

Assuming a uniform relative velocity  $\vec{v}$ ,  
 the relative trajectory is  $\vec{r}(t) = \vec{r}_2 + \vec{v}t$  (■), where  
 $\vec{r}_2$  is the location at  $t=0$ ;  $\vec{r}_1$  is the location at time  $\Delta t$   
 before  $t=0$ .

Then: 
$$\vec{v} = \frac{\vec{r}_2 - \vec{r}_1}{|\Delta t|}$$

•) Convert ranges and angles into Cartesian coordinates  $(x, y, z)$ :

$$\vec{r}_1 = (x_1, y_1, z_1) \quad \text{and} \quad \vec{r}_2 = (x_2, y_2, z_2)$$

•) Calculate Cartesian components of velocity:

$$v_x = \frac{x_2 - x_1}{|\Delta t|}, \quad v_y = \frac{y_2 - y_1}{|\Delta t|}, \quad v_z = \frac{z_2 - z_1}{|\Delta t|}$$

CPA occurs at time  $t=T$ , when distance  $s(t) = |\vec{r}(t)|$  is minimum.

Since  $|\vec{r}(t)| = \sqrt{(x_2 + v_x t)^2 + (y_2 + v_y t)^2 + (z_2 + v_z t)^2}$  (▲)

which is minimum when  $\frac{d|\vec{r}(t)|}{dt} \Big|_{t=T} = 0$ , we solve a linear equation for  $t=T$ , the time of CPA

$$(x_2 + v_x T)v_x + (y_2 + v_y T)v_y + (z_2 + v_z T)v_z = 0$$

•) Find time  $T$  (since observing  $\vec{r}_2$ ):

$$T = - \frac{x_2 v_x + y_2 v_y + z_2 v_z}{v_x^2 + v_y^2 + v_z^2}$$

•) Plug the solution for  $T$  into Equation (▲) to get range at CPA.

•) Plug the solution for  $T$  into Equation (■) and use it to find the direction at CPA. In 2-D that's one angle, in 3-D that will be 2 angles.