A.M.L.

POSITION LINE SLIDE RULE

BYGRAVE SLIDE RULE

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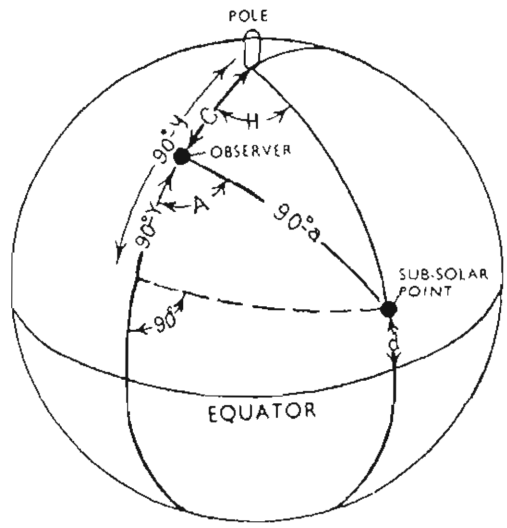

H = Hour Angle.
A = Azimuth.
C = Co-Latitude.
d = Declination.
a = Altitude.

Dotted Line—Perpendicular from Sub-Solar point to Observer's Meridian.

This slide rule has been designed to calculate the altitude of a celestial object as it would be seen from a given point on the earth's surface at a given time.

THEORY OF THE METHOD.

The three points—the Pole, the observer's position, and the sub-solar (or sub-stellar) point—determine a spherical triangle, sufficient elements of which are known to enable an unique solution to be obtained ; this triangle is usually solved by direct logarithmic calculation or by the use of special tables based on logarithmic functions.

In order to solve the triangle by a slide rule it was necessary to re-arrange the formulæ involved in the solution so that each step involved not more than four logarithms or three numbers.

The method adopted is illustrated in Fig. 1, which shows the theoretical diagram and the formulæ employed. From the sub-solar point a perpendicular is drawn to the observer's meridian forming two right-angled spherical triangles. These two triangles have the perpendicular and the right angle at the meridian in common ; the angles opposite the common side are the Hour Angle and the Azimuth respectively; the sides opposite the right angle are the complements of the declination and the altitude ; the parts of the meridian forming the third sides are the complements of the auxiliary angles *y* and Y respectively, the difference of these being the co-latitude.

By applying Napier's Rules of Circular Parts to the triangle containing the Hour Angle we get

$$\tan y = \frac{\tan d}{\cos H}$$

enabling the second auxiliary angle Y to be obtained by the use of the co-latitude.

The tangent of the perpendicular drawn, the common side of both triangles, can now be determined in terms of the elements of both triangles and these values equated to each other, giving

$$\tan A = \frac{\tan H \cos y}{\cos Y}$$

From the triangle containing the azimuth angle A the relation $\tan a = \cos A \tan Y$ is derived.

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On examination of the original spherical triangle it will be at once seen that the altitude could have been found equally well if the perpendicular had been drawn from the observer's position to the meridian through the sub-solar point, in which case the corresponding angle to the azimuth in the above case would be meaningless as regards the Position Line Problem. This enables a check to be applied to any problem worked on the slide rule by interchanging the latitude and declination in which case the final altitudes should agree, although the second false azimuth of the check must be disregarded. As the true azimuth was used to determine the altitude, this has been checked by checking the altitude.

DESCRIPTION OF THE SLIDE RULE.

Two scales are printed on two cylinders sliding with reference to each other. The inner cylinder, on which all results are read, is graduated with log tangents and the spiral scale is about twenty-four feet long. It is divided into minutes of arc throughout its length, and the smallest degree division, occurring at the middle of the scale, is over an inch in length. The outer scale is graduated with log cosines. Two pointers are provided, one for each scale, and are attached to a sliding ring, a stop being provided to register the cosine pointer on the zero of its scale The pointer, which has to be used for each setting, is clearly marked, and the full instructions for dealing with all possible cases are printed at the bottom of the outer cylinder. After a little practice, the calculation can be performed in about two minutes, and the result should be accurate to one minute of arc, with careful use.

A third cylinder sliding inside the others is used for carrying notes, secured by rubber bands, and can be partly withdrawn from below.

On the back of the slide rule are given scales of dip, refraction, moon's parallax, and for converting time into arc, so that the whole of the reduction of a sight can be done without any reference book whatever, other than the abridged Nautical Almanac.

DIRECTIONS FOR USE.

