of latitude and meridian angle interchanged as in the Weems' *New Line of Position Tables.*

Myerscough and Hamilton. The *Rapid Navigation Tables*, by W. Myerscough and W. Hamilton, were published in London in 1939. A perpendicular is dropped from the zenith to the hour circle of the celestial body. With slight modification, the altitude formulas of Souillagouet and the azimuth formula of Gingrich are used. Six quantities are tabulated in a single table of 90 pages. Both declination and latitude are limited to 70°. In the 1950 edition, the limits of declination and latitude were extended to 89°. A revision of the 1950 edition was published in 1965 as *Rapid Navigation Tables for Mariners*, a table of 195 pages.

Ageton's *Manual of Celestial Navigation*, published in 1942, combines the first table of Weems' *New Line of Position Tables* as table I, and H.O. Pub. No. 211 (art. 2111) as table II. The basic formulas are restated in terms of secants and cosecants. The result is a short, easy solution without interpolation, involving four book openings, eight table entries, and four mathematical steps. Since the H.O. Pub. No. 211 table is included, the book can be used for Ageton's earlier method.

Benest and Timberlake. The *Astro-Navigation Tables for the Common Tangent Method* by two British professors, E. E. Benest and E. M. Timberlake, were published in 1946. In three tables of 61, 18, and 12 pages is given a logarithmic solution for altitude only, by dropping a perpendicular from the zenith. The formulas are slight modifications of those of Ogura.

The location of the line of position is somewhat similar to the method sometimes used in longitude method solutions such as H.O. Pubs. Nos. 203 and 204 (art. 2106). Two assumed positions are selected, usually 1° apart on the same meridian. The altitude intercept at each position is determined, and a circle, or arc of a circle, is drawn with the assumed position as the center, and the altitude intercept as the radius. The line of position is the common tangent to the two circles. Since there are four common tangents, the general direction of the body is required. Where doubt exists as to which of two or more answers is the correct one, additional solutions from other assumed positions may resolve the ambiguity. If the celestial body is near the meridian, the two assumed positions are better taken on the same parallel of latitude. Even with these precautions, there is danger of selection of the wrong line.

Tavole H *(I. I. 3113)*, published by the Istituto Idrografico della Marina of Italy in 1947, combines table I of Ogura and table II of Weems' *New Line of Position Tables*, including, also, the Rust azimuth diagram (art. 2106). This table is a modification of an earlier *Tavole F*.

Čumbelić. In 1969 Captain Petar Čumbelić of Yugoslavia published his single volume and compact *Nautičke Tablice*. This method is based upon a triangle divided by dropping a perpendicular from the zenith. The table includes an English explanation.

2111. Altitude methods, perpendicular from body.—Figure 2111 is a diagram on the plane of the celestial meridian (art. 1432), with the navigational triangle shown in heavy lines. A perpendicular from the celestial body, *M*, to the celestial meridian divides the triangle into two right spherical triangles. In figure 2111 the length of the perpendicular is designated *v* and the two parts of the colatitude are designated *w* and *x*. By means of Napier's rules (art. 142, vol. II), the following basic formulas can be derived:

$$\sin v = \cos d \sin t \quad (1)$$

$$\cos w = \sin d \sec v, \text{ or } \sin w = \cot t \tan v \quad (2)$$

$$\sin h = \cos v \cos x \quad (3)$$

$$\sin Z = \sin v \sec h, \text{ or } \cos Z = \tan x \tan h \quad (4)$$

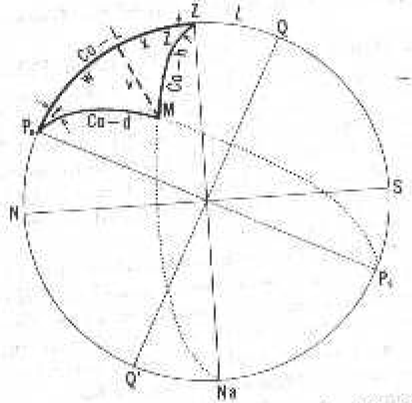

FIGURE 2111.—Navigational triangle with perpendicular from celestial body to celestial meridian.

Since $x=90°-(w+L)$, formula (3) can be written in terms of latitude, and w foun[d] from equation (2). Thus, both h and Z can be determined by means of t, d, and L an[d] auxiliary functions found from them.

