient in a restricted space.

If the astronomical triangle be divided into two right triangles, as in Figure 19, the following simple formulas may be used in solving for the altitude and azimuth:

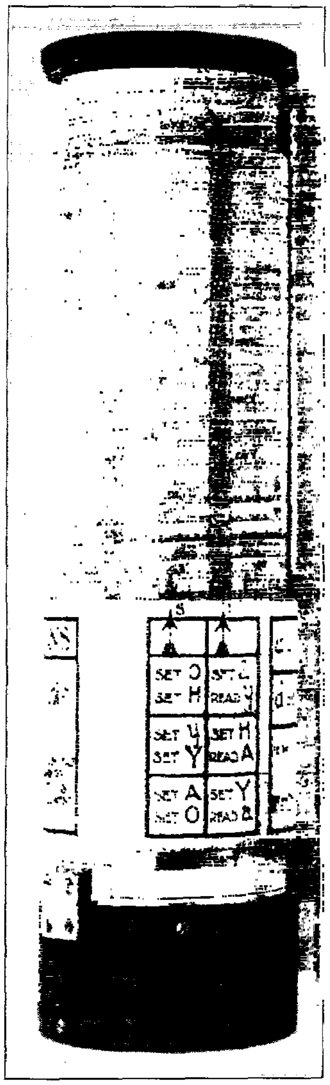

FIG. 30.—Bygrave slide-rule

$$\tan y = \frac{\tan d}{\cos H}$$

If l and d are the same name, $Y = (90° - l) + y$.
If l and d are opposite names, $Y = (90° - l) - y$.

$$\tan A = \frac{\cos y \tan H}{\cos Y}$$

$$\tan a = \cos A \tan Y,$$

where a = altitude.
A = azimuth.
l = latitude.
d = declination.
H = hour angle.
and y and Y are auxiliary angles.

The Bygrave slide rule [43] (fig. 30) is an English instrument which solves the astronomical triangle by the above formulas. (The notation is that used on the rule and is shown in fig. 30.) The rule consists of three concentric tubes. The inner bears a scale of logarithmic tangents, the intermediate tube a scale of logarithmic cosines, and the outer tube two pointers, one for each scale. Complete instructions are given on the instrument itself. The slide rule is about 9 inches in length and 2½ inches in diameter. An accuracy of 1′ or 2′ of arc is attainable in almost every case. The procedure is straightforward, simple, and rapid; and since there are but two scales, the chances for error are reduced to a minimum. The procedure must be changed when the azimuth lies between 85° and 95°, when the hour angle is less than 20′ of arc, or when the declination is less than 30′. The rules for these exceptional cases are not involved and are printed on the instrument, so that no difficulties need be anticipated.

THE REDUCTION OF OBSERVATIONS ON BOARD AIRCRAFT

In marine navigation, a high degree of accuracy is essential in the reduction of astronomical observations. Other factors, however, are of equal or greater importance in aerial navigation. Since aircraft travel at great speeds, position must be fixed at frequent intervals, at least every hour, and perhaps even more often. Also the aerial navigator must do all of the work of navigating himself, and he can spare but a few minutes of very valuable time for any one operation. Any method, therefore, which is to be of any use in aerial navigation must be rapid. The element of time is of prime importance.

Simplicity follows as a second essential. To be rapid, a method of computation must be as straightforward and direct as possible, with the number of operations reduced to a minimum. Furthermore, the same method and the same procedure should be applicable on all occasions; there is no time to choose between methods. Also the necessary equipment of maps, instruments, books of tables, and so on, must be reduced to the lowest terms.

[43] The Engineer, Mar 3, 1922. "A Position Line Slide Rule."

Accuracy is required, although not to the same degree as at sea. If an observation has a possible error of 5′ of arc and the reduction of the observation introduces another possible error of the same amount, the final result may be off 10′. An accuracy of 2′or 3′ of arc is desirable in aerial navigation, and a higher degree of precision is warranted if the speed and simplicity of the work are not impaired.

A further requirement is that of convenience. When space and time are limited, the navigator can not handle cumbersome books of tables, inconveniently arranged and requiring interpolations for every quantity to be found. His tools, whether books, charts, or instruments, must not only be convenient in themselves but must be arranged in systematic order.

The above considerations apply most particularly to airplanes. Here the time factor is of greatest importance and the navigator works under the greatest disadvantages. Experience alone will demonstrate the most suitable method. It would seem, however, that the Bygrave slide rule is admirably adapted for the purpose, since it fulfills all the conditions of a practicable method. Printed forms for computation arranged to facilitate the work are a necessity. These forms after use may be kept as more or less permanent records of the work.

In airships, more space is available and consequently the navigator may provide himself with more equipment. It would seem that the Bygrave slide rule and printed computation forms would be advantageous for all ordinary purposes. However, supplementary methods may be provided for use on special occasions, if the value of these methods fully compensates for the added equipment required. Additional tables and the two-star diagrams, for example, may prove to be helpful.

IX. MAPS AND ACCESSORIES

The aerial navigator requires two distinct types of maps. His problem is somewhat similar to that confronting the marine navigator who has one set of charts covering rather large areas of the sea on which he lays out his route and plots his courses and position lines, and another set drawn to larger scales presenting the features of coast lines, bays, and straits, and harbors in more or less detail as may be necessary.

Thus for the purposes of piloting, or, in other words, the recognition of his position by the topographical features of the country over which he is flying, the aerial navigator employs route maps. These are usually in the form of long strips showing an area from 50 to 100 miles in width from the point of departure to the destination. The route map pictures the distinctive features of the land surface as completely as possible. On the one hand, the natural features, as rivers, lakes, coasts, and mountains, are clearly indicated, together with information regarding forests, the elevation of the ground surface, and terrain dangerous for landing. On the other, railroad lines, highways, cities, and towns, lighthouses and beacons, and landing fields of all kinds are indicated. In short, the route map must show features of the country which may aid the navigator in keeping to his course and help him to find safe landing fields for emergencies as well as those fields at which he wishes to land. The scales used vary; the most common are 1 to 200,000 and 1 to 500,000. Much work has been done with a view to the perfecting of the route map, but, although in everyday use, no generally recognized standard has as yet been produced.

For long-distance flights over the sea and over the land, in particular, regions devoid of distinctive landmarks, the route map is not sufficient in itself. General maps covering larger areas are required. These may serve several purposes; for example, by their aid the proper route may be selected; on them can be plotted the course pursued as determined by dead reckoning and the position lines found by radio bearings or astronomical observations. The route maps are not suitable for these purposes, as the areas shown are too limited, the scale is unnecessarily large, and the great amount of detail is confusing.

To be of service in the plotting of astronomical observations, the general map should have certain characteristics, and it so happens that these are convenient, if not essential, for the other uses to which the map may be put. The meridians should be straight lines, or very nearly so, in order to facilitate the plotting of azimuths. Furthermore, all great circles should be