The data required are :—

The Dead Reckoning or Assumed Position of the observer.

The exact time of the observation.

The Right Ascension and Declination of the body observed.

From these the Local Hour Angle is calculated by the usual methods. Turning to the Slide Rule, and holding it by the lower corrugated fibre ring at the bottom in the left hand, the value of y is found by the following operations :—

Set pointer S to zero.

Set pointer L to the declination by sliding and turning the inner cylinder secured to the top end cap of the slide rule.

Set pointer S to the Hour Angle H by turning and sliding the outer cylinder.

Read off the value of y by the pointer L. As two figures (supplementary) are shown to each mark on the scales it is necessary to remember that y is greater or less than 90° according to whether H was greater or less than 90°.

Next form Y from y and c, the co-latitude, adding y to c if the declination and latitude are of the same name and subtracting if they are of opposite names.

The Azimuth A is now found from the Slide Rule by setting the pointer S to y, then the pointer L to the hour angle H, the pointer S to the value of Y, and then reading off the azimuth from the pointer L. Here again we have the rule that A is greater or less than 90° according to whether Y is greater or less than 90°. The Azimuth is noted for future use and the altitude found from the Slide Rule by a third series of settings.

Set Pointer S to the Azimuth A, pointer L to the value of Y, return pointer S to zero, and read off the altitude by pointer L.

The calculation is now complete, having found both Azimuth and Altitude.

As described in the Theory of the Method, the calculation can be checked by re-working with the declination and latitude interchanged, in which case the new azimuth is to be rejected but the altitude will be correct.

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THE OBSERVED ALTITUDE.

Before the observed altitude can be compared with the calculated altitude, it must be corrected for Dip of the Horizon (height of eye) Refraction, semi-diameter, and Parallax, and this is done either by the usual nautical tables or by the Auxiliary tables found on the slide rule ; while the latter are sufficiently accurate for aircraft observations, for marine work the nautical tables are to be preferred.

Here it is essential to note that if the observations were made with a bubble or artificial horizon, then the corrections for Height of Eye (Dip) and semi-diameter must not be applied.

The Azimuth and the difference of the calculated and corrected observed altitudes are now used to draw the position line.

CALCULATION OF GREAT CIRCLE COURSES.

Reference to the figure shows at once that the Slide Rule can be used to calculate Great Circle Courses by the following method :—

The latitude and longitude of the point of departure are used in place of the Dead Reckoning Position.

The latitude of the point of arrival is used instead of the declination.

The difference of the longitude S is used for the Hour Angle.

The Azimuth so found is the Initial Great Circle Course, and the zenith distance or 90° — altitude found is the length of the Great Circle Course which, when reduced to minutes of arc, is the length in Geographical Miles.

Accuracy of the Results obtained with the Slide Rule.

A number of examples have been worked with the Slide Rule, and it is found that the accuracy of the altitude obtained is one minute of arc.

For the convenience of the user, a concise summary of the directions for use and the correction tables for the observed altitude are printed on the slide rule cursor, available for instant reference when required.

EXAMPLES.

I.

Latitude 36° 51′.6 N.
Hour Angle 53° 15′.3 E.
Declination 10° 27′ S.
C = 53° 8′.4
y = 17° 8′

Y = 36° 0′
Azimuth = S.57° 42′ E.
Altitude = 21° 13′

By five figure logarithms the values are S.57° 42′.4 E. and 21° 13′.1.

II. What is the Great Circle Course and Distance between Lat. 36° 51′.6 N., Long. 70° W., and Lat. 10° 27′ S., Long. 16° 44′.7 W.

The first position is the observer's position.

The second Latitude is used as the declination.

The difference of Longitude is used as the Hour Angle.

This gives :—

Latitude 36° 51′.6 N.
Hour Angle 53° 15′.3 E.
Declination 10° 27′ S.

which is exactly the same as the first example, and therefore gives Course S.57° 42′ E. and distance 5,400—1,273 miles=4,127 miles.

III. Check on Example I.

New Latitude 10° 27′ S.
Hour Angle 53° 15′.3 E.
New Declination 36° 51′.6 N.
c = 79° 33′
y = 51° 25′
Y = 28° 8′
False Azimuth 43° 26′
Altitude 21° 13′ showing the original working is correct.

Height of Slide Rule, 9 inches ; Diameter of Slide Rule, 2½ inches ; Weight in metal case, under 2 lbs.

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