William Thomson (Lord Kelvin) was the first to divide the navigational triang[le] as shown in figure 2111 for sight reduction, but his method (art. 2106) was for dete[r]mination of longitude. Various later methods made such a division for determinatio[n] of altitude.

Fuss. The *Tables to Find Altitudes and Azimuths*, devised by V. E. Fuss, an astr[on]omer at the Kronstadt (Russia) Naval Observatory, were published in 1901. In the[se] tables a perpendicular is dropped from the celestial body, the following notation bei[ng] used (fig. 2111):

y is designated a
w is designated $90°-b$
x is designated $B-90°$
$B=90°-L+b$ (if y falls between Z and Q).

Solution is by the following formulas:

$$\sin a = \cos d \sin t$$
$$\cot b = \cot d \cos t$$
$$\sin h = \cos a \sin B$$
$$\cot Z = \cot a \cos B.$$

The assumed latitude is selected to provide the nearest 15′ value of B. The assum[ed] longitude is selected so that t will be the nearest whole 1^m (0°25). The tables are ent[ered] twice, first with t and d to find a and b, interpolating for d, and then with B a[nd a] to find h and Z, interpolating for a. The method involves two book openings, [...] table entries, four interpolations, and ten mathematical steps. There are 144 p[ages] of tables.

Aquino. The *Altitude and Azimuth Tables* of Radier de Aquino, a Brazilian n[aval] officer, were first published in 1909. These were followed the next year by his Se[...] *Air Navigation Tables*. Several later editions of both publications appeared with [...]

modification, principally of the auxiliary material given. Aquino dropped a perpendicular from the celestial body to the celestial meridian, and used the same formulas as Fuss and generally the same arrangement, except that longitude is assumed so as to provide a meridian angle to the nearest whole degree.

H.O. Pub. No. 209 (Pierce), *Position Tables for Aerial and Surface Navigation,* was published by the U. S. Navy Hydrographic Office in 1930. These tables were devised by Commander M. R. Pierce, USN, in 1925, when he was navigator of the dirigible USS *Los Angeles.* The method is based upon a triangle divided by a perpendicular from the celestial body. It is generally similar to those designed by Fuss and Aquino, but the arrangement is somewhat different, requiring 206 pages of tables. This method was never widely used, and is now out of print.

H.O. Pub. No. 211 (Ageton), *Dead Reckoning Altitude and Azimuth Table,* was published by the U. S. Navy Hydrographic Office in 1931. This method, designed by Lieutenant Arthur A. Ageton, USN, while a student of the Post Graduate School, then at Annapolis, Maryland, is based upon a triangle divided by dropping a perpendicular from the celestial body. It is generally similar to those of Fuss and Aquino. However, Ageton modified the formulas so as to include only secants and cosecants. In terms of figure 2111, his notation is as follows:

v is designated R
w is designated $90°-K$
x is designated $K \sim L$
$K = x + L.$

Ageton's formulas are

$$\csc R = \csc t \sec d$$

$$\csc K = \frac{\csc d}{\sec R}$$

$$\csc h = \sec R \sec (K \sim L)$$

$$\csc Z = \frac{\csc R}{\sec h}.$$

A single table of 36 pages gives five-place log cosecants (labeled A) and log secants (labeled B), both multiplied by 100,000 to eliminate the decimal. These values are given in parallel columns for each 0.′5 of angle from 0° to 180°. The table is well arranged and indexed for quick reference. The rules are relatively simple and well presented. The method can be used for solution from the dead reckoning or any other assumed position. The method is intended for use without interpolation. These features combined to make this a popular method, although solution is somewhat tedious, and large errors may be encountered if t is near 90°. The method has been largely superseded by those tables constituting a list of computed answers. However, the method is now published as table 35.

If a celestial body near the visible horizon is observed, it may be *below* the celestial horizon (zenith distance greater than 90°), because of refraction and dip. Under these conditions the *computed* altitude, Hc, is negative (art. 2023). In the solution by H.O. Pub. No. 211, Hc is negative if K is of the same name as L and greater than $(90°+L)$, or if K is of contrary name to L and greater than $(90°-L)$. Under the second of these conditions, Z is less than 90° and should be taken from the top of the table if K is greater than $(180°-L)$.

Fontoura da Costa and Penteado's *Tabuas de Altura e Azimute* were published in Lisboa, Portugal, in 1936. These consist of 26 pages of log secant and log cosecant tables similar to those of H.O. Pub. No. 211. The method and formulas are slight modifications of those of H.O. Pub. No. 211 (table 35).