http://fer3.com/arc/m2.aspx/LADEE-Moon-Probe-Puzzle-Stuart-sep-2013-g25123

## LADEE Moon Probe Puzzle

**From:** Robin Stuart
**Date:** 2013 Sep 12, 15:57 -0700

On 6 September,2013 on schedule at 23:27 EDT the LADEE mission was launched from Wallops Island, Virginia and its ascent was visible from the East Coast of the US. Details of the launch trajectory can be found at
http://www.orbital.com/NewsInfo/MissionUpdates/MinotaurV/files/LADEEMission-
Overview.pdf I took the attached photographs from an undisclosed location. … The second photo shows Stage 3 burn out against the background stars and also the rocket plume that was barely visible to the naked eye. The relatively bright star above the trail end is Fomalhaut. Information from the link above indicates that Stage 3 burn out was to occur at T +207.62s, Latitude 37.41, Longitude -69.43, Altitude 161.99km. From measurements of the photograph the last point visible of the rocket's trail appears close to R.A. 23h5m47.4s, Declination -31d46.1' (J2000.0). Where was the photo taken from?

Robin Stuart

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http://fer3.com/arc/m2.aspx/LADEE-Moon-Probe-Puzzle-Bernecky-sep-2013-g25131

**From:** Robert Bernecky
**Date:** 2013 Sep 13, 17:44 -0700

LADEE Moon Probe Sighting

I have my doubts on how accurate this approach is, but the idea is to find where the line from the direction defined by
R.A. 23h05m47.4s, Declination -31d46.1'at 2013 Sept 7, 03h 30m 28s Z through the point
Latitude 37.41, Longitude -69.43, Altitude 161.99km
intersects the earth's surface. See figure.

With the given numbers, I come up with N 42.0477, W 71.1326

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http://fer3.com/arc/m2.aspx/LADEE-Moon-Probe-Puzzle-Stuart-sep-2013-g25132

**From:** Robin Stuart
**Date:** 2013 Sep 14, 07:49 -0700

Robert,

The photos were taken from the North East corner of Lake Massapoag at 42d 06’39” N, 71d 10’ 35”. Your result of 42.0477 N, 71.1326 W lies 4.25nm in a direction roughly towards the rocket. Well done! My own calculation yielded 42.0346 N, 71.1172 W which is about a 1nm further out than yours but along the same direction. The difference between the true and computed positions translates into around a 10’ error in declination so perhaps with a bit more care in the measurements one might be able to do better,

Regards,
Robin Stuart

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http://fer3.com/arc/m2.aspx/LADEE-Moon-Probe-Puzzle-Bernecky-sep-2013-g25134

**From:** Robert Bernecky
**Date:** 2013 Sep 14, 17:59 -0700

I am surprised at the result. I was thinking that if I was within 50 nm I was doing good. As David Fleming and possibly others might be interested in the details, I wrote it up. In the process I found a few mistakes that I had made in the calculations. The new location is N 42.0983, W 71.1420 which is an astounding 1.7 nm from truth.

--Bob.

**Attached File:**
LADEE-Moon-Probe-Writeup.pdf

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http://fer3.com/arc/m2.aspx/LADEE-Moon-Probe-Puzzle-Stuart-sep-2013-g25140

**From:** Robin Stuart
**Date:** 2013 Sep 15, 12:08 -0700

[...]

There are a few of additional effects that could be accounted for. The first is that I stated the burn out position in J2000.0 coordinates. These should be converted to the apparent position for the equinox of date and changes the direction of the rocket slightly. In those coordinates the Stage 3 burn out position is R.A. 23h 6m 34.2s, Declination -31d 41.6' which moves my estimated position a bit to the South West and more in line with the rocket's azimuth.

In principle of course the local elevation which is around 75m should be accounted for. There is also some parallax induced by atmospheric refraction. Refraction means that the observer actually seeing rays that would pass over his head if atmosphere were removed. Due to the resulting parallax the rocket appears lower than its true geometric position with respect to the background stars. I believe that this phenomenon is something that needs to be accounted for in very precise eclipse timings,

Regards,

Robin Stuart

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Conversions between ellipsoidal coordinates $(\varphi, \lambda, h)$ and ECEF Cartesian coordinates $(x, y, z)$ can be performed by the methods described at
http://www.navipedia.net/index.php/Ellipsoidal_and_Cartesian_Coordinates_Conversion

Define $\alpha = \left(\dfrac{e^2}{1-e^2}\right)$ where $e^2 = 0.00669454$ is the square of the eccentricity of the geoid.
All coordinates are expressed in units of the Earth's equatorial radius, $a$ (6378.16 km). The observer's position lies somewhere along the line $\mathbf{x} = \mathbf{p} + t\mathbf{u}$. Points, $\mathbf{x}$, on the surface of the geoid can be shown to satisfy the equation

$$\mathbf{x}^2 + \alpha\,(\mathbf{x}\cdot\hat{\mathbf{z}})^2 = 1$$

which can be expanded to

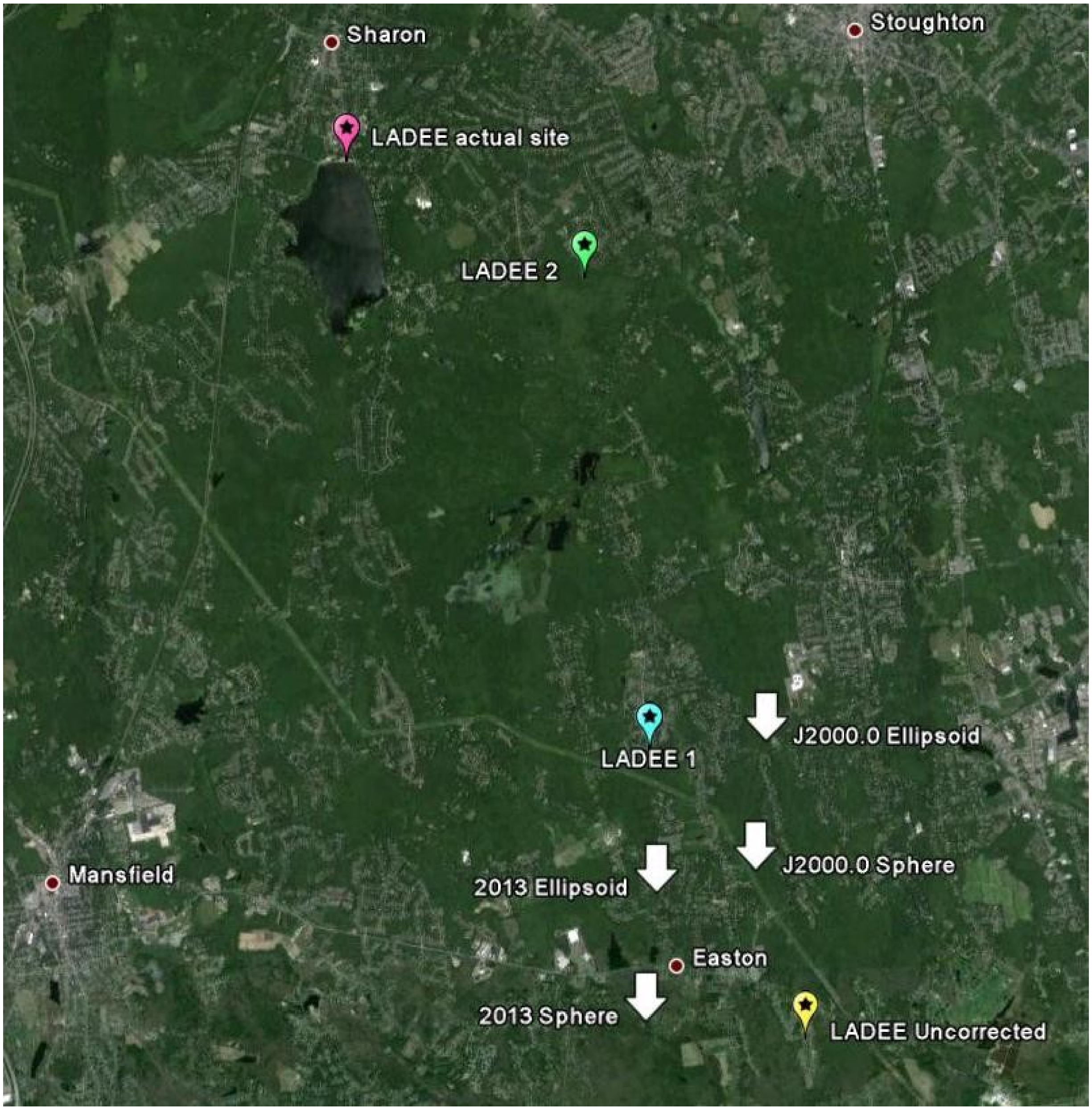
$$\left[1 + \alpha\,(\mathbf{u}\cdot\hat{\mathbf{z}})^2\right]t^2 + 2\left[(\mathbf{p}\cdot\mathbf{u}) + \alpha\,(\mathbf{u}\cdot\hat{\mathbf{z}})(\mathbf{p}\cdot\hat{\mathbf{z}})\right]t + \left[|\mathbf{p}|^2 + \alpha\,(\mathbf{p}\cdot\hat{\mathbf{z}})^2 - 1\right] = 0$$

where $\hat{\mathbf{z}}$ is a unit vector in the $z$ direction. This allows observer's ECEF Cartesian coordinates for zero altitude to be solved for.

This gives

Spherical Earth using J2000.0 coordinates for direction: 42.0346°, -71.1172°
Ellipsoidal Earth using J2000.0 coordinates for direction: 42.0485°, -71.1157°
Spherical Earth using 2013 coordinates for direction: 42.0184°, -71.1331°
Ellipsoidal Earth using 2013 coordinates for direction: 42.0322°, -71.1316°

The last of these is 5.1 nautical miles from the actual site.




Corrections for the observer's altitude and for refraction both tend move the estimated position toward the rocket and away from the true position.

Robin Stuart
September 2013